Unit-1

PROJECT SCHEDULING AND MONITORING

Introduction

Solicitation of information, ability, implements and procedures in the project environment leads to sustainable project management. Now a days, project management had introduced a new methodologies to reach great heights in sustainable development of projects. It has developed as an unmistakable territory of the management practices to address the difficulties of new monetary condition, globalization process, fast innovative changes, and quality concerns of the stakeholders.

Project Definition

Project in broad sense states that a novel attempt with precise intention and diverges so extensively that it is a great challenge to exactly define. It is also defined as unique development consists of a group of synchronized and well-ordered events with start and finish dates, assumed to achieve an objective confirming to precise desires which comprises the time and resource constraints.

These projects exhibit different meanings in different streams like completion of a survey and analysis in management streams, construction of irrigation structures, buildings, bridges etc in case of civil engineering, development of a software or execution of a output for software professionals, invention of a new machinery for mechanical engineers, evaluation of new circuits for Electrical and electronics people, establishment of new industries or new business for business people. So by these examples we can conclude that the perception of project is different for different type of people, but whatever may be the type of project, it should require a proper start and finish time i.e perfect planning and scheduling.

Project Characteristics

The following common characteristics will exhibit by all projects

- Distinctive in environment.

- Have certain goals to accomplish.
- Involves set of resources.
- Have a definite time frame for conclusion with a certain start and finish.
- Comprises risk and ambiguity.
- Entails cross-functional teams and interdisciplinary methodology.

Project Life Cycle

Every project, from origin to conclusion, go through several phases of a life cycle substitute to life cycle of stake holders. There is no common harmony on the number of phases in a project life cycle. An understanding of the life cycle is significant to effective conclusion of the project as it enables to recognize the logical sequence of events in the continuum of growth from start to finish. Classic project consists of four phases- Conceptualization, Planning, Execution and Termination. Each phase is striking by one or more deliverables such as Thought reminder, Feasibility report, Implementation Plan, HRD plan, Resource allocation plan, Evaluation report etc.

Conceptualization Phase

Conception phase, preliminarily starts with the initiation of an thought, selection of the product / service, Pre-choice, Feasibility studies Assessment and Sanction. The idea is hypothesized with preliminary thoughts of all possible substitutes for attaining the project objectives.As the thought built for setting out basis, technique, evaluated costs, benefits and different approaches forappreciating the stake holders,we have to arrive for conclusion by considering all the possibilities in detail.

Planning Phase

In this phase the project organization is planned based on project assessment and sanctions. Comprehensive plans for action, finance, and resources are developed and combined to the quality factors. In the procedure major objectives need to be accomplished in this phase are

- Selection of activities and theirsequencing

- Time frame forimplementation
- Assessment and Finance
- Recruitment

A Detailed Project Report (DPR) specifying various aspects of the project is finalized to enableimplementation in this phase.

Execution Phase

This phase of the project observes the intense activity where the plans are placed intoprocess. Each activity is observed, organized and synchronized to attain project objectives. Important actions in this phaseare

- Collaborating withstakeholders
- Appraisingprogress
- Observing cost andtime
- Monitoringquality
- Handlingchanges

Termination Phase

This phase results in closure of project where the acceptableoutcomes are attained and project is put in to force with suitable monitoring.

As planning stage in any project is like heart in human being, so let us discuss in detail about this phase.

Principles of Planning:

Planning includes decision-making and action that will lead to the attainment of precise phenomena or objectives, the occurrence of which could not be extemporaneous. The main principle of the planning is to complete a project with in a stipulated time by minimum resources with maximum output. For achieving this the following mission has to be set

- definingthe tentative plan
- Assuming the scope statement;
- Identifying the planning team;

- Classifying the deliverables and generating the work breakdown structure;
- Selecting the activities needed to complete those deliverables and networking the activities by a logical sequence;
- Assessing the resource necessities for the activities;
- Approximating time and cost for activities;
- developing the schedule;
- finalization the budget;
- risk forecasting;
- Getting formal sanction to start work.

Such a planning makes it easier to consolidate design and familiarizeourselves with its various mechanismsrequires for successful completion of the project.

Advantages of planning

- Simplifies the objectives
- Reduces the uncertainties
- Enables the coordination
- Develops the employee moral
- Helps in budgeting
- Facilitates controlling
- Affords competitive edge
- Encourages novelties

Limitations of Project planning

The Limitations are of two types

- Internal Limitations
- External limitations

Internal Limitations: These are further divided into

- Administration inflexibility
- Misapplied Planning
- Time intense

- Prospect in work planning
- Improper financial planning

External Limitations: These are mainly classified as

- Political instability
- Labour formalities
- Natural hazards
- Technological advancements
- Competitors' policies
- Changes in requirements of stakeholders

Stages of Planning:

The basic stages of project planning are:

- Scope – determining the in-scope necessities for the undertaking to encourage the work breakdown structure
- Preparation of the work breakdown structure – dividing the breakdown of the project into tasks and sub-tasks
- Project schedule development – listing the complete schedule of the tasks and detailing their sequence of execution
- Resource planning – indicating who will do what work, at which time, and if any special skills are needed to accomplish the project tasks
- Budget planning – specifying the budgeted cost to be incurred at the completion of the project
- Procurement planning – focusing on vendors outside your company and subcontracting
- Risk management – planning for possible risks and considering optional contingency plans and mitigation strategies
- Quality planning – assessing quality criteria to be used for the project
- Communication planning – designing the communication strategy with all project stakeholders

Scheduling:

It is defined as the graphical representation of the growth rate of the project activities from the initial stage to end stage by developing a coordination among all the activities.

Preparation of Schedules:

The following steps are involved in preparation of schedules:

1. Initially the total project should be planned in such a way that all the operations should be divided as activities.

2. The approximate time for each activity should be estimated and from that the total time for project should be estimated based on the quantity of work

3. The sequential order should be planned perfectly to avoid the delay in work.

Let me explain the above concept by considering the example of designing a multi storey structure in construction industry

Initially to build a multi storey structure, the following steps are required

- Reconnaissance survey to be carried out
- Surveying of the area should be done
- Testing of soil samples should be done
- Planning and designing of the structure should be carried out based on the soil SBC
- Laying of foundations
- Construction of super structure
- Finishing's
- Paintings etc

For completing these stages, the approximate time required should be estimated.

In completing this project, a sequential order of construction should be carried out one by one and in cases one parallel to other activities will also be taken up.

So finally the total time needed for completion of the activities in all aspects should be estimated, that is the final project completion time and this process of planning in a pictorial representation is called as scheduling.

Methods of Scheduling:

Based on the size of the project, various methods of scheduling are adopted, they are:

1. Gantt Charts or Bar charts

2. Mile stone charts

3. Network Analysis

Gantt Charts or Bar charts:

A Gantt or bar chart is a “graphical representation of project activities, shown in a time-scaledbar line with no links shown between activities’’

(Popescu and Charoenngam 1995).

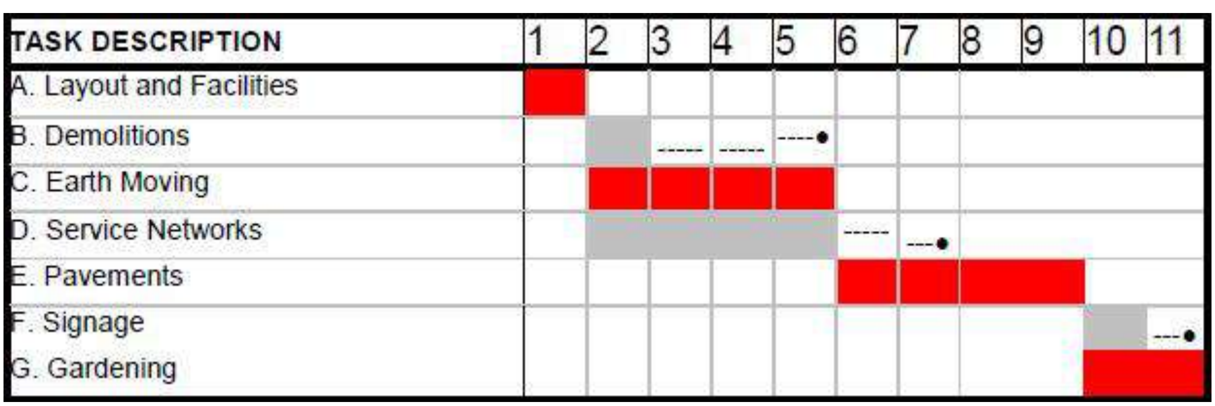
The Gantt chart or bar chart was originally developed by Henry L. Gantt in 1917.

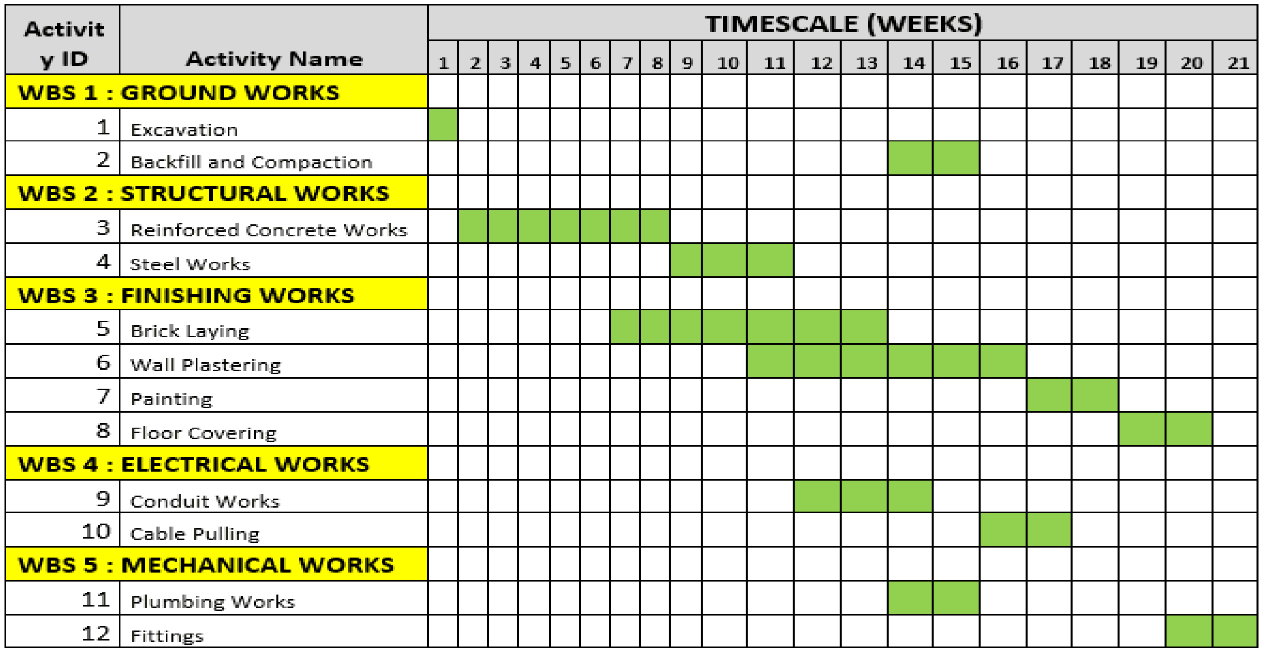
It became popular especially in the construction industry because of its ability to graphically represent a project’s activities on a time scale.

As an initial step for construction of Gantt chart for a project, it must be broken into smaller, usually homogeneous activities or a task.

There is no any correct or absolutely in correct method to divide a project exists; however, the scheduler should take a balanced approach and break it down into a reasonable number of activities that are easily measured and controlled without being overly detailed.

An activity, or a task, may be as large as laying the foundation of a building, as small as erecting the formwork of one footing, or anywhere in between. The duration of each activity must be estimated. Bars are then drawn to show, for each activity, the duration and the starting and ending points. As mentioned previously, links between activities are not usually shown.





ADVANTAGES OF BAR CHARTS

- Bar charts have gained wide recognition and reputation mainly because of their Ease and comfort of preparation and understanding.
- No ‘‘theory’’ or complex calculations are involved.
- Easy to understand
- They can be constructed anywhere with just a pencil and paper.
- Unlike networks, bar charts are time-scaled; that is, the length of a bar representing certain activity is proportional to the duration of that activity.
- Another advantage is that bar charts particularly appeal to persons who do not require a technical background.

DISADVANTAGES OF BAR CHARTS

- The main disadvantage of bar charts is absence of logical depiction (relationships)
- Why did this activity start on that date? Bar charts do not reveal the answer. It could be a logical relationship, a resource constraint, or a subjective decision by the project manager.
- Although some software programmers tried to depict logical relationships

on bar charts, the result was not always clear. The logic lines would get twisted, and different networks, bar charts do not allow the length of the bars to be subjectively changed or the bars to be moved around to make the chart look or read better.

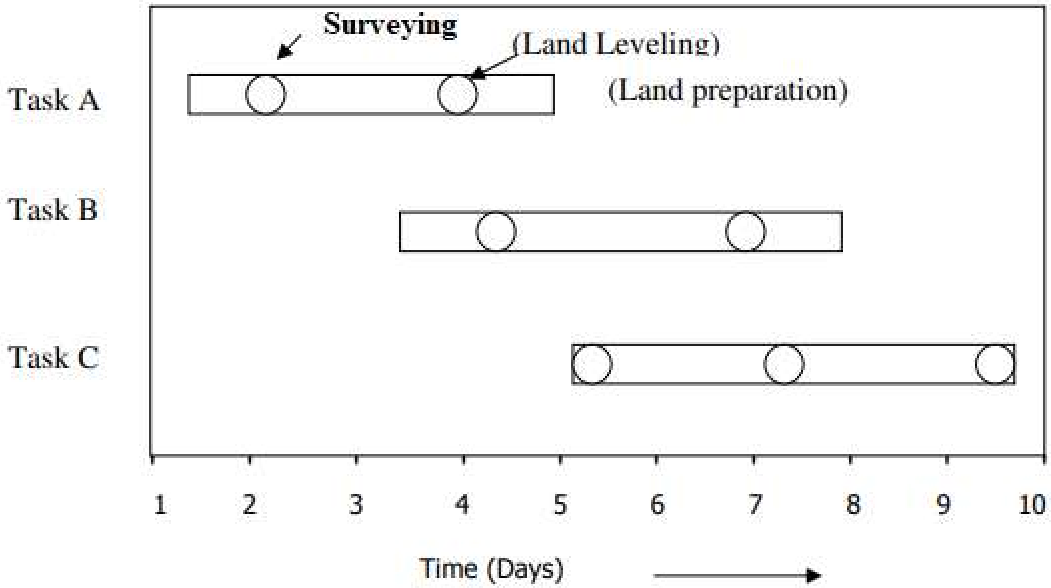
- Another limitation, rather than a disadvantage, of bar charts is the size and difficulty of projects. Bar charts may not be practical for projects with large numbers of activities—unless you use them in two ways:

Milestone Charts:

Milestone chart is an upgrading over the bar chart (Gantt chart) by presenting the concept of milestone.

The milestone, characterized by a circle or Triangle over a task in the bar chart specifies conclusion of a specific phase of the task.

For example land preparation (Task A) includes surveying and leveling. From the simple bar chart it is difficult to monitor progress of the surveying. Introduction of a milestone on day 3 would specify that the surveying would be completed by day 3 of the project. In a milestone chart a task is broken down in to specific phases (activities) and after accomplishment of each of the specific activity a milestone is reached or in other words an event occurs. The chart also shows the sequential relationship among the milestones or events within the same task but not the relationship among milestones contained in different tasks. For example in figure , the milestone 2 of task A cannot be reached until the milestone 1 is traversed and the activity between milestone 1 and 2 is over. Similarly, in task B the milestone 4 can begin only after completion of milestone 3. But the relationship between the milestone of task A and task B is not indicated in the milestone chart.



Limitations:

- Does not show interrelation between activities.
- Does not specify critical activities.
- Does not reflect the concept of indecision in completing the task.
- Very cumbrous to draw the chart for big projects.

Network Analysis:

The network is a logical extension of Gantt's milestone chart incorporating the alterations so as to illustrate interrelationship between and among all the milestones in an entire project.

The two techniques for network analysis are

1. Programme Evaluation and review Technique (PERT) and
2. Critical Path Method (CPM).

These two techniques were developed almost simultaneously during 1956-1958. PERT was established for US navy for scheduling the research and development events for Polaris missiles programme.

CPM was established by E.I. du Pont de Nemours & Company as an application to construction project.

Though these two methods were developed simultaneously they have striking similarity and the significant difference is that the time estimates for activities is assumed deterministic in CPM and probabilistic in

PERT.

There is also little distinction in terms of application of these concepts.

PERT is used where emphasis is on scheduling and monitoring the project and CPM is used Task A Task B Task C ' ' ' ' ' ' ' ' ' ' 1 2 3 4 5 6 7 8 9 10 Time (Days)

Controlling:

Project managers must confirm that they should control their exclusive, transient and unstable projects in order to attain their goals.

The project manager also wants to employ ‘hard’, quantitative control processes and it is these that are the main focus of completion of their task.

These procedures address all three project dimensions – quality, time and cost, and therefore include all of the following:

- Monitoring the scope of the project – controlling change.
- Confirming that the project’s products/deliverables fulfil their requirements – controlling quality.
- Confirming that events happen on time – scheduling.
- Ensuring that work is completed within budget – cost control.
- Handling risks.
- Handling problems and recognizing issues
- Project should ends with benefits for clients.

Job Layout:

A job layout is equipped with a potential that work continues easily without any obstacle. .

Job layout is defined as a site drawing of the planned construction showing the locations of entry, exit, temporary services, material stores and stocks, plant or equipment and site offices.

Factors affecting Job layout:

The following factors should be considered in preparing site layout:

- Nature, Scope and Type of work: the nature of site layout depends majorly on the type of work to be carried out. E.g. the site layout for a dam would be much more complicated and extensive as compared to that of a residential building.

- Topography of the site: - the topography of the site i.e. location, size, nature of terrain etc. of the site also consequences the site layout. E.g. the site layout in case of rocky mountains terrain would be vastly different from that in case of a plain terrain
- Methods adopted for execution of work: - the method chosen for the execution of work deeply affects the nature of site layout e.g. if the work is carried out mostly mechanically i.e. using more equipment and less labour, more emphasis has to be placed on providing facilities for the machinery than for the workers.

Work break down Structure (WBS):

Successful project management mainly depends on a large degree on the project manager's capability to specify the work content of projects in terms of its deliverables and activities.

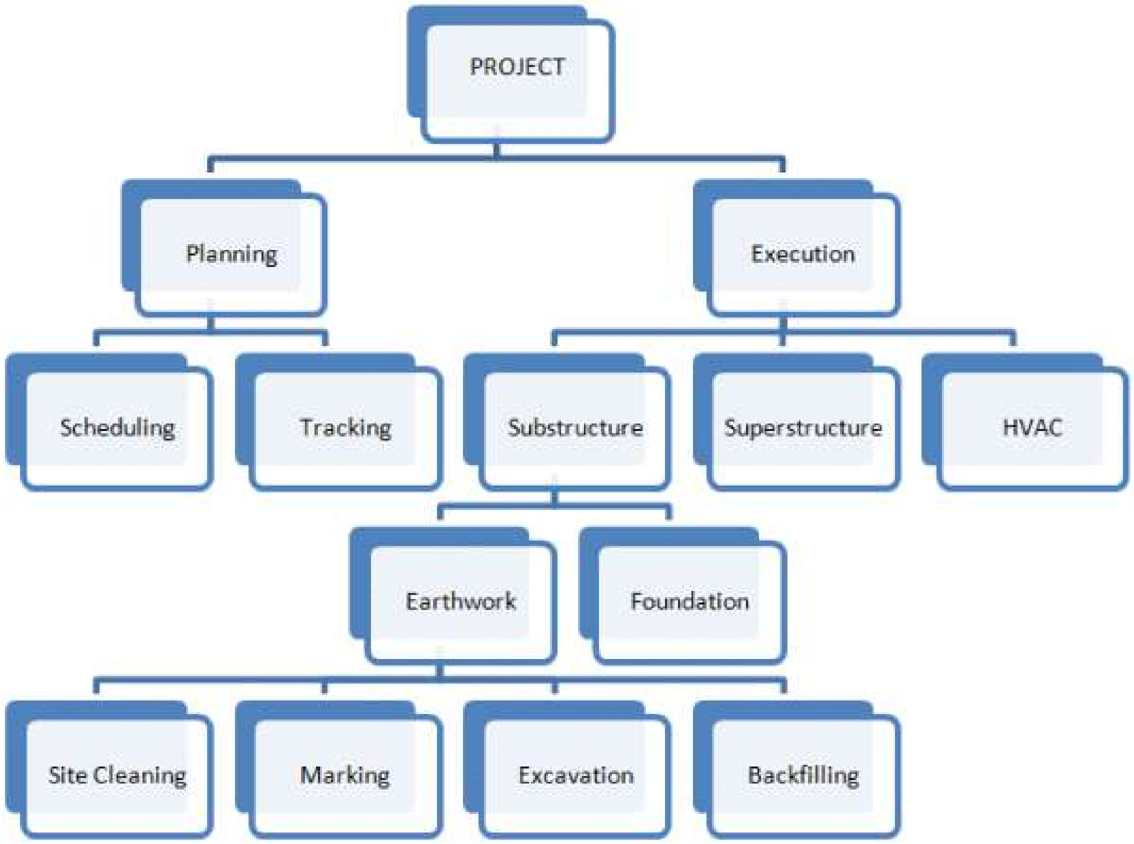
One of the major tools for planning and controlling the work content is the WBS.

The WBS is usually used at the beginning of a project for defining project scope, establishing Gantt schedules and estimating costs.

It lives on, throughout the project, in the project schedule.

A WBS is a product oriented family tree of phases, activities and tasks which classifies, describes and graphically exhibits the total work to be accomplished in order to achieve the final objectives of a project.

It is a fundamental project management technique for describing and forming the total scope of a project, using a hierarchical tree structure. Each descending level represents an increasingly detailed definition of the project. It is a system for subdividing a project into manageable work tasks, components or elements to provide a common framework for scope schedule, costs, providing of responsibility, communications, risk assessment monitoring and control.



Example of a Work break down Structure

Causes for project failures:

1. Poor Planning

Poor planning will always be a major cause for the project failure. If proper planning is failed it will increase the cost of project, total administrative system disturbs, progress of work lapsed.

2. Inadequate Documentation and Tracking

This is the responsibility of the project manager. Tracking milestones is how you are going to know whether you are meeting expectations. Proper recording and monitoring lets the PM identify where more resources are needed to complete a project on time.

3. Bad Team

The people in team should have a responsibility to ensure that the project is effective. Management should not micromanage but provide support to confirm that the PM can follow through with the expectations placed upon them.

4. Inexperienced Project In charges

A project In charge has a lot of responsibility. You need to allot people to management roles who have higher education and good experience. In some cases, and perhaps more often than not, inexperienced in charges are given projects.

5. Inaccurate budget Estimations

Inaccurate budget estimations may lead to enhancement of project cost and in sometimes the total project may stop.

6. Culture or Ethical Misalignment

Company culture must be included of capability, pro-activeness, and proficiency. If it isn't, team members will not be inspired to do their best. Basically, everyone complicated must be invested in their part of the project to successfully complete it.

Project failure can be avoided by having adequate employee training, project management software and management transparency

UNIT - 2

Introduction:

- Any project involves planning, scheduling and controlling a number of interrelated activities with use of limited resources, namely, men, machines, materials, money and time.
- The projects may be extremely large and complex such as construction of housing, a highway, a shopping complex etc.
- introduction of new products and research and development projects.
- It is required that managers must have a dynamic planning and scheduling system to produce the best possible results and also to react immediately to the changing conditions and make necessary changes in the plan and schedule.
- A convenient analytical and visual technique of PERT and CPM prove extremely valuable in assisting the managers in managing the projects.
- PERT stands for Project Evaluation and Review Technique developed during 1950's. The technique was developed and used in conjunction with the planning and designing of the Polaris missile project.
- CPM stands for Critical Path Method which was developed by DuPont Company and applied first to the construction projects in the chemical industry.
- Though both PERT and CPM techniques have similarity in terms of concepts, the basic difference is; CPM has single time estimate and PERT has three time estimates for activities and uses probability theory to find the chance of reaching the scheduled time.

Project management generally consists of three phases.

Planning:

Planning involves setting the objectives of the project. Identifying various activities to be performed and determining the requirement of resources such as men, materials, machines, etc.

The cost and time for all the activities are estimated, and a network diagram is developed showing sequential interrelationships (predecessor and successor) between various activities during the planning stage.

Scheduling:

Based on the time estimates, the start and finish times for each activity are worked out by applying forward and backward pass techniques, critical path is identified, along with the slack and float for the non-critical paths.

Controlling:

Controlling refers to analyzing and evaluating the actual progress against the plan. Reallocation of resources, crashing and review of projects with periodical reports are carried out.

COMPONENTS OF PERT/CPM NETWORK

PERT / CPM networks contain two major components

i. Activities, and

ii. Events

Activity: An activity represents an action and consumption of resources (time, money, energy) required to complete a portion of a project. Activity is represented by an arrow.

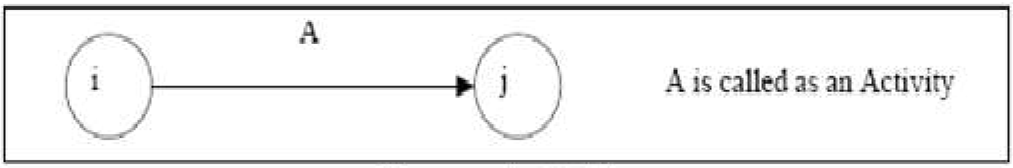


Figure : An Activity

Event: An event (or node) will always occur at the beginning and end of an activity. The event has no resources and is represented by a circle. The ith event and jth event are the tail event and head event respectively.

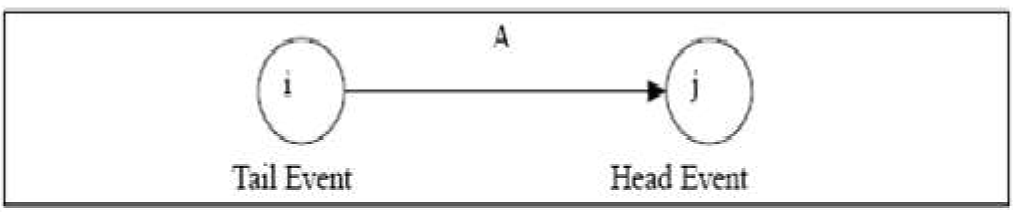


Figure : An Event

<u>Merge and Burst Events</u>

One or more activities can start and end simultaneously at an event.

<u>Preceding and Succeeding Activities</u>

Activities performed before given events are known as preceding activities, and activities performed after a given event is known as succeeding activities.

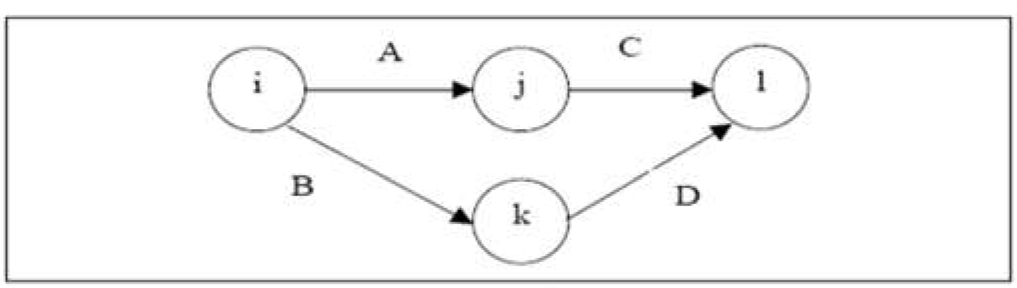


Figure : Preceding and Succeeding Activities

Activities A and B precede activities C and D respectively.

Dummy Activity

An imaginary activity which does not consume any resource and time is called a dummy activity. Dummy activities are simply used to represent a connection between events in order to maintain logic in the network. It is represented by a dotted line in a network.

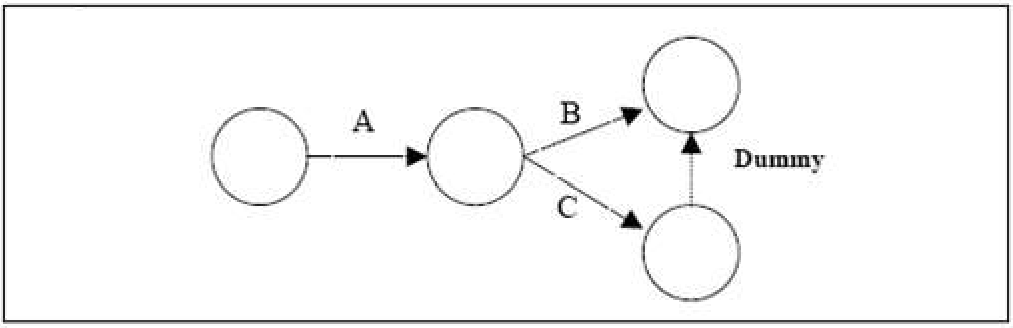


Figure : Dummy Activity

ERRORS TO BE AVOIDED IN CONSTRUCTING A NETWORK

a. Two activities starting from a tail event must not have a same end event. To ensure this, it is absolutely necessary to introduce a dummy activity, as shown in Figure.

b. Looping error should not be formed in a network, as it represents performance of activities repeatedly in a cyclic manner, as shown below in Figure.

c. In a network, there should be only one start event and one ending event as shown below, in Figure.

d. The direction of arrows should flow from left to right avoiding mixing of direction as shown in Figure.

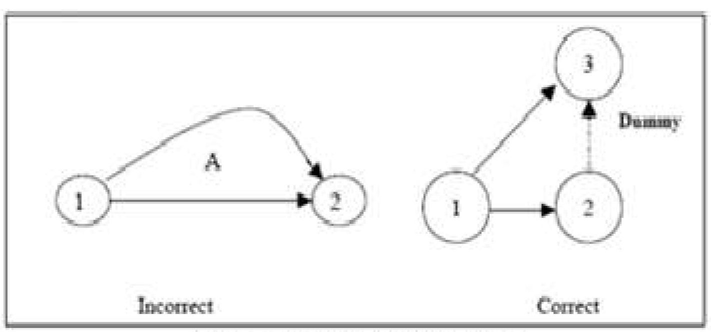


Figure : Correct and Incorrect Activities

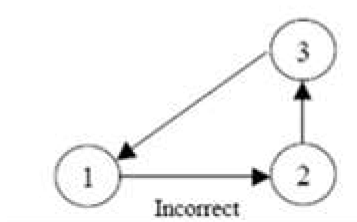


Figure : Looping Error

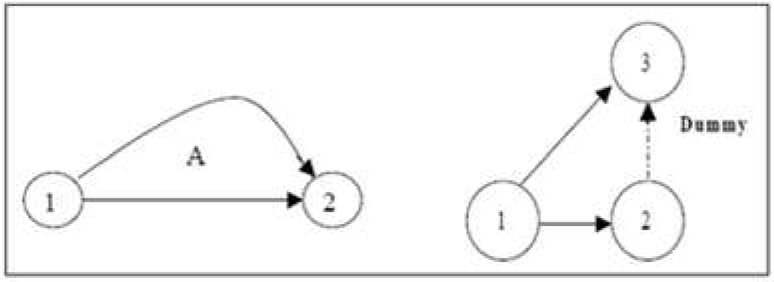


Figure : Only One Start and End Event

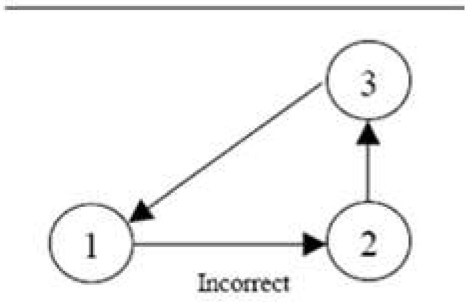


Figure : Wrong Direction of Arrows

RULES IN CONSTRUCTING A NETWORK

1. No single activity can be represented more than once in a network. The length of an arrow has no significance.

2. The event numbered 1 is the start event and an event with highest number is the end event. Before an activity can be undertaken, all activities preceding it must be completed. That is, the activities must follow a logical sequence (or – interrelationship) between activities.

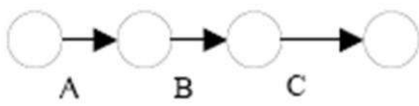
3. In assigning numbers to events, there should not be any duplication of event numbers in a network.

4. Dummy activities must be used only if it is necessary to reduce the complexity of a network.

5. A network should have only one start event and one end event.

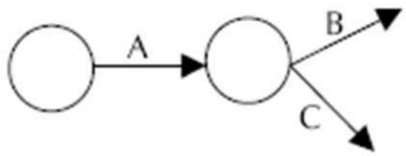
Some conventions of network diagram are shown in Figure (a), (b), (c), (d) below:

(a)



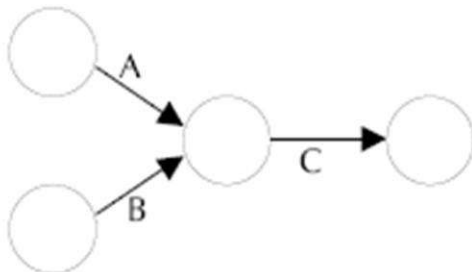
Activity B can be performed only after completing activity A, and activity C can be performed only after completing activity B.

(b)



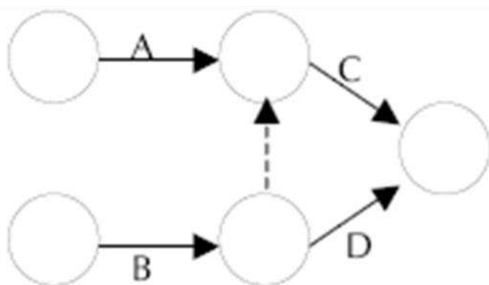
Activities B and C can start simultaneously only after completing A.

(c)



Activities A and B must be completed before start of activity C.

(d)



Activity C must start only after completing activities A and B. But activity D can start after completion of activity B.

<u>PROCEDURE FOR NUMBERING THE EVENTS USING FULKERSON'S RULE</u>

Step1: Number the start or initial event as 1.

Step2: From event 1, strike off all outgoing activities. This would have made one or more events as initial events (event which do not have incoming activities). Number that event as 2.

Step3: Repeat step 2 for event 2, event 3 and till the end event. The end event must have the highest number.

Example 1:

Draw a network for a house construction project. The sequences of activities with their predecessors are given in Table below.

Table : Sequence of Activities for House Construction Project

Name of the activity	Starting and finishing event	Description of activity	Predecessor	Time duration (days)
A	(1,2)	Prepare the house plan	--	4
B	(2,3)	Construct the house	A	58
C	(3,4)	Fix the door / windows	B	2
D	(3,5)	Wiring the house	B	2
E	(4,6)	Paint the house	C	1
F	(5,6)	Polish the doors / windows	D	1

Solution:

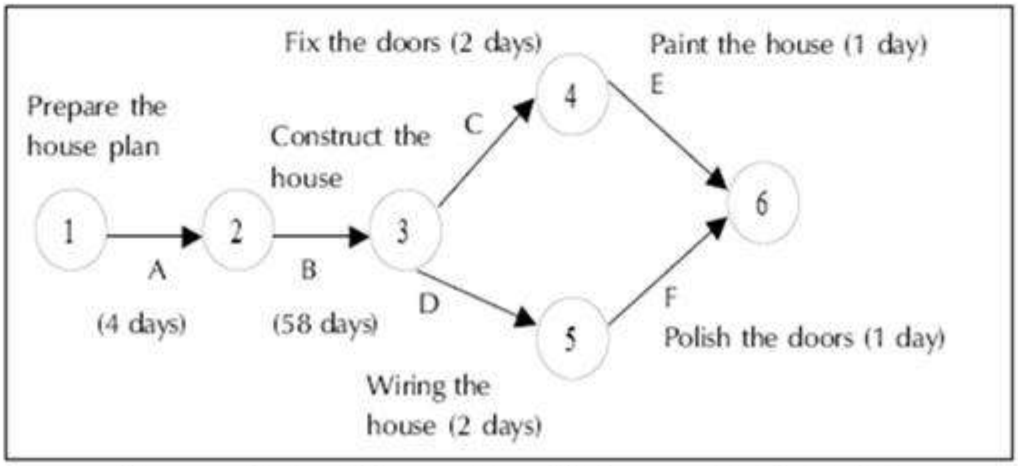


Figure : Network diagram representing house construction project.

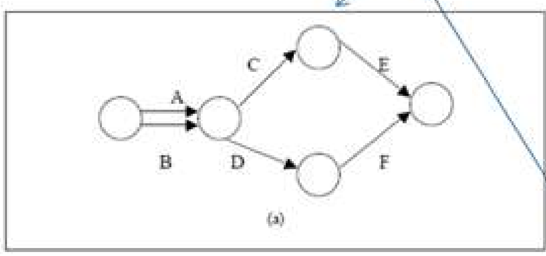
The network diagram in Figure shows the procedure relationship between the activities. Activity A (preparation of house plan), has a start event 1 as well as an ending event 2. Activity B (Construction of house) begins at event 2 and ends at event 3. The activity B cannot start until activity A has been completed. Activities C and D cannot begin until activity B has been completed, but they can be performed simultaneously. Similarly, activities E and F can start only after completion of activities C and D respectively. Both activities E and F finish at the end of event 6.

Example 2: Consider the project given in Table and construct a network diagram. Table: Sequence of Activities for Building Construction Project

Activity	Description	Predecessor
A	Purchase of Land	-
B	Preparation of building plan	-
C	Level or clean the land	A
D	Register and get approval	A, B
E	Construct the building	C
F	Paint the building	D

Solution:

The activities C and D have a common predecessor A. The network representation shown in Figure (a), (b) violates the rule that no two activities can begin and end at the same events. It appears as if activity B is a predecessor of activity C, which is not the case. To construct the network in a logical order, it is necessary to introduce a dummy activity as shown in Figure.



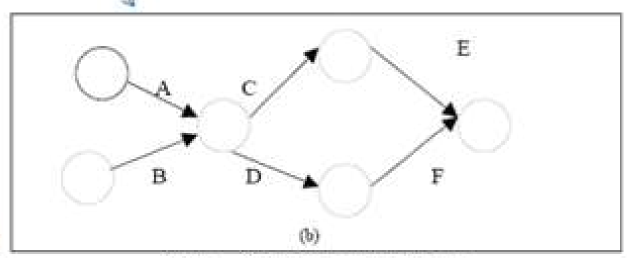


Figure :: Network representing the Error

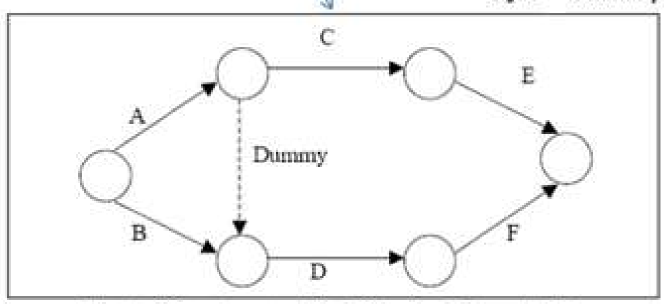


Figure : Correct representation of Network using Dummy Activity

Activity	Description	Predecessor
A	Purchase of Land	-
B	Preparation of building plan	-
C	Level or clean the land	A
D	Register and get approval	A, B
E	Construct the building	C
F	Paint the building	D

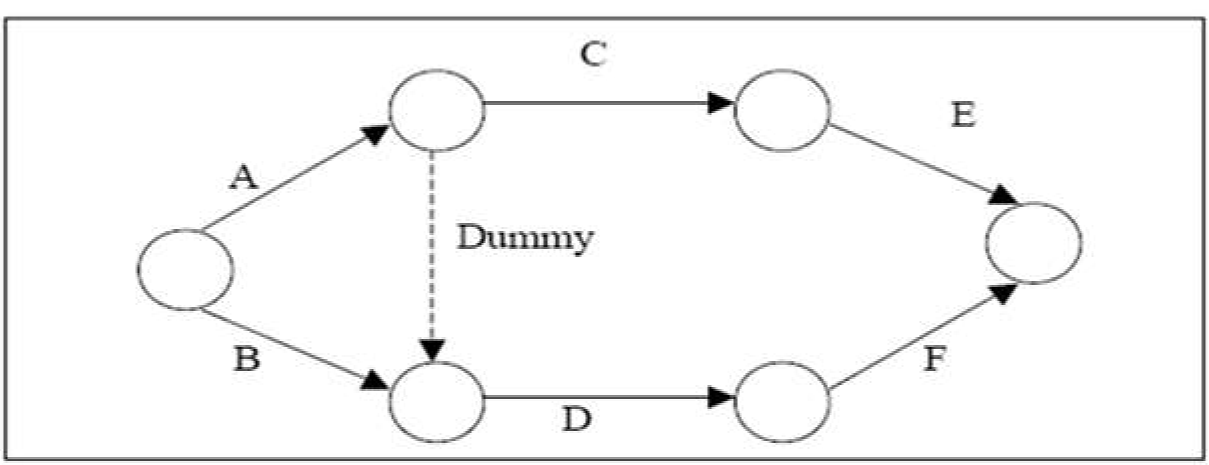


Figure : Correct representation of Network using Dummy Activity

Example 3:

Construct a network for a project whose activities and their predecessor relationship are given in Table.

Table : Activity Sequence for a Project

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor	-	-	-	A	B	B	C	D	E	H, I	F, G

Solution: *The network diagram for the given problem is shown in Figure with* activities A, B and C starting simultaneously.

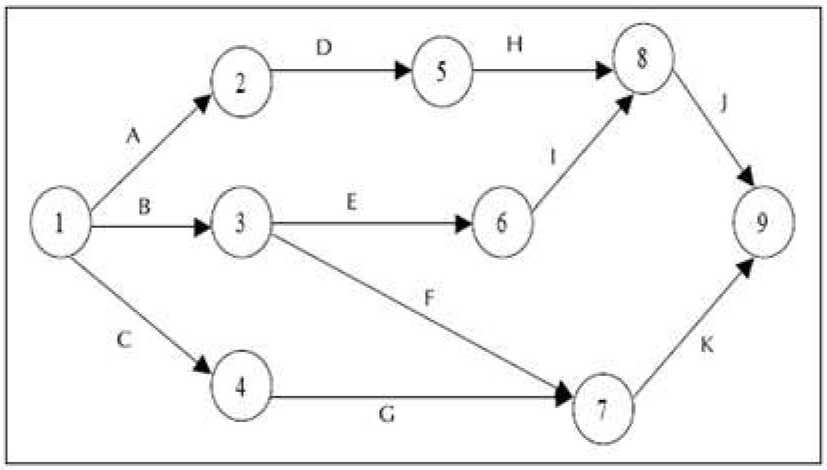


Figure : Network Diagram

Example 4: Draw a network diagram for a project given in Table.

Table : Project Activity Sequence

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Immediate Predecessor	-	A	B	A	D	C, E	D	D	H	H	F, H	G, J

***Solution: An activity network diagram describing the project is shown in Figure*, below:**

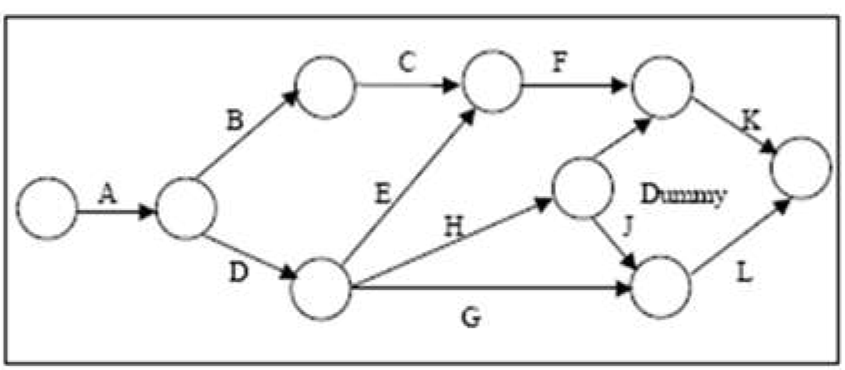


Figure : Network Diagram

CRITICAL PATH ANALYSIS

- The critical path for any network is the longest path through the entire network.
- Since all activities must be completed to complete the entire project, the length of the critical path is also the shortest time allowable for completion of the project.
- Thus if the project is to be completed in that shortest time, all activities on the critical path must be started as soon as possible. These activities are called critical activities.
- If the project has to be completed ahead of the schedule, then the time required for at least one of the critical activity must be reduced.
- Further, any delay in completing the critical activities will increase the project duration. The activity, which does not lie on the critical path, is called non-critical activity. These non-critical activities may have some slack time.
- The slack is the amount of time by which the start of an activity may be delayed without affecting the overall completion time of the project. But a critical activity has no slack.
- To reduce the overall project time, it would require more resources (at extra cost) to reduce the time taken by the critical activities to complete. Scheduling of Activities: Earliest Time (TE) and Latest Time (TL).
- Before the critical path in a network is determined, it is necessary to find the earliest and latest time of each event to know the earliest expected time (TE) at which the activities originating from the event can be started and to know the latest allowable time (TL) at which activities terminating at the event can be completed.

Forward Pass Computations (to calculate Earliest, Time TE)

Step 1: Begin from the start event and move towards the end event.

Step 2: Put TE = 0 for the start event.

Step 3: Go to the next event (i.e node 2) if there is an incoming activity for event 2, add calculate TE of previous event (i.e event 1) and activity time.

Note: If there are more than one incoming activities, calculate TE for all incoming activities and take the maximum value. This value is the TE for event 2.

Step 4: Repeat the same procedure from step 3 till the end event.

Backward Pass Computations (to calculate Latest Time TL)

Procedure:

Step 1: Begin from end event and move towards the start event. Assume that the direction of arrows is reversed.

Step 2: Latest Time TL for the last event is the earliest time. TE of the last event.

Step 3: Go to the next event, if there is an incoming activity, subtract the value of TL of previous event from the activity duration time. The arrived value is TL for that event. If there are more than one incoming activities, take the minimum TE value.

Step 4: Repeat the same procedure from step 2 till the start event.

DETERMINATION OF FLOAT AND SLACK TIMES

As discussed earlier, the non – critical activities have some slack or float. The float of an activity is the amount of time available by which it is possible to delay its completion time without extending the overall project completion time.

t_{ij} = duration of activity

TE = earliest expected time

TL = latest allowable time

ES_{ij} = earliest start time of the activity

EF_{ij} = earliest finish time of the activity

LS_{ij} = latest start time of the activity

LF_{ij} = latest finish time of the activity

Total Float TF_{ij}: The total float of an activity is the difference between the latest start time and the earliest start time of that activity.

$TF_{ij} = LS_{ij} – ES_{ij}$ ------------- (1)

or

$TF_{ij} = (TL – TE) – t_{ij}$ ---------- (2)

Free Float FF_{ij}: The time by which the completion of an activity can be delayed from its earliest finish time without affecting the earliest start time of the succeeding activity is called free float.

$FF_{ij} = (E_j – E_i) – t_{ij}$ ---------- (3)

FF_{ij} = Total float – Head event slack

Independent Float IF_{ij}: The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time.

$IF_{ij} = (E_j – L_i) – t_{ij}$.................... (4)

IF_{ij} = Free float – Tail event slack Where tail event slack = $L_i – E_i$

The negative value of independent float is considered to be zero.

Critical Path:

After determining the earliest and the latest scheduled times for various activities, the minimum time required to complete the project is calculated. In a network, among various paths, the longest path which determines the total time duration of the project is called the critical path. The following conditions must be satisfied in locating the critical path of a network.

An activity is said to be critical only if both the conditions are satisfied.

1. TL – TE = 0
2. TLj – tij – TEj = 0

Example:

A project schedule has the following characteristics as shown in Table

i. Construct PERT network.
ii. Compute TE and TL for each activity.
iii. Find the critical path.

Table : Project Schedule

Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

Table : Project Schedule

Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

(i) From the data given in the problem, the activity network is constructed as shown in Figure given below

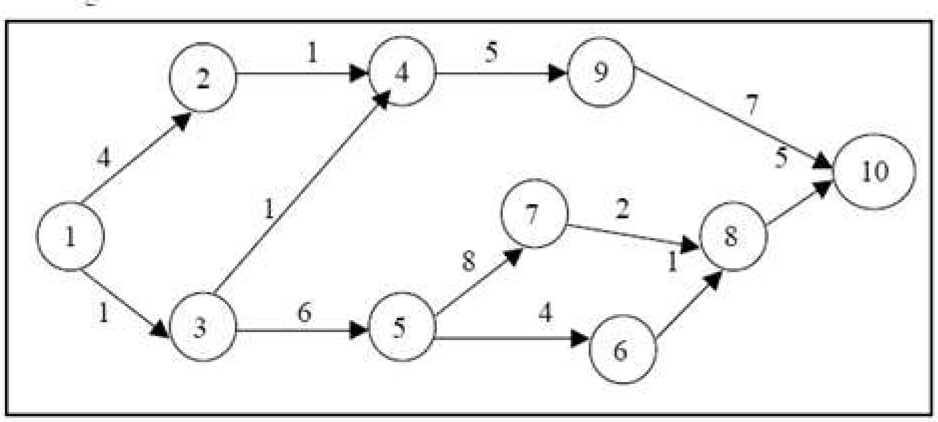


Figure : Activity Network Diagram

(ii) To determine the critical path, compute the earliest time TE and latest time TL for each of the activity of the project. The calculations of TE and TL are as follows:

To calculate TE for all activities

TE1 = 0

TE2 = TE1 + t1, 2 = 0 + 4 = 4

TE3 = TE1 + t1, 3 = 0 + 1 =1

TE4 = max (TE2 + t2, 4 and TE3 + t3, 4) = max (4 + 1 and 1 + 1) = max (5, 2) = 5 days

TE5 = TE3 + t3, 6 = 1 + 6 = 7

TE6 = TE5 + t5, 6 = 7 + 4 = 11

TE7 = TE5 + t5, 7 = 7 + 8 = 15

TE8 = max (TE6 + t6, 8 and TE7 + t7, 8) = max (11 + 1 and 15 + 2) = max (12, 17) = 17 days

TE9 = TE4 + t4, 9 = 5 + 5 = 10

TE10 = max (TE9 + t9, 10 and TE8 + t8, 10) = max (10 + 7 and 17 + 5) = max (17, 22) = 22 days

To calculate TL for all activities

TL10 = TE10 = 22

TL9 = TE10 – t9, 10 = 22 – 7 = 15

TL8 = TE10 – t8, 10 = 22 – 5 = 17

TL7 = TE8 – t7, 8 = 17 – 2 = 15

TL6 = TE8 – t6, 8 = 17 – 1 = 16

TL5 = min (TE6 – t5, 6 and TE7 – t5, 7) = min (16 – 4 and 15 –8) = min (12, 7) = 7 days

TL4 = TL9 – t4, 9 = 15 – 5 =10

TL3 = min (TL4 – t3, 4 and TL5 – t3, 5) = min (10 – 1 and 7 – 6) = min (9, 1) = 1 day

TL2 = TL4 – t2, 4 = 10 – 1 = 9

TL1 = Min (TL2 – t1, 2 and TL3 – t1, 3) = Min (9 – 4 and 1 – 1) = 0

Table : Various Activities and their Floats

Activity	Activity Name	Normal Time (tij)	Earliest Time (TE)		Latest Time (TL)		Total Float
			Start (ES)	Finish (EF)	Start (LS)	Finish (LF)	
1-2	A	4	0	4	5	9	5
1-3	B	1	0	1	0	1	0
2-4	C	1	4	5	9	10	5
3-4	D	1	1	2	9	10	8
3-5	E	6	1	7	1	7	0
4-9	F	5	5	10	10	15	5
5-6	G	4	7	11	12	16	5
5-7	H	8	7	15	7	15	0
6-8	I	1	11	12	16	17	5
7-8	J	2	15	17	15	17	0
8-10	K	5	17	22	17	22	0
9-10	L	7	10	17	15	22	5

(iii) From the Table , we observe that the activities 1 – 3, 3 – 5, 5 – 7,7 – 8 and 8 – 10 are critical activities as their floats are zero.

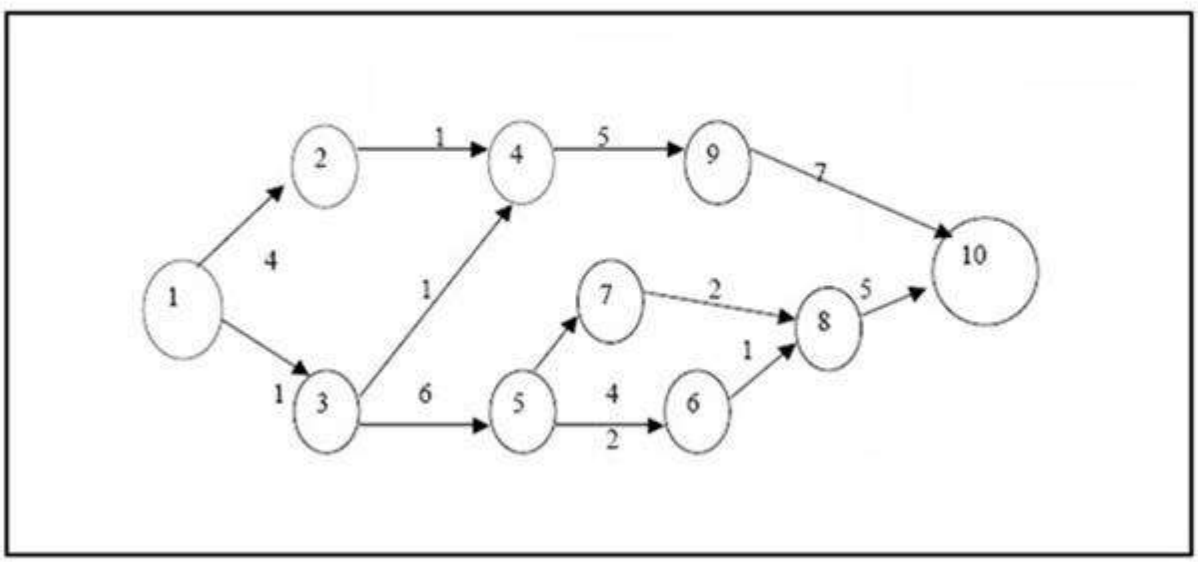


Figure : Critical Path of the Project

PROJECT EVALUATION REVIEW TECHNIQUE, (PERT)

In the critical path method, the time estimates are assumed to be known with certainty. In certain projects like research and development, new product introductions, it is difficult to estimate the time of various activities.

Hence PERT is used in such projects with a probabilistic method using three time estimates for an activity, rather than a single estimate, as shown in Figure.

Optimistic time to:

It is the shortest time taken to complete the activity. It means that if everything goes well then there is more chance of completing the activity within this time.

Most likely time tm:

It is the normal time taken to complete an activity, if the activity were frequently repeated under the same conditions.

Pessimistic time tp:

It is the longest time that an activity would take to complete. It is the worst time estimate that an activity would take if unexpected problems are faced.

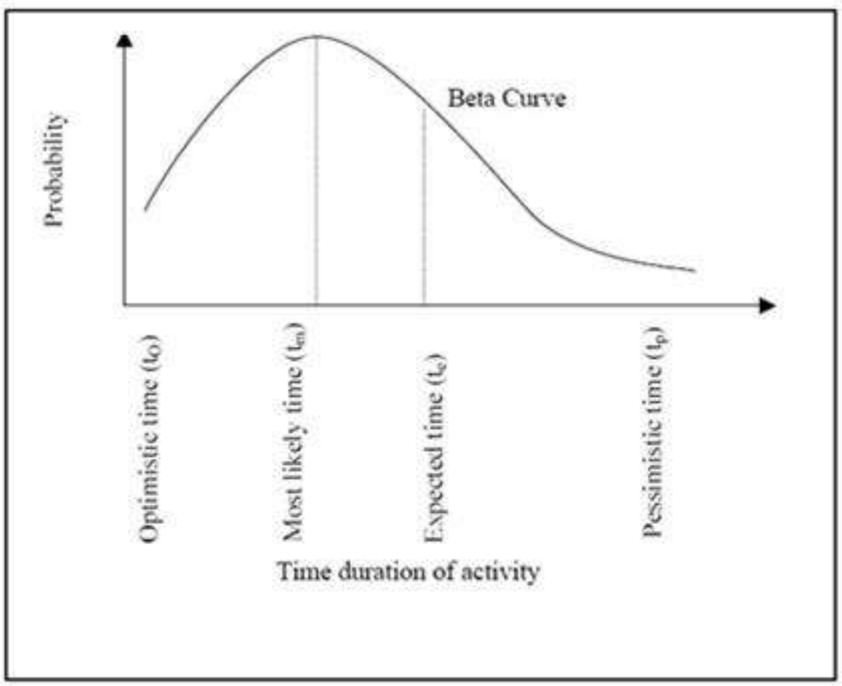


Figure : PERT Using Probabilistic Method with 3 Time Estimates

Taking all these time estimates into consideration, the expected time of an activity is arrived at.

The average or mean (ta) value of the activity duration is given by,

$$T_a = \frac{t_0 + 4t_m + t_p}{6}$$

The variance of the activity time is calculated using the formula.

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6} \right)^2$$

Probability for Project Duration

The probability of completing the project within the scheduled time (Ts) or contracted time may be obtained by using the standard normal deviate where Te is the expected time of project completion.

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma^2} \text{ in critical path}}$$

Probability of completing the project within the scheduled time is,

$$P\,(T \leq T_s) = P\,(Z \leq Z_0)$$

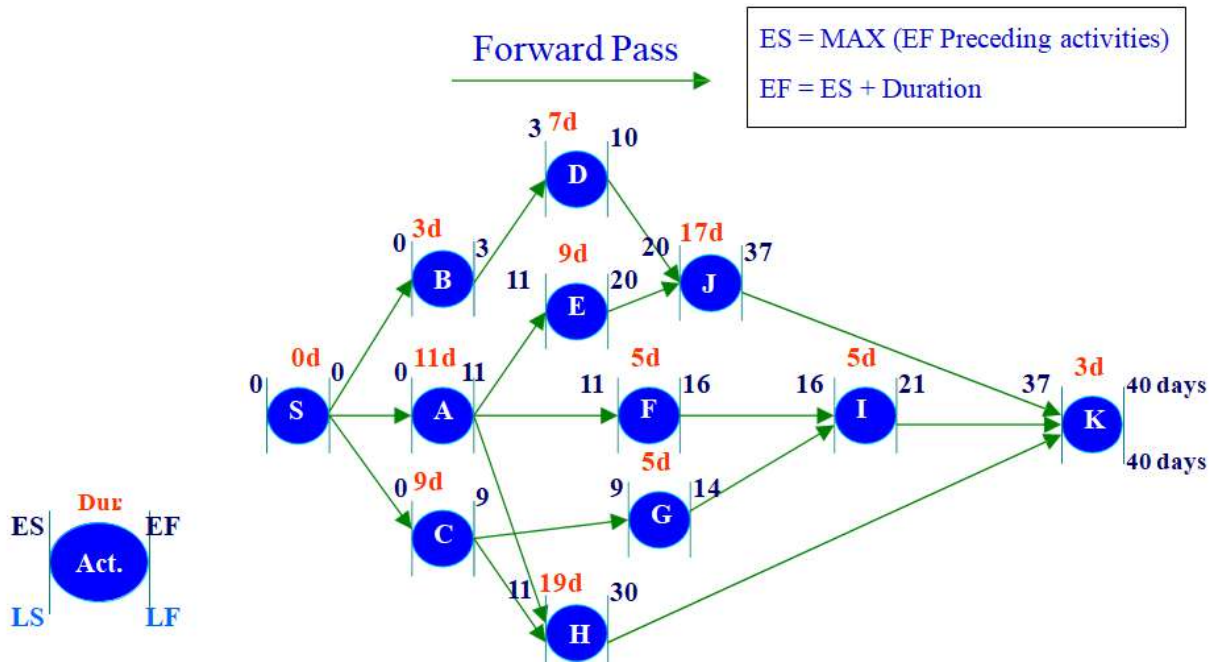
Exercise – Network Analysis & Parameter Calculation

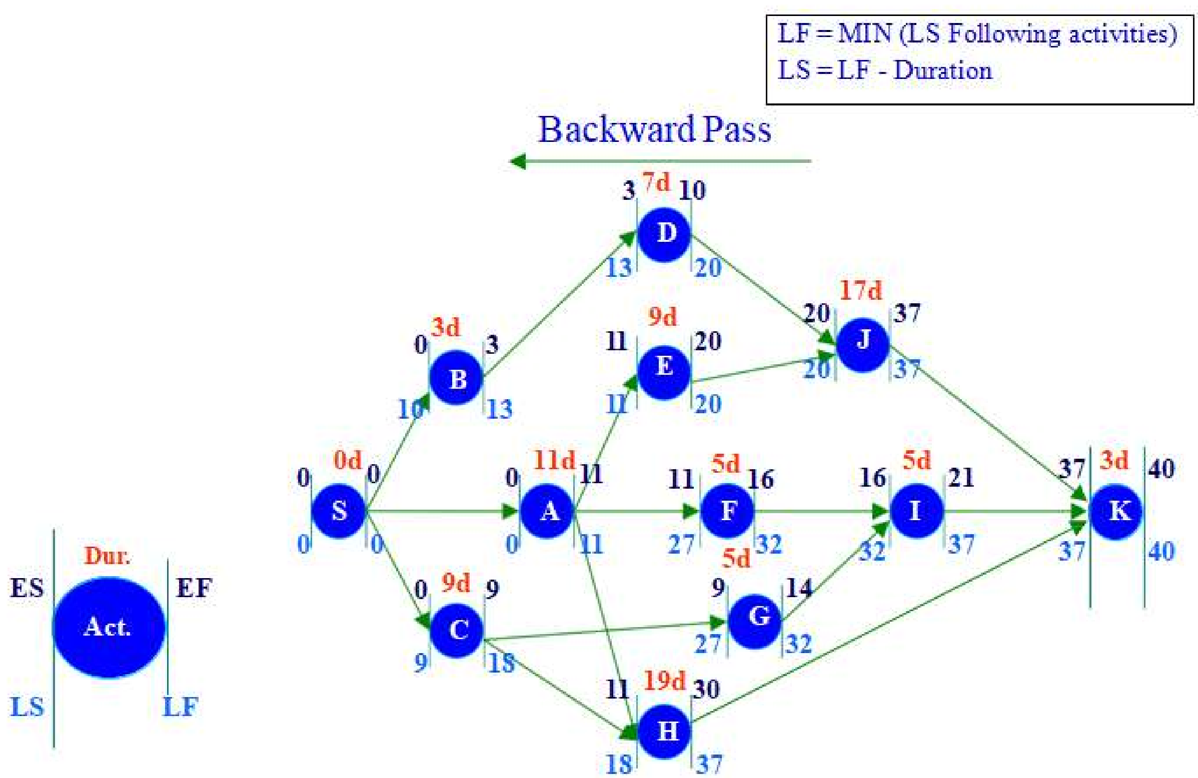
Activity	Dur	Preced
A	11	S
B	3	S
C	9	S
D	7	B
E	9	A
F	5	A
G	5	C
H	19	A,C
I	5	F,G
J	17	E,D
K	3	H,I,J

Draw AON Network & Calculate

1. ES, EF
2. Project Duration
3. LF, LS
4. TF, FF,
5. Critical activities
6. INTF, INDF

Activity	Duration	Predecessor	Early Start	Early Finish	Late Start	Late Finish	TF	Critical	FF	INTF	INDF
A	11	Start									
B	3	Start									
C	9	Start									
D	7	B									
E	9	A									
F	5	A									
G	5	C									
H	19	A,C									
I	5	F,G									
J	17	E,D									
K	3	H,I,J									


Forward Pass
ES = MAX (EF Preceding activities)
EF = ES + Duration
Dur.
ES
Act.
EF
LS
LF
S
A
B
C
D
E
F
G
H
I
J
K
0d
3d
11d
9d
7d
9d
5d
5d
19d
5d
17d
3d
40 days
40 days


LF = MIN (LS Following activities)
LS = LF - Duration
Backward Pass
Dur.
ES
Act.
EF
LS
LF
S
A
B
C
D
E
F
G
H
I
J
K
0d
3d
11d
9d
7d
9d
5d
5d
19d
5d
17d
3d

Critical Path Identification → TF=0 for Critical Activities

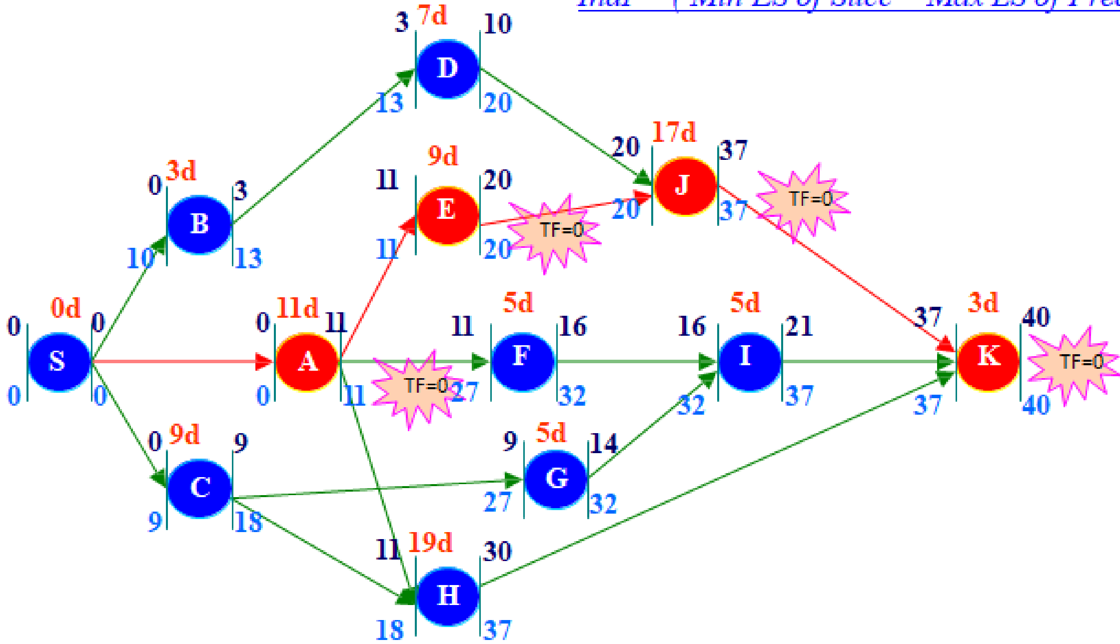
Float Calculation →

TF = (LS-ES) = (LF-EF)

FF = (Min ES of Successor – EF)

IntF = (TF- FF)

IndF =(Min ES of Succ – Max LS of Pred – Dur)



Activity	Duration	Predecessor	Early Start	Early Finish	Late Start	Late Finish	TF	Critical	FF	INTF	INDF
A	11	Start	0	11	0	11	0	y	0	0	0
B	3	Start	0	3	10	13	10	n	0	10	0
C	9	Start	0	9	9	18	9	n	0	9	0
D	7	B	3	10	13	20	10	n	10	0	0
E	9	A	11	20	11	20	0	y	0	0	0
F	5	A	11	16	27	32	16	n	0	16	0
G	5	C	9	14	27	32	18	n	2	16	0
H	19	A,C	11	30	18	37	7	n	7	0	0
I	5	F,G	16	21	32	37	16	n	16	0	0
J	17	E,D	20	37	20	37	0	y	0	0	0
K	3	H,I,J	37	40	37	40	0	y	0	0	0

INTRODUCTION TO PRECEDENCE DIAGRAM METHOD

History of Precedence Diagram Method

- Developed in 1960s by Zachry Construction & IBM
- Attempt to get advantages of AOA's events with a AON type representation.
- The default representation in all popular scheduling software.
- Not fully accepted by all professional schedulers as software algorithms are not standardized.

Rationale for Precedence Diagram Method

- Consider a Pipeline Project
- Key Activities
  - Excavate
  - Lay pipe
  - Backfill
- Develop Project Network
  - How many activities?
  - What is the sequence?

NEED FOR PDM

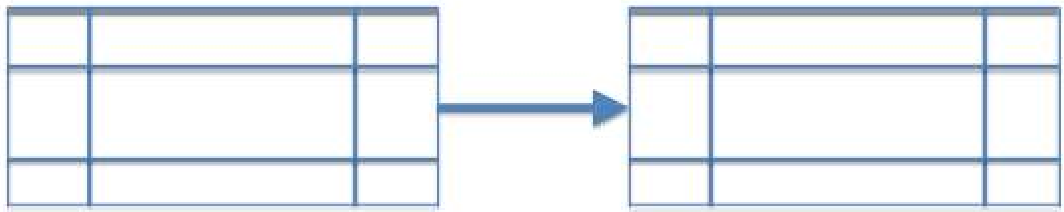
- Above network clearly says that after completion of excavation; pipe laying will be done; After pipe laying is complete - backfilling will be done.

Does this represent real sequence??

1. Pipe laying may start after 3 or 4 days of excavation.
2. Backfilling may start after 3 or 4 days of pipe laying.

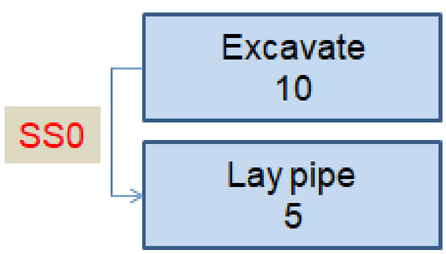
Precedence Diagram Method Notation

- AON Type Notation

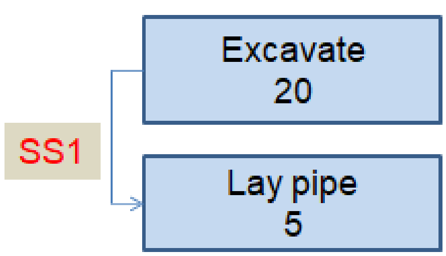


- 4 Types of Relationships
    - Start to Start relationship
    - Finish to Start relationship
    - Finish to Finish relationship
    - Start to Finish relationship
- Leads & Lags

Start to Start Relationship

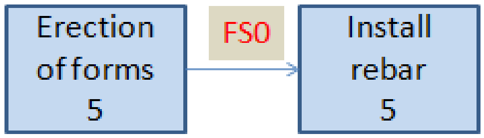


- **Zero lag**: Laying pipe can start no earlier than excavation.

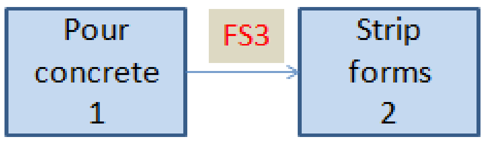


- **Positive lag:** Laying pipe can start no earlier than one day after excavation starts.

Finish to Start Relationship

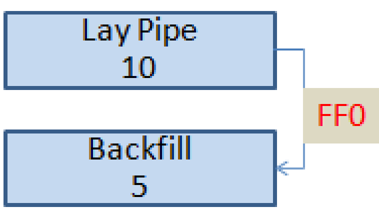


- **Zero lag:** Installation of reinforcement can start only after the erection of forms is completed.

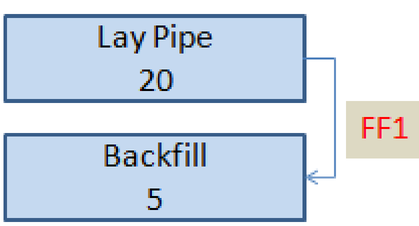


- **Positive lag:** The striping of forms can start no earlier than 3 days after completion of concrete pouring.

Finish to Finish Relationship

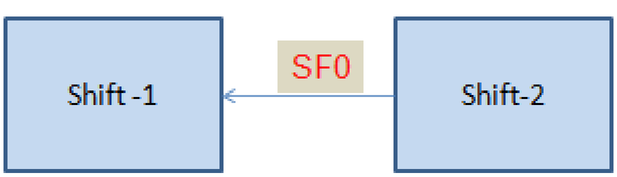


- **Zero lag:** The completion of backfilling can be no earlier than the completion of laying pipe.



- **Positive lag:** Backfilling can be completed no earlier than 1 day after the laying of pipe is completed.

Start to Finish Relationship



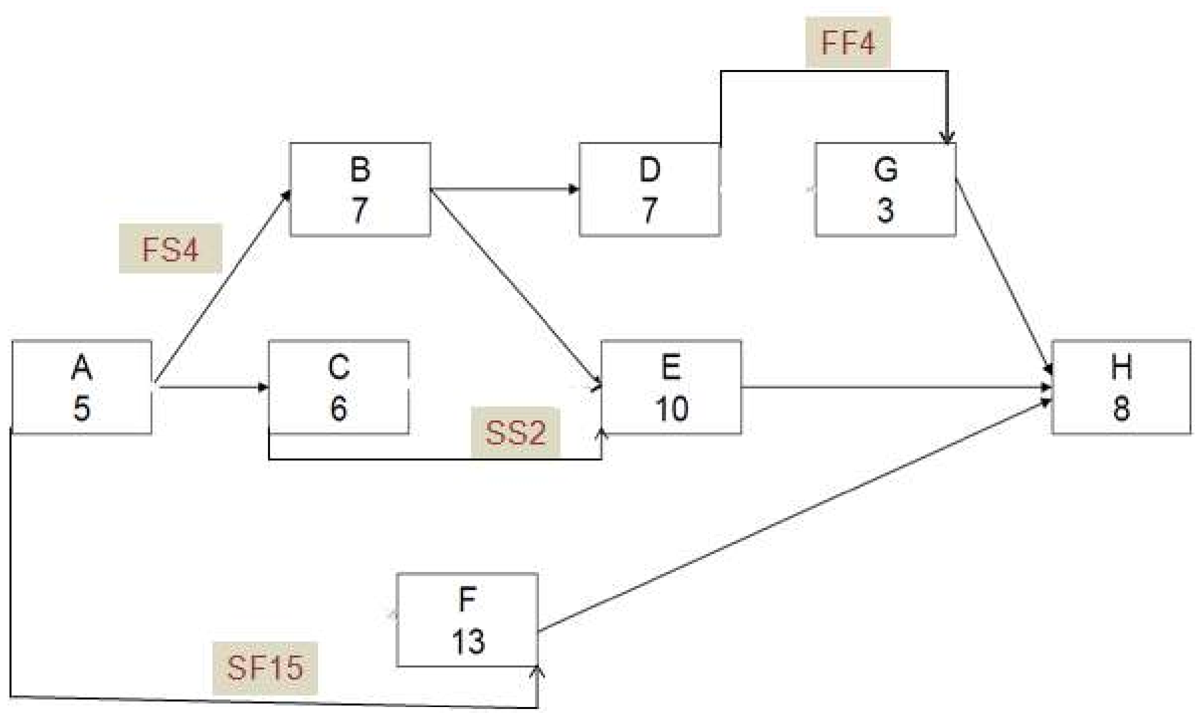
- Shift 1 can end their work only after Shift-2 has started. Not a common relationship – rarely used.

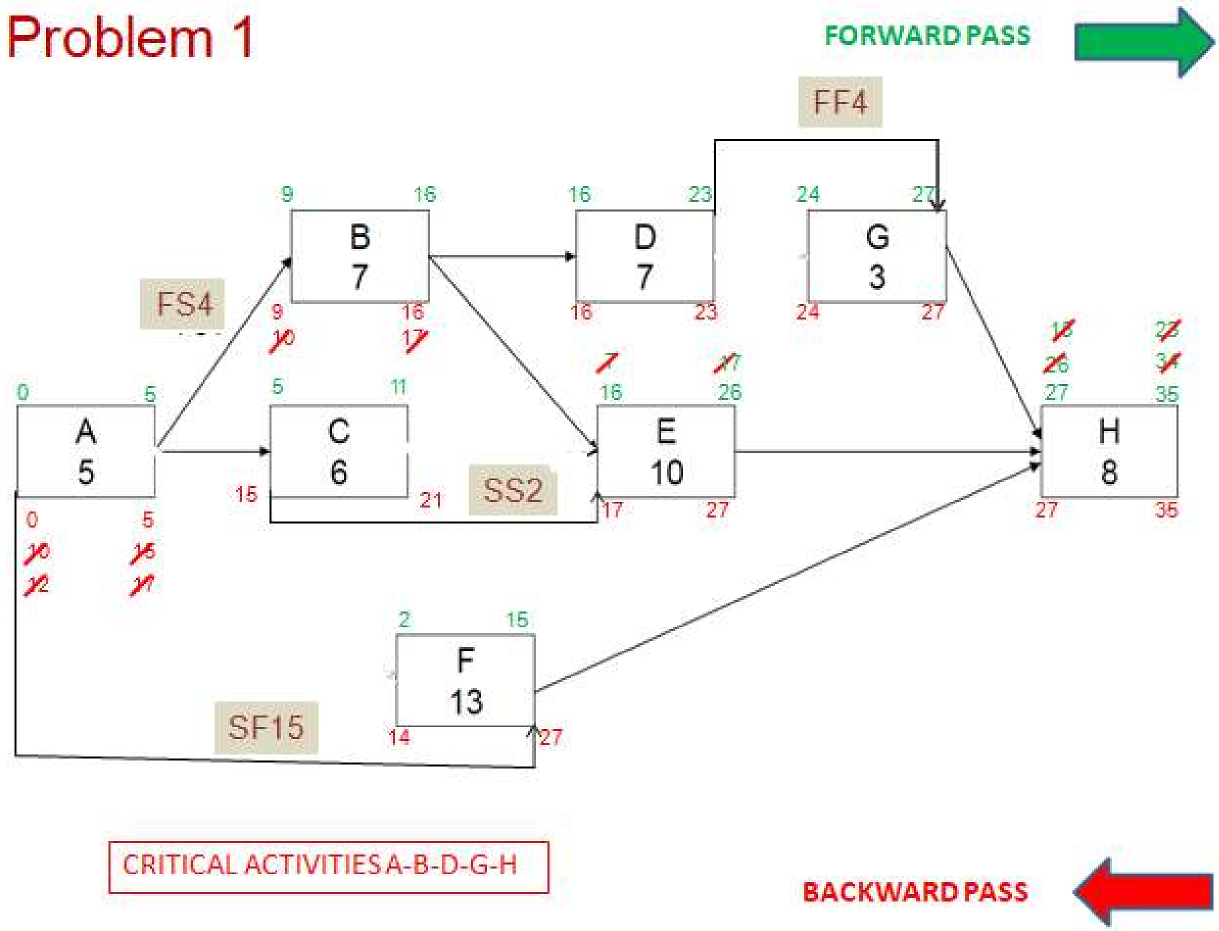
PDM – Problem-1

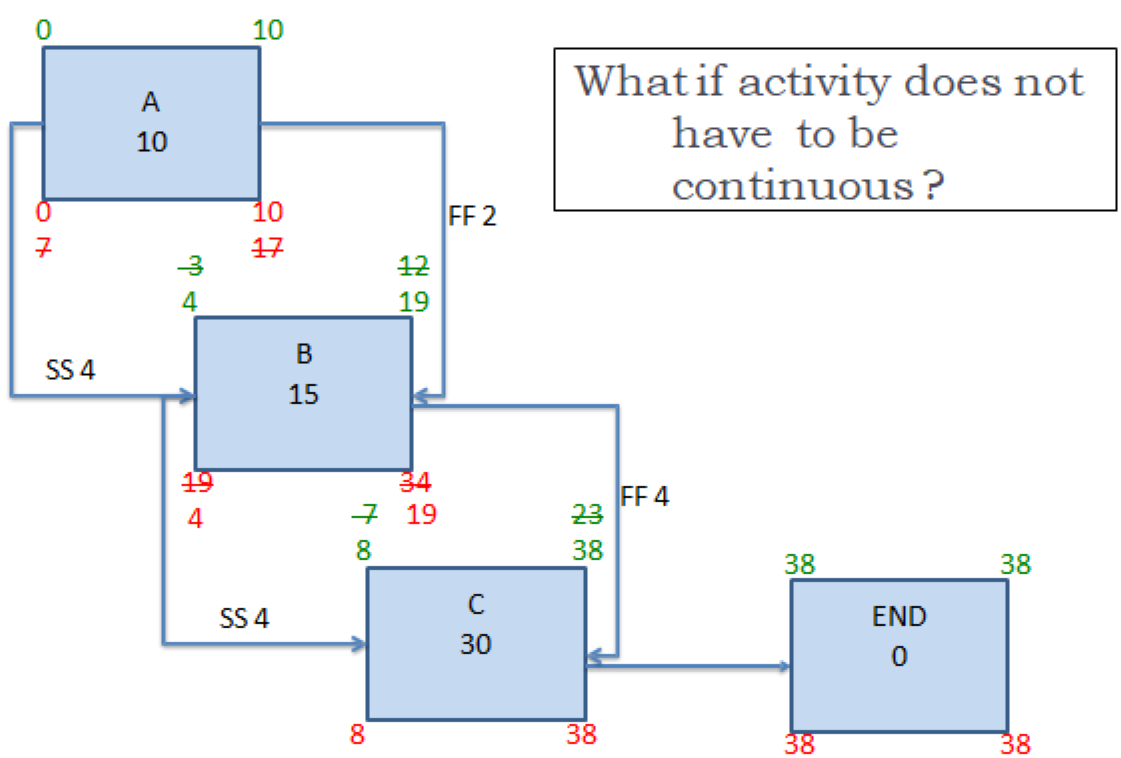
Activity	Predecessor	Duration
A	**-**	**5**
B	**A (FS +4)**	**7**
C	**A**	**6**
D	**B**	**7**
E	**C (SS +2), B**	**10**
F	**A (SF+15)**	**13**
G	**D (FF+4)**	**3**
H	**E,F,G**	**8**

Draw PDM Network & Find ES, EF, LS, LF & CRITICAL PATH

Problem 1: PDM Representation







Comparison between AON and PDM

AON

- Allows only one kind of logical relationship between activities.

- The preceding activity must be completed before any succeeding activity can begin (lag = 0).
- Cannot create complex schedule.
- Easy to interpret

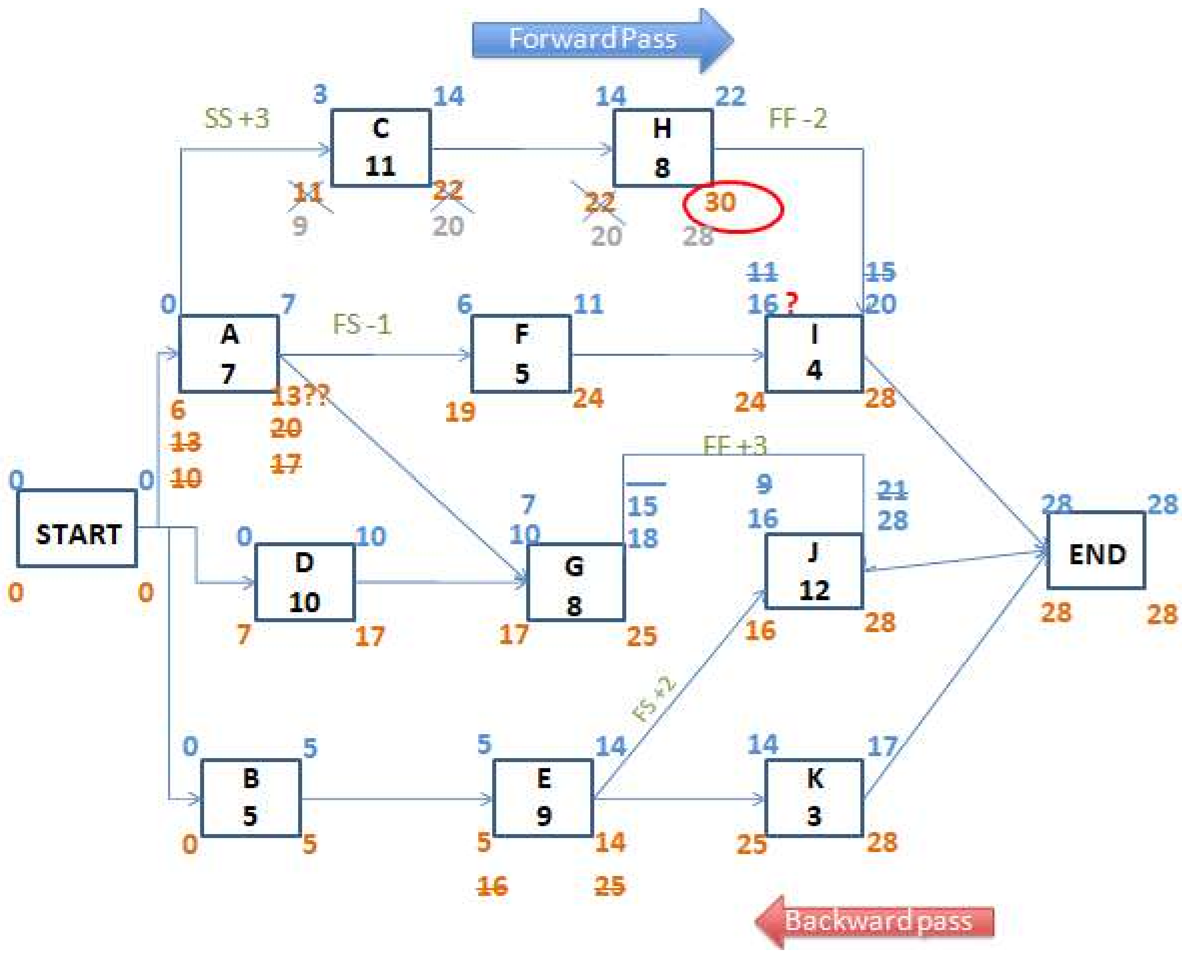
PDM

- Allows four kind of logical relationship between activities (FS, SS, SF, and FF).
- The preceding activity can have lag (zero, +ve, -ve) between succeeding activity.
- Can be used as flexible scheduling tool.
- Not easy to interpret

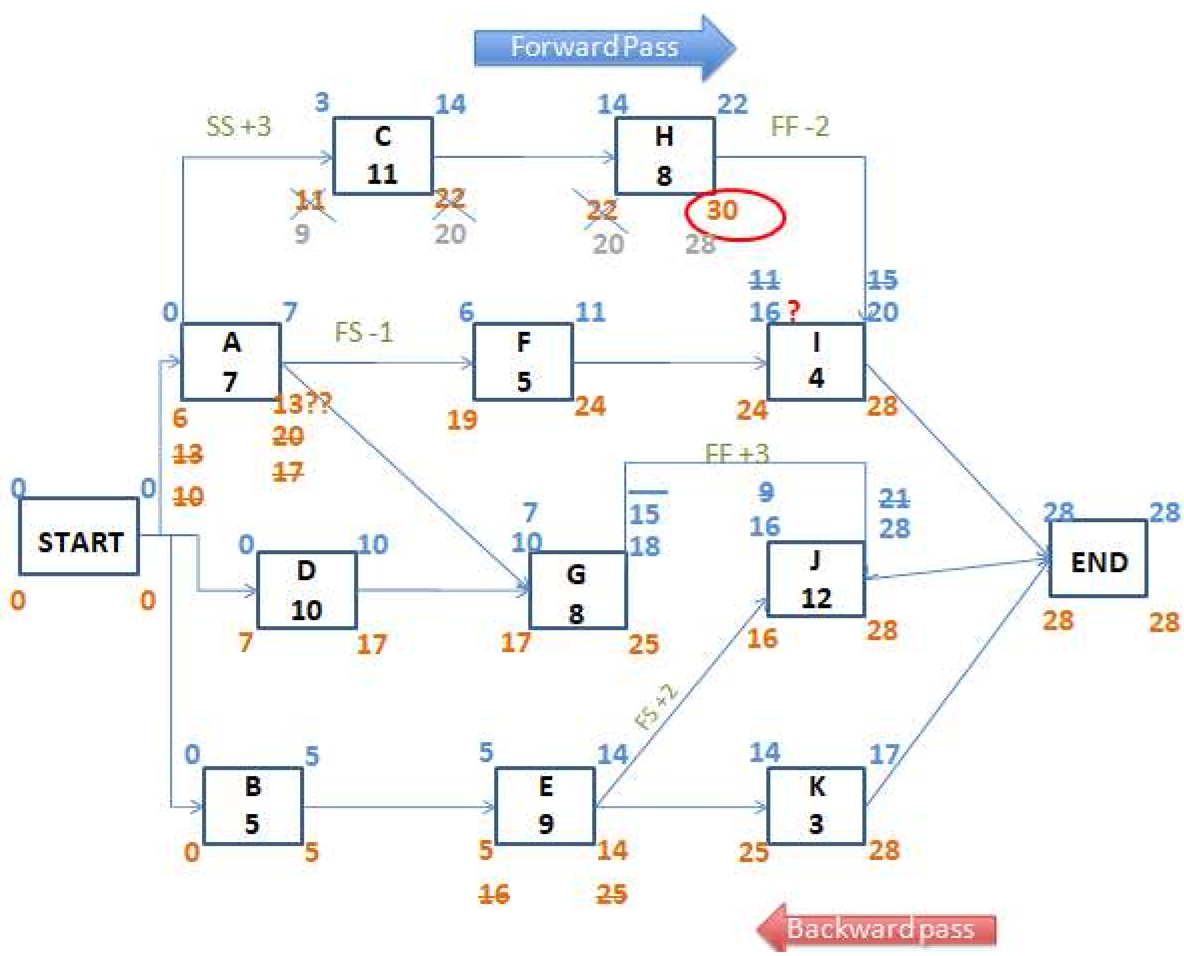
PDM – Problem – 2

Activity	Predecessor	Duration
Start	-	0
A	Start	7
B	Start	5
C	A (SS +3)	11
D	Start	10
E	B	09
F	A (FS -1)	5
G	A,D	8
H	C	8
I	F H (FF -2)	4
J	E (FS +2) G (FF +3)	12
K	E	3
End	I,J,K	0

Draw PDM Network & Find ES, EF, LS, LF & CRITICAL PATH



Forward Pass
SS +3
C
11
H
8
FF -2
30
A
7
FS -1
F
5
I
4
13??
FF +3
START
D
10
G
8
J
12
END
FS +2
B
5
E
9
K
3
Backward pass



Forward Pass
SS +3
C
11
H
8
FF -2
30
A
7
FS -1
F
5
I
4
13??
FF +3
START
D
10
G
8
J
12
END
FS +2
B
5
E
9
K
3
Backward pass

Unit-3

Comparison of alternatives:-

For most of the engineering projects, equipments etc., there are more than one feasible alternative. It is the duty of the project management team (comprising of engineers, designers, project managers etc.) of the client organization to select the best alternative that involves less cost and results more revenue. For this purpose, the economic comparison of the alternatives is made. The different cost elements and other parameters to be considered while making the economic comparison of the alternatives are initial cost, annual operating and maintenance cost, annual income or receipts, expected salvage value, income tax benefit and the useful life. When only one, among the feasible alternatives is selected, the alternatives are said to be mutually exclusive.

As already mentioned in module-1, the cost or expenses are generally known as cash outflows whereas revenue or incomes are generally considered as cash inflows. Thus in the economic comparison of alternatives, cost or expenses are considered as negative cash flows. On the other hand the income or revenues are considered as positive cash flows. From the view point of expenditure incurred and revenue generated, some projects involve initial capital investment i.e. cash outflow at the beginning and show increased income or revenue i.e. cash inflow in the subsequent years. The alternatives having this type of cash flow are known as investment alternatives. So while comparing the mutually exclusive investment alternatives, the alternative showing maximum positive cash flow is generally selected. In this case, the investment is made at the beginning to gain profit at the future period of time. Example for such type alternatives includes purchase of a dozer by a construction firm. The construction firm will have different feasible alternatives for the dozer with each alternative having its own initial investment, annual operating and maintenance cost, annual income depending upon the production capacity, useful life, salvagevaluesetc.Thusthealternativewhichwillyieldmoreeconomicbenefitwillbe selected by the construction firm. There are some other projects which involve only costs or expenses throughout the useful life except the salvage value if any, at the end of the useful life. The alternatives having this type of cash flows are known as cost alternatives. Thus while comparing mutually exclusive cost alternatives, the alternative showing minimum negative cash flow is generally selected. Example for such type alternatives includes construction of a government funded national highway stretch between two regions. For this project there will be different feasible alternatives depending upon length of the stretch, type of pavement, related environmental, social and regulatory aspects etc. Each alternative will have its initial cost of construction, annual repair and maintenance cost and some major repair cost if any, at some future point of time. The alternative that will exhibit lowest cost will be selected for the construction of the highwaystretch.

The differences in different parameters namely initial capital investment, annual operation cost, annually generated revenue, expected salvage value, useful life, magnitude of output and its quality, performance and operational characteristics etc. may exist among the mutually exclusive alternatives. Thus the economic analysis of the mutually exclusive alternatives is generally carried out on the similar or equivalent basis since each of the feasible alternatives will meet the desired requirements of the project, if selected.

The economic comparison of mutually exclusive alternatives can be carried out by different equivalent worth methods namely present worth method, future worth method and annual worth method. In these methods all the cash flows i.e. cash outflows and cash inflows are converted into equivalent present worth, future worth or annual worth considering the time value of money at a given interest rate per interest period.

Comparison of alternatives by present worth method:

In the present worth method for comparison of mutually exclusive alternatives, the future amounts i.e. expenditures and incomes occurring at future periods of time are converted into equivalent present worth values at a certain rate of interest per interest period and are added to present worth occurring at „0" time. The converted equivalent present worth values are always less than the respective future amounts since the rate of interest is normally greater than zero. The cash flow of the mutually exclusive alternatives may consist of future expenditures and incomes in different forms namely randomly placed single amounts, uniform amount series commencing from end of year 1, randomly placed uniform amount series i.e. commencing at time period other than end of year 1, positive and negative uniform gradient series starting either from end of year 1 or at different time periods and geometric gradient series etc. The different compound interest factors namely single payment present worth factor, uniform series present worth factor and present worth factors for arithmetic and geometric gradient series etc. will be used to convert the respective future amounts to the equivalent present worth values for different alternatives.

The methodology for the comparison of mutually exclusive alternatives by the present worth method depends upon the magnitude of useful lives of the alternatives. There are two cases; a) the useful lives of alternatives are equal and b) the useful lives of alternatives are not equal. The alternatives having equal useful lives are designated as equal life span alternatives whereas the alternatives having unequal life spans are referred as different life span alternatives.

a) Equal life span alternatives

The comparison of mutually exclusive alternatives having equal life spans by present worth method is comparatively simpler than those having different life spans. In case of equal life span mutually exclusive alternatives, the future amounts as already stated are converted into the equivalent present worth values and are added to the present worth occurring at time zero. Then the alternative that exhibits maximum positive equivalent present worth or minimum negative equivalent present worth is selected from the considered feasible alternatives.

a) Different life span alternatives

In case of mutually exclusive alternatives, those have different life spans; the comparison is generally made over the same number of years i.e. a common study period. This is because; the comparison of the mutually exclusive alternatives over same period of time is required for unbiased economic evaluation of the alternatives. If the comparison of the alternatives is not made over the same life span, then the cost alternative having shorter life span will result in lower equivalent present worth i.e. lower cost than the cost alternative having longer life span. Because in this case, the cost of the short span alternative is considered only for a shorter period of time, even though this alternative may not be economical. In case of mutually exclusive investment alternatives, the alternative with longer life span will result in higher equivalent present worth i.e. higher positive equivalent worth, as the costs, revenues, savings through reduced costs is considered over a longer period of time than the alternative with shorter life span. Thus in order to minimize the effect of such kind of discrepancy on the selection of best alternative from the considered feasible alternatives, the comparison is made over the same lifespan.

The two approaches used for economic comparison of different life span alternatives are as follows;

i) Comparison of mutually exclusive alternatives over a time period that is equal to least common multiple (LCM) of the individual lifespans

ii) Comparison of mutually exclusive alternatives over a study period which is not necessarily equal to the life span of any of thealternatives.

In the first approach the comparison is made over a time period equal to the least common multiple of the life spans of mutually exclusive alternatives. The cash flow of the alternatives i.e. cash flow of the first cycle is repeated and the number of repetitions depends upon the value of least common multiple of life spans between the mutually exclusive alternatives. It may be noted here that the cash flow i.e. all the costs and revenues of the alternatives in the successive cycle will be exactly same as that in the first cycle.Forexampleiftherearetwoalternativeswithusefullivesof4yearsand5years.

Then the alternatives will compared over a period of 20 years (least common multiple of life spans) at the given rate of interest per year. Thus the cash flow of the alternative having the life span of 4 years will be repeated 5 times including the first cycle whereas the cash flow of the alternative with life span of 5 years will be repeated 4times including the first cycle. After that the most economical alternative will be selected. Taking another example, there are two alternatives with life spans of 5 years and 10 years. In this case the alternatives will be compared over a period of 10 years (LCM). Thus the alternative with life span of 5 years will be analyzed for 2 cycles whereas the alternative with 10 year life span will be analyzed for one cycle only at the given rate of interest peryear.

In the second approach, a study period is selected over which the economic comparison of mutually exclusive alternatives is carried out. The length of the study period will depend on the overall benefit of the project i.e. it may be shorter or longer (as compared to useful lives of the individual alternatives) depending upon the short-term or long-term benefits as desired for the project. Thus the cash flows of the alternatives occurring during the study period are only considered for the economic comparison. However if any alternative possesses salvage value at the end of its useful life and that occurs after the study period, then its equivalent value must be included in the economic analysis. The values of equivalent present worth of the mutually exclusive alternatives are calculated over the selected study period and the alternative showing maximum positive equivalent present worth or minimum negative equivalent present worth isselected.

1. Present worth method:-

Now some examples showing the use of present worth method for comparison of mutually exclusive alternatives are presented. First the comparison of equal life span mutually exclusive alternatives by present worth method will be illustrated followed by comparison of different life span alternatives. The following examples are formulated only to demonstrate the use of different methods for comparison of alternatives. The values of different cost and incomes mentioned in the examples are not the actual ones pertaining to a particular item. In addition it may also be noted here that the cash flow diagrams have been drawn not to the scale. These are merely graphical representations.

Example -1

There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.3,00,000, Annual operating and maintenance cost = Rs.20,000, Expected salvage value = Rs.1,25,000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.2,00,000, Annual operating and maintenance cost = Rs.35,000, Expected salvage value = Rs.70,000, Useful life = 5 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

Solution:

Since both alternatives have the same life span i.e. 5years, the present worth of the alternatives will be compared over a period of 5 years. The cash flow diagram of Alternative-1 is shown in Fig. 2.1.

As already mentioned Module-1, the cash outflows i.e. costs or expenditures are represented by vertically downward arrows whereas the cash inflows i.e. revenue or income are represented by vertically upward arrows. The same convention is adopted here.

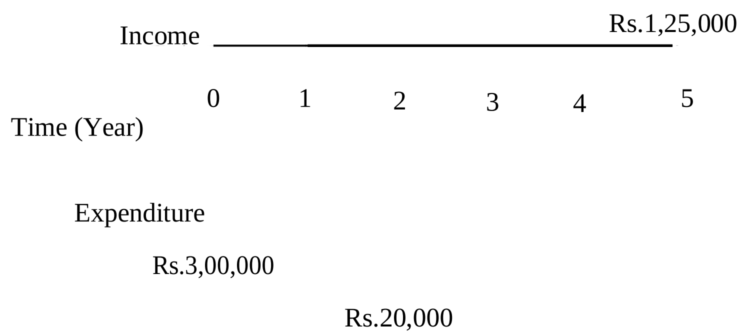


Fig. 2.1 Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows;

The initial cost, P = Rs.3,00,000 (cash outflow),

Annual operating and maintenance cost, A = Rs.20,000 (cash outflow),

Salvage value, F = Rs.1,25,000 (cash inflow).

$PW_1 = -3,00,000 - 20,000(P/A, i, n) + 1,25,000(P/F, i, n)$

$PW_1 = -3,00,000 - 20,000(P/A, 10\%, 5) + 1,25,000(P/F, 10\%, 5)$

Now putting the mathematical expressions of different compound interest factors (as mentioned in Module-1) in the above expression for PW_1 (in Rs.) results in the following;

$$PW_1 = -3,00,000 - 20,000\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + 1,25,000\left[\frac{1}{(1+i)^n}\right]$$

$$PW_1 = -3,00,000 - 20,000\left[\frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5}\right] + 1,25,000\left[\frac{1}{(1+0.1)^5}\right]$$

$$PW_1 = -3,00,000 - 20,000 \times 3.7908 + 1,25,000 \times 0.6209$$

$PW_1 = -3,00,000 - 75,816 + 77,613$

PW_1 = - Rs.2,98,203

The cash flow diagram of Alternative-2 is shown in Fig. 2.2.

Rs.70,000

5

Time (Year)

Rs.35,000

Fig. 2.2 Cash flow diagram ofAlternative-2

Now the equivalent present worth of Alternative-2 i.e. PW_2(in Rs.) is calculated as follows;

The initial cost, P = Rs.2,00,000 (cash outflow),

Annual operating and maintenance cost, A = Rs.35,000 (cash outflow),

Salvage value, F = Rs.70,000 (cash inflow).

PW_2= - 2,00,000 – 35,000(P/A, i, n) + 70,000(P/F, i, n)

PW_2= - 2,00,000 – 35,000(P/A, 10%, 5) + 70,000(P/F, 10%, 5)

$$PW_2 = -2,00,000 - 35,000\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + 70,000\left[\frac{1}{(1+i)^n}\right]$$

$$PW_2 = -2,00,000 - 35,000\left[\frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5}\right] + 70,000\left[\frac{1}{(1+0.1)^5}\right]$$

$$PW_2 = -2,00,000 - 35,000 \times 3.7908 + 70,000 \times 0.6209$$

PW_2= - 2,00,000 – 1,32,678 + 43,463

PW_2= - Rs.2,89,215

Comparing the equivalent present worth of both the alternatives, it is observed that Alternative-2 will be selected as it shows lower negative equivalent present worth compared to Alternative-1 at the interest rate of 10% per year.

The equivalent present worth of both the alternatives can also be calculated by using the values of compound interest factors from interest tables. The equivalent present worth of Alternative-1 i.e. $PW_1$is calculated as follows;

$PW_1 = -\ 3{,}00{,}000 - 20{,}000(P/A,\ i,\ n) + 1{,}25{,}000(P/F,\ i,\ n)$

PW_1= - 3,00,000 – 20,000(*P/A, 10%, 5*) + 1,25,000(*P/F, 10%, 5*)

The values of compound interest factors i.e. *(P/A, i, n)* and *(P/F, i, n)* can be obntained from the interest tables (discrete compounding) available in texts cited in the list of references [1, 7, 14]. Now referring to the interest table for 10% interest rate, the values of compound interest factors i.e. *(P/A, 10%, 5)* and *(P/F, 10%, 5)* at interest rate („*i'*) of 10% and for interest period („*n'*) of 5 years are obtained at the intersection of these factors and interest period „*n*" equal to 5 i.e. the values are obtained from *P/A* columnand *P/F* column at „*n*" equal to 5 from the interest table (discrete compounding) correspondingto10%interestrate.Theobtainedvaluesof*(P/A,10%,5)*and*(P/F,10%,*
5) are 3.7908 and 0.6209 respectively (same as those obtained using mathematical expressions of these factors).

Now putting the values of compound interest factors in the above expression, the equivalent present worth of Alternative-1 i.e. $PW_1$is calculated as follows;

$$PW_1 = -3,00,000 - 20,000 \times 3.7908 + 1,25,000 \times 0.6209$$

PW_1= - 3,00,000 -75,816 + 77,613

PW_1= - Rs.2,98,203

Now the calculation of equivalent present worth of Alternative-2 i.e. PW_2(in Rs.) is presented below.

PW_2= - 2,00,000 – 35,000(*P/A, i, n*) + 70,000(*P/F, i, n*)

PW_2= - 2,00,000 – 35,000(*P/A, 10%, 5*) + 70,000(*P/F, 10%, 5*)

Now putting the values of compound interest factors in the above expression (same as above) the equivalent present worth of Alternative-2 i.e. $PW_2$is calculated as follows;

$$PW_2 = -2,00,000 - 35,000 \times 3.7908 + 70,000 \times 0.6209$$

PW_2= - 2,00,000 – 1,32,678 + 43,463

PW_2= - Rs.2,89,215

It may be noted that in the above example only cost components and the salvage value of the alternatives were considered for comparison. In the next example, same problem as mentioned in Example-1 will be discussed by taking into account the annual revenues of the alternatives.

Example -2

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by concrete mixer) from Alternative-1 and Alternative-2 are Rs.50000 and Rs.45000 respectively. Compute the equivalent present worth of the alternatives at the same rate of interest as in Example-1 i.e. 10% per year and find out the economical alternative.

Solution:

The cash flow diagram of Alternative-1 is shown in Fig. 2.3.

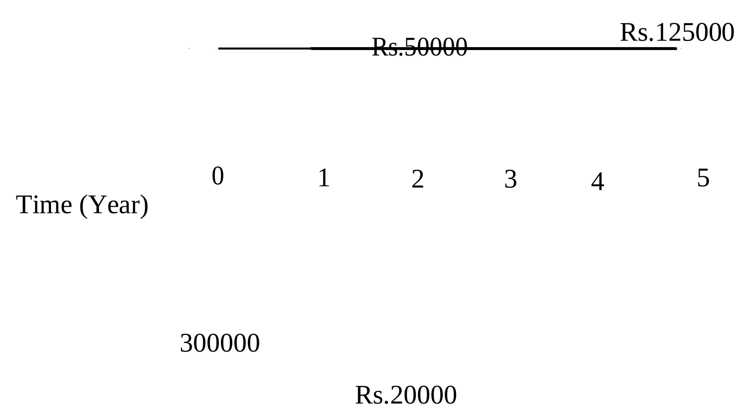


Fig. 2.3 Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 is calculated as follows;

PW_1= - 300000 - 20000(P/A, i, n) + 50000(P/A, i, n) + 125000(P/F, i, n)

PW_1= - 300000 - 20000(P/A, 10%, 5) + 50000(P/A, 10%, 5) + 125000(P/F, 10%, 5)

PW_1= - 300000 + (50000 – 20000) (P/A, 10%, 5) + 125000(P/F, 10%, 5)

PW_1= - 300000 + 30000(P/A, 10%, 5) + 125000(P/F, 10%, 5)

$$PW_1 = -300000 + 30000\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + 125000\left[\frac{1}{(1+i)^n}\right]$$

$$PW_1 = -300000 + 30000\left[\frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5}\right] + 125000\left[\frac{1}{(1+0.1)^5}\right]$$

$PW_1 = -300000 + 30000 \times 3.7908 + 125000 \times 0.6209$

$PW_1 = -300000 + 113724 + 77613$

***PW*₁= - Rs.108663**

The cash flow diagram of Alternative-2 is shown in Fig. 2.4.

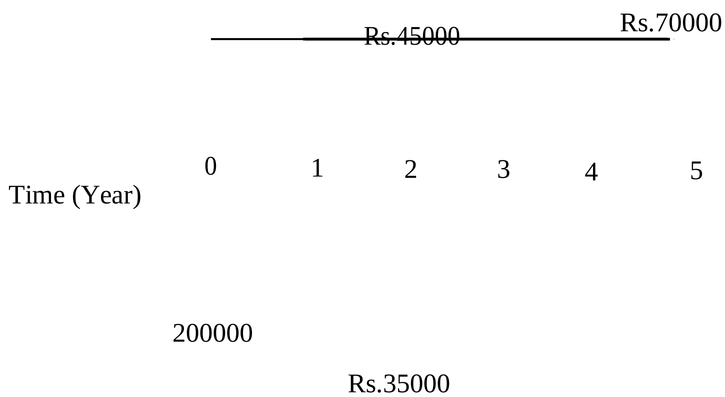


Fig. 2.4 Cash flow diagram of Alternative-2

Fig. 2.4 Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative-2 i.e. PW_2(in Rs.) is calculated as follows;

PW_2= - 200000 - 35000(P/A, i, n) + 45000(P/A, i, n) + 70000(P/F, i, n)

PW_2= - 200000 - 35000(P/A, 10%, 5) + 45000(P/A, 10%, 5) + 70000(P/F, 10%, 5)

PW_2= - 200000 + (45000 – 35000) (P/A, 10%, 5) + 70000(P/F, 10%, 5)

PW_2= - 200000 + 10000(P/A, 10%, 5) + 70000(P/F, 10%, 5)

$$PW_2 = -200000 + 10000\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + 70000\left[\frac{1}{(1+i)^n}\right]$$

$$PW_2 = -200000 + 10000\left[\frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5}\right] + 70000\left[\frac{1}{(1+0.1)^5}\right]$$

$$PW_2 = -200000 + 10000 \times 3.7908 + 70000 \times 0.6209$$

PW_2= - 200000 + 37908 + 43463

***PW*₂= - Rs.118629**

Comparing the equivalent present worth of the both the alternatives, it is observed that Alternative-1 will be selected as it shows lower cost compared to Alternative-2. The annual revenue to be generated by the alternatives made the difference as compared to the outcome obtained in Example-1.

When there are more than two alternatives for the selection of the best economical alternative by present worth method, the same procedure as mentioned earlier for the case of two alternatives is followed and illustrated in the next example.

Example -3

A construction contractor has three options to purchase a dump truck for transportation and dumping of soil at a construction site. All the alternatives have the same useful life. The cash flow details of all the alternatives are provided as follows;

Option-1: Initial purchase price = Rs.2500000, Annual operating cost Rs.45000 at the end of 1^{st} year and increasing by Rs.3000 in the subsequent years till the end of useful life, Annual income = Rs.120000, Salvage value = Rs.550000, Useful life = 10years.

Option-2: Initial purchase price = Rs.3000000, Annual operating cost = Rs.30000, Annual income Rs.150000 for first three years and increasing by Rs.5000 in the subsequent years till the end of useful life, Salvage value = Rs.800000, Useful life = 10 years.

Option-3: Initial purchase price = Rs.2700000, Annual operating cost Rs.35000 for first 5 years and increasing by Rs.2000 in the successive years till the end of useful life, Annual income = Rs.140000, Expected salvage value = Rs.650000, Useful life = 10 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 8% per year.

Solution:

The cash flow diagram of Option-1 is shown in Fig. 2.5.

Rs.120000 Rs.550000

Time (Year)

Fig. 2.5 Cash flow diagram of Option-1

Rs.72000

For Option-1, the annual operating cost is in the form of a positive uniform gradient series with gradient starting from end of year „2‟. The operating cost at the end of different years can be split into the uniform base amount of Rs.45000 and the gradient amount in multiples of Rs.3000 as shown in Fig. 2.6.

hown in Fig. 2.6.

Rs.120000 Rs.550000

Time (Year)

Rs.27000

Fig. 2.6 Cash flow diagram of Option-1
with annual operating cost split into uniform base amount and gradient amount

The present worth of the uniform gradient series will be located at the beginning i.e. in year „0‟ i.e. 2 years before the commencement of the uniform gradient.

Now the equivalent present worth (in Rs.) of Option-1 is calculated as follows;

PW_1= - 2500000 - 45000(*P/A, i, n*) - 3000*(P/G, i, n)* + 120000*(P/A, i, n)* + 550000(*P/F, i, n*)

PW_1= - 2500000 - 45000*(P/A, 8%, 10)* - 3000*(P/G, 8%, 10)* + 120000*(P/A, 8%, 10)* + 550000*(P/F, 8%, 10)*

PW_1= - 2500000 + (120000 - 45000) *(P/A, 8%, 10)* - 3000*(P/G, 8%, 10)* + 550000*(P/F, 8%, 10)*

Now putting the values of different compound interest factors (the expressions in terms of *'i'* and *'n'* already stated in Module-1) in the above expression for PW_1 results in the following;

$$PW_1 = -2500000 + 75000 \times 6.7101 - 3000 \times 25.9768 + 550000 \times 0.4632$$

PW_1= - 2500000 + 503258 - 77930 + 254760

PW_1= - Rs.1819912

The cash flow diagram of Option-2 is shown in Fig. 2.7.

Rs.800000

Time (Year)

Rs.30000

Fig. 2.7 Cash flow diagram of Option-2

For Option-2, the annual income is in the form of a positive uniform gradient series with gradient starting from end of year „4‟. The annual income can be split into the uniform base amount of Rs.150000 and the gradient amount in multiples of Rs.5000 starting from end of year „4‟ and is shown in Fig. 2.8.

Rs.800000

Time (Year)

Rs.30000

Fig. 2.8 Cash flow diagram of Option-2 with annual income split into uniform base amount and gradient amount

The equivalent present worth of the gradient series (of the annual income) starting from end of year „4‟ will be located at the end of year „2‟ i.e. 2 years before the start of the gradient. Further the present worth of this amount at beginning i.e. at time „0‟ will be obtained by multiplying the equivalent present worth „P_g' (shown in Fig. 2.8) at the end of year „2‟ (which is a future amount) with the single payment present worth factor *(P/F, i, n)*.

Now the equivalent present worth (in Rs.) of Option-2 is determined as follows;

PW_2= - 3000000 - 30000(*P/A, 8%, 10*) + 150000*(P/A, 8%, 10)* + P_g *(P/F, 8%, 2)* + 800000(*P/F, 8%, 10*)

Now in the above expression, P_g will be replaced by *G (P/G, i, n)* i.e. 5000(*P/G, 8%, 8*).

PW_2= - 3000000 - 30000(*P/A, 8%, 10*) + 150000*(P/A, 8%, 10)* + 5000*(P/G, 8%, 8) (P/F, 8%, 2)* + 800000(*P/F, 8%, 10*)

PW_2= - 3000000 + (150000 - 30000) *(P/A, 8%, 10)* + 5000*(P/G, 8%, 8) (P/F,* *8%, 2) +*

800000(*P/F, 8%, 10*)

Rs.650000

Now putting the values of different compound interest factors in the above expression for

PW_2 results in the following;

$$PW_2 = -3000000 + 120000 \times 6.7101 + 5000 \times 17.8061 \times 0.8573 + 800000 \times 0.4632$$

PW_2= - 3000000 + 805212 + 76326 + 370560

PW_2= - Rs.1747902

The cash flow diagram of Option-3 is shown in Fig. 2.9.

Rs.140000

Time (Year)

Rs.45000

Fig. 2.9 Cash flow diagram ofOption-3

For Option-3, the annual operating cost is in the form of a positive uniform gradient series with gradient starting from end of year „6‟. The annual operating cost can thus be split into the uniform base amount of Rs.35000 and the gradient amount in multiples of Rs.2000 starting from end of year „6‟ (shown in Fig.2.10).

The equivalent present worth of the gradient series for the annual operating cost starting from end of year „6‟ will be located at the end of year „4‟. Further the present worth of this amount at time „0‟ will be determined by multiplying the equivalent present worth „P_g' (shown in Fig. 2.10) at the end of year „4‟ with the single payment present worth factor *(P/F, i, n)*.

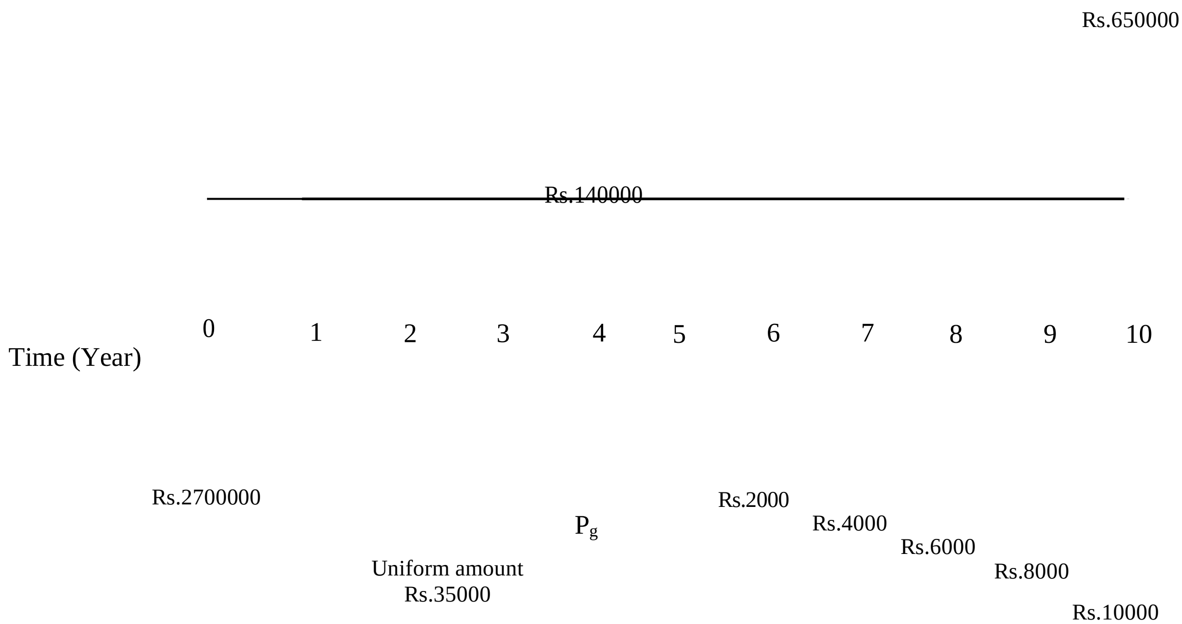


Fig. 2.10 Cash flow diagram of Option-3 with annual operating cost split into uniform base amount and gradient amount

The equivalent present worth (in Rs.) of Option-3 is obtained as follows;

PW_3= - 2700000 - 35000(*P/A, 8%, 10*) - P_g(*P/F, 8%, 4)* + 140000*(P/A, 8%, 10)* + 650000(*P/F, 8%, 10*)

Now in the above expression, P_g will be replaced by *G (P/G, i, n)* i.e. 2000(*P/G, 8%, 6*).

PW_3= - 2700000 - 35000(*P/A, 8%, 10*) - 2000*(P/G, 8%, 6) (P/F, 8%, 4)* + 140000*(P/A, 8%, 10)* + 650000(*P/F, 8%, 10*)

PW_3= - 2700000 + (140000 - 35000) *(P/A, 8%, 10)* - 2000*(P/G, 8%, 6) (P/F, 8%, 4)* + 650000(*P/F, 8%, 10*)

Now putting the values of different compound interest factors in the above expression, the value of PW_3 is givenby;

$$PW_3 = -2700000 + 105000 \times 6.7101 - 2000 \times 10.5233 \times 0.7350 + 650000 \times 0.4632$$

PW_3= - 2700000 + 704561 - 15469 + 301080

***PW*₃= - Rs.1709828**

From the comparison of equivalent present worth of all the three mutually exclusive alternatives, it is observed that Option-3 shows lowest negative equivalent present worth as compared to other options. Thus Option-3 will be selected for the purchase of the dumptruck.

2.Annual worth analysis:

In this method, the mutually exclusive alternatives are compared on the basis of equivalent uniform annual worth. The equivalent uniform annual worth represents the annual equivalent value of all the cash inflows and cash outflows of the alternatives at the given rate of interest per interest period. In this method of comparison, the equivalent uniform annual worth of all expenditures and incomes of the alternatives are determined using different compound interest factors namely capital recovery factor, sinking fund factor and annual worth factors for arithmetic and geometric gradient series etc. Since equivalent uniform annual worth of the alternatives over the useful life are determined, same procedure is followed irrespective of the life spans of the alternatives i.e. whether it is the comparison of equal life span alternatives or that of different life span alternatives. In other words, in case of comparison of different life span alternatives by annual worth method, the comparison is not made over the least common multiple of the life spans as is done in case of present worth and future worth method. The reason is that even if the comparison is made over the least common multiple of years, the equivalent uniform annual worth of the alternative for more than one cycle of cash flow will be exactly same as that of the first cycle provided the cash flow i.e. the costs and incomes of the alternative in the successive cycles is exactly same as that in the first cycle. Thus the comparison is made only for one cycle of cash flow of the alternatives. This serves as one of greater advantages of using this method over other methods of comparison of alternatives. However if the cash flows of the alternatives in the successive cycles are not the same as that in the first cycle, then a study period is selected and then the equivalent uniform annual worth of the cash flows of the alternatives are computed over the study period.

Now the comparison of mutually exclusive alternatives by annual worth method will be illustrated in the following examples. First the data presented in Example-2 will be used for comparison of the alternatives by the annual worth method.

Example -4

There are two alternatives for purchasing a concrete mixer and following are the cash flow details;

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by concrete mixer) from Alternative-1 and Alternative-2 are Rs.50000 and Rs.45000 respectively. Compute the equivalent uniform annual worth of the alternatives at the interest rate of 10% per year and find out the economical alternative.

Solution:

The cash flow diagram of Alternative-1 i.e. Fig. 2.3 is shown here again for ready reference.

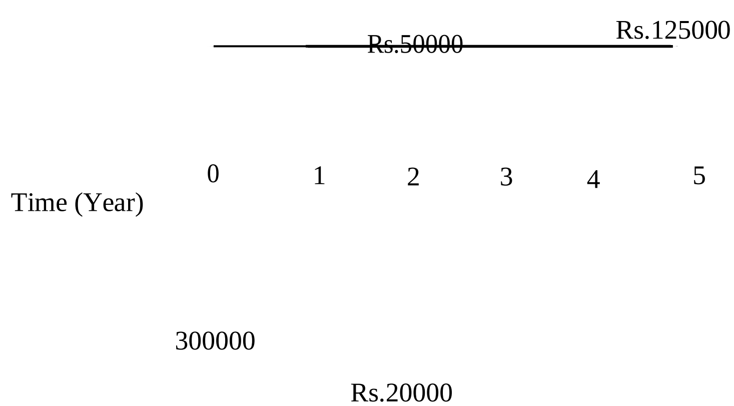


Fig. 2.3 Cash flow diagram of Alternative -1

The equivalent uniform annual worth of Alternative-1 i.e. AW_1 is computed as follows;

$$AW_1 = -300000(A/P, i, n) - 20000 + 50000 + 125000(A/F, i, n)$$

$$AW_1 = -300000(A/P,10\%,5) - 20000 + 50000 + 125000(A/F,10\%,5)$$

Here Rs.20000 and Rs.50000 are annual amounts.

Now putting he values of different compound interest factors ; AW_1

$$= -300000(0.2638) + (50000 - 20000) + 125000(0.1638)$$

$$AW_1 = -79140 + 30000 + 20475$$

***AW_1* = - Rs.28665**

The cash flow diagram of Alternative-2 is shown here again for ready reference.

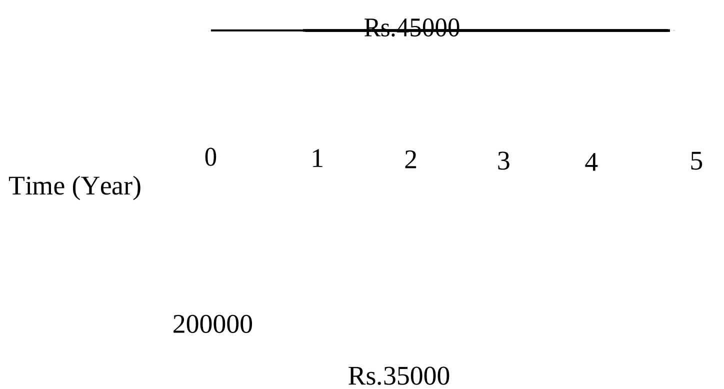


Fig. 2.4 Cash flow diagram of Alternative -2

Now the equivalent uniform annual worth of Alternative-2 i.e. AW_2 is calculated as follows;

$$AW_2 = -200000(A/P,i,n) - 35000 + 45000 + 70000(A/F,i,n)$$

$$AW_2 = -200000(A/P,10\%,5) - 35000 + 45000 + 70000(A/F,10\%,5)$$

For alternative-2, Rs.35000 and Rs.45000 are annual amounts.

Now putting the values of different compound interest factors in the above expression;

$$AW_2 = -200000(0.2638) + (45000 - 35000) + 70000(0.1638)$$

$$AW_2 = -52760 + 10000 + 11466$$

***AW_2* = - Rs.31294**

From this comparison, it is observed that Alternative-1 will be selected as it shows lower negative equivalent uniform annual worth compared to Alternative-2. This outcome is in consistent with the outcome obtained by present worth method in Example-2.

Example -5

A material supply contractor has two options (i.e. from two different manufacturing companies, Company-1 and Company-2) to purchase a tractor for supply of construction materials. The details of cash flow of the two options are given below;

Company-1 Tractor: Initial purchase cost = Rs.2000000, Annual operating cost including labor and maintenance = Rs.50000, Cost of new set of tires to be replaced at the end of year „3‟, year „6‟and year „9‟ = Rs.110000 each, Expected salvage value = Rs.520000, Useful life = 10 years.

Company-2 Tractor: Initial purchase cost = Rs.2200000, Annual operating cost including labor and maintenance = Rs.27000, Cost of new set of tires to be replaced at the end of year „4‟ and year „8‟ = Rs.120000 each, Expected salvage value = Rs.700000, Useful life = 10 years.

Determine which company tractor should be selected on the basis of equivalent uniform annual worth at the interest rate of 12% per year.

Solution:

The cash flow diagram of Company-1 tractor is shown in Fig. 2.19.

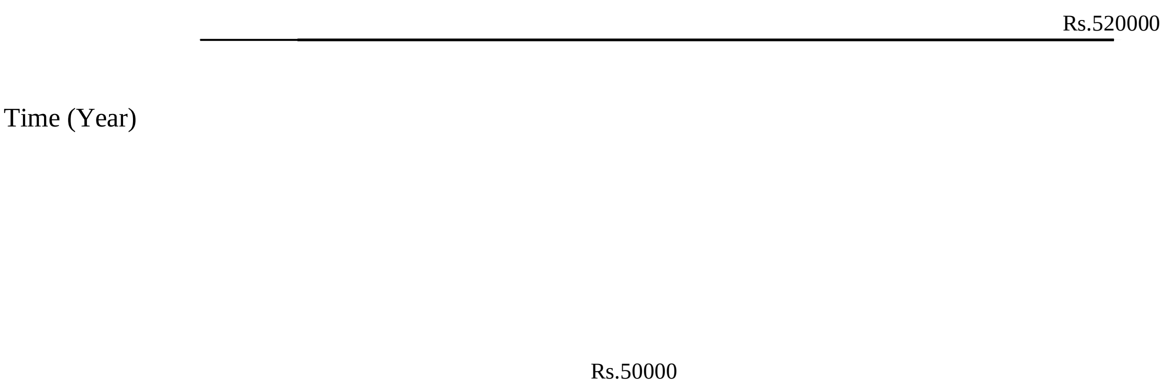


Fig. 2.19 Cash flow diagram of Company-1 Tractor

From the cash flow diagram it is noted that three single amounts i.e. Rs.110000 each are located at the end of year „3‟, year „6‟ and year „9‟. For these amounts, first the equivalent present worth at time „0‟ is determined and then equivalent annual worth of this present worth is computed using the appropriate compound interest factor.

The equivalent uniform annual worth of Company-1 tractor is determined as follows;

$$AW_1 = -2000000(A/P,12\%,10) - 50000 - 110000(P/F,12\%,3)(A/P,12\%,10) - 110000(P/F,12\%,6)(A/P,12\%,10) - 110000(P/F,12\%,9)(A/P,12\%,10) + 520000(A/F,12\%,10)$$

Now putting the values of different compound interest factors in the above expression;

$$AW_1 = -2000000 \times 0.1770 - 50000 - 110000 \times 0.7118 \times 0.1770 - 110000 \times 0.5066 \times 0.1770 - 110000 \times 0.3606 \times 0.1770 + 520000 \times 0.0570$$

$$AW_1 = -354000 - 50000 - 13859 - 9864 - 7021 + 29640$$

***AW*₁= - 405104**

Time (Year)

Rs.27000

Fig. 2.20 Cash flow diagram of Company-2 Tractor

From Company-2 tractor, two single amounts i.e. Rs.120000 each are located at the end of year „4‟, and year „8‟. Similar to first alternative, first the equivalent present worth at time„0‟oftheseamountsisdeterminedandthenequivalentannualworthiscomputed.

The equivalent uniform annual worth of Company-2 tractor is computed as follows;

$$AW_2 = -2200000(A/P,12\%,10) - 27000 - [120000(P/F,12\%,4)(A/P,12\%,10)] - [120000(P/F,12\%,8)(A/P,12\%,10)] + 700000(A/F,12\%,10)$$

Now putting the values of different compound interest factors in the above expression;

$$AW_2 = -2200000 \times 0.1770 - 27000 - 120000 \times 0.6355 \times 0.1770 - 120000 \times 0.4039 \times 0.1770 + 700000 \times 0.0570$$

$$AW_2 = -389400 - 27000 - 13498 - 8579 + 39900$$

AW₂= - 398577

From the above comparison, it is observed that Company-2 Tractor shows lower negative equivalent uniform annual worth as compared to Company-1 tractor. Thus the contractor should select Company-2 Tractor for purchase.

3.Future worth analysis:

In the future worth method for comparison of mutually exclusive alternatives, the equivalent future worth (i.e. value at the end of the useful lives of alternatives) of all the expenditures and incomes occurring at different periods of time are determined at the given interest rate per interest period. As already mentioned, the cash flow of the mutually exclusive alternatives may consist of expenditures and incomes in different forms. Therefore the equivalent future worth of these expenditures and incomes will be determined using different compound interest factors namely single payment compound amount factor, uniform series compound amount factor and future worth factors for arithmetic and geometric gradient seriesetc.

The use of future worth method for comparison of mutually exclusive alternatives will be illustrated in the following examples. Similar to present worth method, first the comparison of equal life span alternatives by future worth method will be illustrated followed by comparison of different life span alternatives. Some of the examples already worked out by the present worth method will be illustrated using the future worth method in addition to some other examples.

Example -6 (Using data of Example-1)

There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

Using future worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

Solution:

The future worth of the mutually exclusive alternatives will be compared over a periodof 5 years. The equivalent future worth of the alternatives can be obtained either by multiplying the equivalent present worth of each alternative already obtained by present worth method with the single payment compound amount factor or determining the future worth of expenditures and incomes individually and adding them to get the equivalent future worth of each alternative.

The equivalent future worth of Alternative-1 is obtained as follows;

$$FW_1 = PW_1 \left(F/P, i, n\right)$$

PW_1 is the equivalent present worth of Alternative-1 which is equal to - Rs.298203 (referring to Example-1). *(F/P, i, n)* is the single payment compound amount factor.

$$FW_1 = -298203\left(F/P, 10\%, 5\right)$$

Now putting the value of single payment compound amount factor in the above expression;

$$FW_1 = -298203 \times 1.6105$$

FW_1 = -Rs.480256

The equivalent future worth of Alternative-1 can also be determined in the following manner (Referring to cash flow diagram of Alternative-1, Fig. 2.1);

$$FW_1 = -300000\left(F/P, 10\%, 5\right) - 20000\left(F/A, 10\%, 5\right) + 125000$$

Now putting the values of different compound interest factors in the above expression;

$$FW_1 = -300000 \times 1.6105 - 20000 \times 6.1051 + 125000$$

$$FW_1 = -483150 - 122102 + 125000$$

FW_1 = -Rs.480252

Now it can be seen that the calculated future worth of Alternative-1 by both ways is same. The minor difference between the values is due to the effect of decimal points in thecalculations.

The equivalent future worth of Alternative-2 is calculated as follows;

$$FW_2 = PW_2 \left(F/P, i, n\right)$$

PW_2 is the equivalent present worth of Alternative-2 which is equal to - Rs.289215 (referring to Example-1).

$$FW_2 = -289215\left(F/P, 10\%, 5\right)$$

Now putting the value of single payment compound amount factor in the above expression;

$$FW_2 = -289215(1.6105)$$

***FW₂*= -Rs.465781**

The equivalent future worth of Alternative-2 can also be determined in the same manner as in case of Alternative-1 and is presented as follows (Referring to cash flow diagram of Alternative-2, Fig. 2.2);

$$FW_2 = -200000(F/P,10\%,5) - 35000(F/A,10\%,5) + 70000$$

Now putting the values of different compound interest factors in the above expression;

$$FW_2 = -200000(1.6105) - 35000(6.1051) + 70000$$

$$FW_2 = -322100 - 213679 + 70000$$

FW_2= -Rs.465779

Thus the future worth of Alternative-2 obtained by both methods is same. In this case also the minor difference between the values is due to the effect of the decimal points in thecalculations.

Comparing the equivalent future worth of the both the alternatives, it is observed that Alternative-2 will be selected as it shows lower negative equivalent future worth as compared to Alternative-1. This outcome of the comparison of the alternatives by future worth method is same as that obtained from the present worth method (Example-1). This is due to the equivalency relationship between present worth and future worth through compound interest factors at the given rate of interest per interest period.

Example -7

There are two alternatives for a construction firm to purchase a road roller which will be used for the construction of a highway section. The cash flow details of the alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.1500000, Annual operating cost = Rs.35000 starting from the end of year „2" (negligible in the first year) till the end of useful life, Annual revenue to be generated = Rs.340000 for first 4 years and then Rs.320000 afterwards till the end of useful life, Expected salvage value = Rs.430000, Useful life = 8 years.

Alternative-2: Initial purchase cost = Rs.1800000, Annual operating cost = Rs.25000, Annual revenue to be generated = Rs.365000, Expected salvage value = Rs.550000, Useful life = 8 years.

Find out the most economical alternative on the basis of equivalent future worth at the interest rate of 9.5% per year.

Solution:

The cash flow diagram of Alternative-1 is shown in Fig. 2.17.

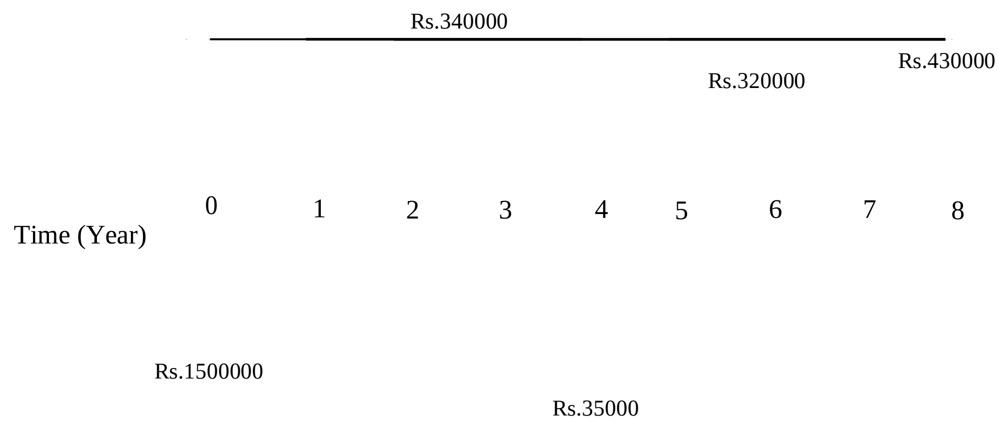


Fig. 2.17 Cash flow diagram of Alternative-1

From Fig. 2.17, it is observed that there are two uniform amount series for the annual incomei.e.firstserieswithRs.340000fromendofyear„1‟tillendofyear„4‟andsecond one with Rs.320000 from end of year „5‟ till end of year „8‟. For the first series, the equivalent present worth at time „0‟ will be calculated first and then it will be multiplied with single payment compound amount factor i.e. *(F/P, i, n)* to calculate its equivalent future worth. For the second uniform series with Rs.320000, the future worth will be calculated by multiplying the uniform amount i.e. Rs.320000 with uniform series compound amount factor by taking the appropriate „*n'* i.e. number ofyears.

The annual operating cost is in the form of a uniform amount series, which starts from end of year „2‟ till the end of useful life i.e. the uniform amount series is shifted by one year.

The equivalent future worth of the Alternative-1 i.e.$FW_1$is computed as follows;

$$FW_1 = -1500000(F/P, 9.5\%, 8) - 35000(F/A, 9.5\%, 7) + 340000(P/A, 9.5\%, 4)(F/$$

$P,9.5\%,8) + 320000(F/A,9.5\%,4) + 430000$

Putting the values of different compound interest factors in the above expression results in thefollowing;

$$FW_1 = -1500000(2.0669) - 35000(9.3426) + 340000(3.2045)(2.0669) + 320000(4.6070) + 430000$$

$$FW_1 = -3100350 - 326991 + 2251950 + 1474240 + 430000$$

FW_1 = Rs.728849

The cash flow diagram of Alternative-2 is shown in Fig. 2.18.

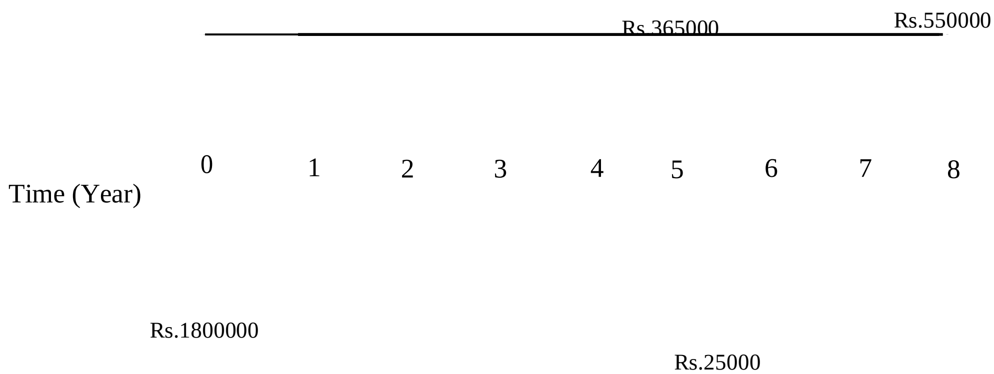


Fig. 2.18 Cash flow diagram of Alternative-2

The equivalent future worth of the Alternative-2 i.e.$FW_2$is calculated as follows;

$$FW_2 = -1800000(F/P,9.5\%,8) - 25000(F/A,9.5\%,8) + 365000(F/A,9.5\%,8) + 550000$$

Now putting the values of different compound interest factors in the above expression results in the following;

$$FW_2 = -1800000(2.0669) + (365000 - 25000)(11.2302) + 550000$$

$$FW_2 = -3720420 + 3818268 + 550000$$

FW_2 = Rs.647848

Comparing the equivalent future worth of the alternatives, it is observed that Alternative-1 shows higher positive equivalent future worth as compared to Alternative-2. Thus Alternative-1 will be selected for purchase of the road roller.

4.Rate of return Analysis (ROR):-

The rate of return technique is one of the methods used in selecting an alternative for a project. In this method, the interest rate per interest period is determined, which equates the equivalent worth (either present worth, future worth or annual worth) of cash outflows (i.e. costs or expenditures) to that of cash inflows (i.e. incomes or revenues) of an alternative. The rate of return is also known by other names namely internal rate of return (IRR), profitability index etc. It is basically the interest rate on the unrecovered balance of an investment which becomes zero at the end of the useful life or the study period. In the following lectures, the rate of return is denoted by"i_r".

Using present worth, the equation for rate of return can be written as follows;

$PW_C = PW_I$ ……………………………… (2.1)

PW_C= Present worth of cash outflows (cost or expenditure)

PW_I= Present worth of cash inflows (income or revenue)

As already stated in earlier lectures, cost or expenditures are considered as negative cash flows whereas income or revenues are considered as positive cash flows.

Equation (2.1) can be rewritten as;

$0 = -PW_C + PW_I$ ………………………….. (2.2)

In the above equation the net present worth is zero.

Now putting the expressions for present worth of cash outflows and that of cash inflows in equation (2.1) results in the followingexpression;

$P_o + F_C(P/F, i_r, n) + A_C(P/A, i_r, n) = F_I(P/F, i_r, n) + A_I(P/A, i_r, n)$ (2.3)

On left hand side of the above equation, $P_o$is the initial cost at time zero and F_C(single amount) and A_C(uniform amount series) are the expenditures occurring at future period of time. Similarly on the right hand side of the equation, F_I(single amount) and A_I(uniform amount series) are the incomes or revenues occurring at future period of time. The value of interest period „n‟ will vary depending upon the occurrence of the future amounts (either expenditure orincome).

Equation (2.3) can be rewritten as follows;

$$0 = -\left[P_o + F_C\left(P/F, i_r, n\right) + A_C\left(P/A, i_r, n\right)\right] + \left[F_I\left(P/F, i_r, n\right) + A_I\left(P/A, i_r, n\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

The value of rate of return „i_r' can be calculated by solving the above equation. The equation (2.4) can be solved either manually through trial and error process or using Microsoft Excel spreadsheet. The first method i.e. trial and error process for determination of the rate of return consumes more time whereas the second method is faster. However the trial and error method gives a clear understanding of the analysis of calculation for the rate of return. Similar to equivalent present worth, the rate of return can also be determined by finding out the interest rate at which the net future worth or net annual worth iszero.

After determination of the rate of return for a given alternative, it is compared with minimum attractive rate of return (MARR) to find out the acceptability of this alternative for the project. If the rate of return i.e. $i_r$is greater than or equal to MARR, then the alternative will be selected or else it will not be selected. The MARR is the minimum rate of return from the investment, which is acceptable. In other words it is the minimum rate of return below which the investment alternatives are economically not acceptable. The minimum attractive rate of return (MARR) serves as an important criteria while selecting a single alternative or comparing mutually exclusive alternatives whenever the investments are made. For an organization, it is governed by various parameters namely availability of financially viable projects, amount of fund available for investment along with the associated risk, and type of organization (i.e. government, public sector, private sector etc.).

The difference between equivalent worth methods (present worth method/future worth method/annual worth method) and rate of return method is that; in case of former, the equivalent worth of the cash inflows and cash outflows are determined at MARR whereas in case of latter, a rate is determined which equates the equivalent worth of cash inflows to that of the cash outflows and the resulting rate is compared against MARR. The rate of return and MARR are expressed in terms of percentage per period i.e. mostly percentage per year.

In the following example, the illustration of the procedure for determination of rate of return for an alternative is presented.

Example -8

A construction firm is planning to invest Rs.800000 for the purchase of a construction equipment which will generate a net profit of Rs.140000 per year after deducting the annual operating and maintenance cost. The useful life of the equipment is 10 years and the expected salvage value of the equipment at the end of 10 years is Rs.200000. Compute the rate of return using trial and error method based on present worth, if the construction firm‟s minimum attractive rate of return (MARR) is 10% peryear.

Solution:

The cash flow diagram of the construction equipment is shown in Fig. 2.29.

Rs.140000 Rs.200000

Time (Year)

Rs.800000

Fig. 2.29 Cash flow diagram of the construction equipment

For determination of rate of return „i_r‟ of the construction equipment, first the equation for net present worth of cash inflows and cash outflows is equated to zero. Then using the trialanderrormethodthevalueof„i_r‟isdetermined.Thenetpresentworthofcash inflows and cash outflows of the construction equipment is given by the following expression. $PW = -800000 + 140000(P/A, i_r, 10) + 200000(P/F, i_r, 10)$

For determining the value of „i_r‟ the net present worth is equated to zero.

$$0 = -800000 + 140000(P/A, i_r, 10) + 200000(P/F, i_r, 10)$$

Now the above equation will be solved through trial and error process to find out the value of i_r. Basically a positive value and a negative value of the net present worth will be determined at rate of return values close to the actual one and then by linear interpolation between these two values, the actual rate of return will be calculated. For finding out the rate of return values (close to the actual one), those will give a positive value and a negative value of net present worth, one has to carry out a number of trial calculations at various values of i_r.

Since MARR is 10%, first assume a value of i_r equal to 8% and compute the net present worth. Now putting the values of different compound interest factors in the expression for net present worth at i_r equal to 8% results in the following;

$$PW = -800000 + 140000(P/A,8\%,10) + 200000(P/F,8\%,10)$$

$$PW = -800000 + 140000(6.7101) + 200000(0.4632)$$

***PW* = Rs.232054**

The above calculated net present worth at i_r equal to 8% is greater than zero, now assume a higher value of i_r i.e. 12% for the next trial and compute the net present worth.

$$PW = -800000 + 140000(P/A,12\%,10) + 200000(P/F,12\%,10)$$

$$PW = -800000 + 140000(5.6502) + 200000(0.3220)$$

***PW* = Rs.55428**

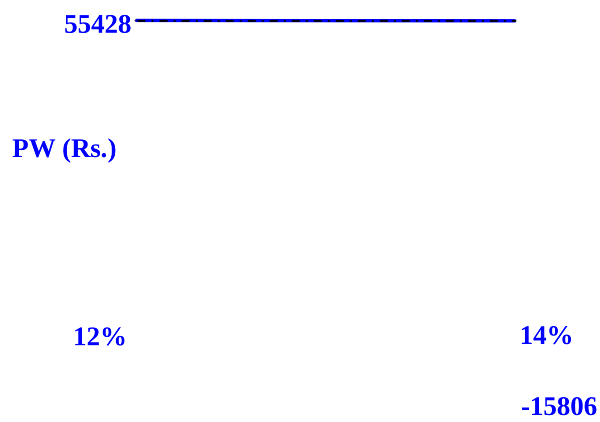
As observed from this calculation, the net present worth is decreased at higher value of i_r. Thus for getting a negative value of net present worth, assume further higher value of i_r than the previous trial and take 14% for the next trial and determine the net present worth.

$$PW = -800000 + 140000(P/A,14\%,10) + 200000(P/F,14\%,10)$$

$$PW = -800000 + 140000(5.2161) + 200000(0.2697)$$

***PW* = -Rs.15806**

Since a negative value of net present worth at i_r equal to 14% is obtained (as above), the actual value of rate of return is less than 14%. The actual rate of return is now obtained by doing linear interpolation either between 8% and 14% or between 12% and 14%. However for obtaining a more accurate value of rate of return, the linear interpolation is carried out between 12% and 14% and is given asfollows;



PW = Rs.55428 at i_r = 12%

PW = - Rs.15806 at i_r = 14%

$$\frac{55428-(-15806)}{14\%-12\%}=\frac{55428-0}{i_r-12\%}$$

On solving the above expression, the value of i_r is found to be 13.55% per year which is greater than MARR (10%). Now using the using Microsoft Excel spreadsheet and entering year-wise cash inflows and cash out flows, the value of rate of return is found to be 13.53% (using the function „IRR"). However this minor difference in the value of i_r obtained from both the methods can be minimized by finding out the net present worth at narrow range of interest rate values and carrying out linear interpolation between these values (trial and error method) to find out the more precise value close to the actual rate ofreturn.

The net present worth of the construction equipment at MARR i.e. 10% is given by;

$$PW = -800000 + 140000(P/A,10\%,10) + 200000(P/F,10\%,10)$$

$$PW = -800000 + 140000(6.1446) + 200000(0.3855)$$

***PW* = Rs.137344 at MARR (10%)**

The net present worth of the construction equipment at MARR is greater than zero and the rate of return is greater than MARR. Thus the purchase of the construction equipment is economically justified. It may be noted here that when the equivalent worth of an investment is greater than zero at interest rate equal to MARR, then the rate of return of the investment is greater thanMARR.

The rate of return „i_r" can also be determined by equating the net annual worth to zero. For the above construction equipment, the net equivalent annual worth at different values of i_r are calculated as follows;

At i_r = 12%

$$AW = -800000(A/P,12\%,10) + 140000 + 200000(A/F,12\%,10)$$

$$AW = -800000(0.1770) + 140000 + 200000(0.0570)$$

***AW* = Rs.9800**

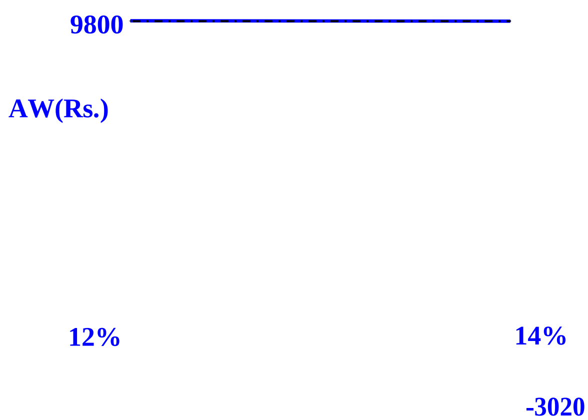
At i_r = 14%

$$AW = -800000(A/P,14\%,10) + 140000 + 200000(A/F,14\%,10)$$

$$AW = -800000(0.1917) + 140000 + 200000(0.0517)$$

***AW* = -Rs.3020**

Now carrying out the linear interpolation between 12% and 14%;



AW = Rs.9800 at i_r = 12%

AW = - Rs.3020 at i_r =14%

$$\frac{9800 - (-3020)}{14\% - 12\%} = \frac{9800 - 0}{i_r - 12\%}$$

On solving the above expression, the value of i_r is found to be 13.52% per year. The minor difference in the values of i_r from present worth and annual worth methods is due to the effect of decimal points in the calculations. Similar to present worth and annual worth methods, the rate of return „i_r" can also be determined by equating the net future worth tozero.

From the above example, a unique value of rate of return was obtained for the construction equipment (on the basis of its cash inflow and cash outflow). This is due to the fact that, there was only one sign change in the cash flows i.e. minus sign at time zero for the cash outflow followed by plus sign for cash inflows during the remaining periods. However in some cases, depending upon the cash flow it is possible to get multiple values of rate of return, those satisfy the rate of return equation of the equivalent worth of cash inflows and cash out flows. This may happen due to more than one sign change in the cash flows e.g. cash outflow (negative) at beginning (time zero) followed cash inflows (positive) at end of year 1 and 2 and then cash outflow (negative) at end of year 3 etc. Thus while selecting an alternative that has multiple values of rate of return (depending on the cash flow), other method of economic evaluation may be adopted to find out the economical suitability of thealternative.

5. Incremental Rate of return:-

When the best alternative (economically suitable) is to be selected from two or more mutually exclusive alternatives on the basis of rate of return analysis, the incremental investment analysis is used. In incremental rate of return method, the alternative with larger investment is selected, provided the incremental (extra) investment over the lower investment alternative produces a rate of return that is greater than or equal to MARR. In other words if the additional benefits i.e. increased productivity, increased income, reduced operating expenditure etc. achieved at the expense of extra investment (associated with larger investment alternative) are more than that could have been obtained from the investment of same amount at MARR elsewhere by the organization, then this additional capital should beinvested.

In incremental rate of return method, the economically acceptable lower investment alternative is considered as the base alternative against which the higher investment alternative is compared. The cash flow of higher investment alternative is considered equal to the cash flow of lower investment alternative plus the incremental cash flow i.e. difference in cash flow between the higher investment and lower investment alternatives.

When using rate of return method for comparing two or more mutually exclusive alternatives, the analysis must be done correctly, otherwise it may lead to incorrect ranking of the alternatives. However this problem is avoided in incremental investment rate of return analysis. In this technique, the individual rate of return values on total cash flow of the mutually exclusive alternatives are not compared against each other rather the rate of return (or IRR) of the mutually exclusive alternatives or the rate of return of the incremental investment is compared against MARR.

The procedures for comparison of mutually exclusive cost alternatives and that of mutually exclusive investment alternatives using incremental investment rate of return analysis are mentioned below. The details about cost and investment alternatives are already stated in Lecture-1 of Module 2.

Steps for comparison of cost alternatives:

i) First arrange the mutually exclusive cost alternatives on the basis of increasing initial capital investment. The lowest capital investment alternative is considered as the base alternative(B).

ii) The incremental cash flow is calculated between the base alternative (B) and the next higher capital investment alternative (H) over the useful life.

iii) Then the rate of return „$i_{r(H-B)}$" of this incremental investment is calculated (procedure as stated earlier) by equating the net equivalent worth (present worth or annual worth or future worth) tozero.

iv) If the calculated „$i_{r(H-B)}$" is greater than or equal to MARR, then alternative „B" is removed from further analysis. Alternative „H" now becomes the new base alternative and is compared against the next higher capital investment alternative. If „$i_{r(H-B)}$" is less than MARR, then alternative „H" is removed from further analysis and alternative „B" remains as the base alternative and is compared against the next higher investment alternative (alternative with investment higher than„H").

v) Steps ii) to iv) are repeated till only one alternative is left i.e. the best alternative which justifies the incremental investment associatedwith.

Steps for comparison of investment alternatives:

i) Arrange the mutually exclusive investment alternatives on the basis of increasing initial capitalinvestment.

ii) Then the rate of return (IRR) on total cash flow of the lowest investment alternative is determined (procedure already stated earlier) to find out its acceptability as the base alternative. If the calculated rate of return is greater than or equal to MARR, this is selected as the base alternative. If the calculated rate of return is less than MARR, then this alternative is not considered for further analysis and the acceptability of the next higher investment alternative as base alternative is found out by calculating the rate of return on its total cash flow and comparing against MARR. This process is continued till the base alternative „B" (acceptable alternative for which rate of return greater than or equal to MARR) is obtained. If no alternative is obtained in this manner i.e. rate of return less than MARR, then do-nothing alternative is selected. The do-nothingalternative indicates that all the investment alternatives are rejected. Similar to the comparison of cost alternatives, the incremental cash flow is now calculated between the base alternative (B) and the next higher investment alternative (H) over the usefullife.

Steps iii) to v) as mentioned above for the comparison of cost alternatives are then followed to select the best alternative.

The comparison of cost alternatives is illustrated in the following example.

Example -9

The development authority of a city has to select a pumping unit from four feasible mutually exclusive alternatives for supply of water to a particular location of the city. The details of cash flow and the useful life of all the alternatives are presented in the following table. The minimum attractive rate of return (MARR) is 20% per year. Select the best alternative using the incremental investment rate of returnanalysis.

Solution:

The cash flow and useful life of all the alternatives are presented in Table 2.1.

Table 2.1 Cash flow of alternatives

Alternative / Cash flow	Alternative-1 A1	Alternative-2 A2	Alternative-3 A3	Alternative-4 A4
Initial capital investment (Rs.)	**7800000**	**6600000**	**8100000**	**7400000**
Annual operating and maintenance cost (Rs.)	**850000**	**1185000**	**800000**	**970000**
Salvage value (Rs.)	**2050000**	**1780000**	**2200000**	**1865000**
Useful life (Years)	**10**	**10**	**10**	**10**

As seen from the above table, these are cost alternatives involving all cash outflows (negative cash flows) except for the salvage value (positive cash flow) at the end of useful life. The alternatives are not in the increasing order of capital investmentas observed from Table 2.1. The alternatives are now arranged in the increasing order of capital investment as shown in Table 2.2 and cash outflows and cash inflows are shown with negative and positive signs respectively.

Table 2.2 Cash flow of alternatives in increasing order of initial capital investment

Alternative / Cash flow	Alternative-2 A2	Alternative-4 A4	Alternative-1 A1	Alternative-3 A3
Initial capital investment (Rs.)	**-6600000**	**-7400000**	**-7800000**	**-8100000**
Annual operating and maintenance cost (Rs.)	**-1185000**	**-970000**	**-850000**	**-800000**
Salvage value (Rs.)	**+1780000**	**+1865000**	**+2050000**	**+2200000**
Useful life (Years)	**10**	**10**	**10**	**10**

After arranging the alternatives in increasing order of capital investment, alternative-2 (A2) now becomes the base alternative (lowest capital investment with Rs.6600000) and it is compared with the next higher investment alternative i.e. alternative-4 (A4) with capital investment of Rs.7400000. The incremental cash flow between the two alternatives A2 and A4

is given as follows;

Incremental capital investment = -Rs.7400000 – (-Rs.6600000) = -Rs.800000 at beginning i.e. at timezero.

Incremental annual operating and maintenance cost from end of year 1 till end of year 10

= -Rs.970000 – (-Rs.1185000) = Rs.215000

Incremental salvage value = Rs.1865000 – Rs.1780000 = Rs.85000 at end of year 10

In order to find out the rate of return (IRR) of this incremental cash flow, the net present worth is equated to zero.

$$PW = 0 = -800000 + 215000\left(P/A, i_r, 10\right) + 85000\left(P/F, i_r, 10\right)$$

The value of rate of return „i_r' is now calculated by solving the above equation either manually through trial and error process with linear interpolation or using Microsoft Excel spreadsheet (already mentioned earlier). For faster calculation, the rate of return is calculated using Microsoft Excel spreadsheet after entering year-wise cash inflows and cashoutflows.Thevalueofrateofreturnisfoundtobe24.06%(usingthefunction „IRR" in Excel spreadsheet). As rate of return of the incremental cash flow is greater than MARR (20%), the incremental investment associated with alternative-4 (A4) is justified and alternative-2 (A2) is now removed from further analysis. Alternative-4 now becomes thenewbasealternativeandiscomparedwithnexthighercapitalinvestmentalternative i.e. alternative-1 (A1) with investment of Rs.7800000. The rate return of this incremental investment is calculated in same manner as above. The entire calculation is now presented in the Table2.3.

Table 2.3 Comparison of cost alternatives using incremental rate of return analysis

Comparison / Incremental cash flow	Between A4 and A2	Between A1 and A4	Between A3 and A1
Incremental capital investment (Rs.)	**-800000**	**-400000**	**-300000**
Incremental annual operating and maintenance cost i.e. savings (Rs.)	**215000**	**120000**	**50000**
Incremental salvage value (Rs.)	**85000**	**185000**	**150000**
Useful life (Years)	**10**	**10**	**10**
Rate of return on incremental investment	**24.06% (> MARR)**	**28.65% (> MARR)**	**14.09% (< MARR)**
Incremental investment justified	**Yes**	**Yes**	**No**
Present worth of incremental cash flow at MARR, 20% (Rs.)	**115115 (> 0)**	**132978 (> 0)**	**-66150 (< 0)**

In Table 2.3 the incremental values between the alternatives indicate the difference in cash flows betweenthem.

The outcomes of the incremental investment analysis for the comparison of cost alternatives as presented in Table 2.3 are briefly described below.

- Comparison between alternatives **A2** (base alternative) and **A4** (next higher capital investment alternative). The obtained rate of return form the incremental investment analysis is 24.06% which is greater than MARR (20%). Alternative-2 **(A2)** is eliminated from further analysis and alternative-4 **(A4)** is the new basealternative.

- Now comparison between alternatives **A4** and **A1** (next higher capital investment alternative). The obtained rate of return form the incremental investment is 28.65% and is greater than MARR. Thus alternative-4 **(A4)** is eliminated from further analysis and alternative-1 **(A1)** is the new basealternative.

- Finally comparison between alternatives **A1** and **A3** (next higher capital investment alternative). The rate of return obtained from the incremental investment analysis is 14.09% which is less than MARR (20%). Thus the incremental investment associated with alternative-3 **(A3)** i.e. largest capital investment alternative is not justified and **hence alternative-1 (A1) is selected as the best alternative,** as no other alternative is left for comparison**.** In addition, the present worth of the incremental investment associated with alternative-1 **(A1)** over alternative-4 **(A4)** at MARR i.e. 20% is greater than zero i.e. Rs.132978 >0.

- It can be seen here that, the largest capital investment alternative **(A3)** is not selected because the incremental investment associated with it results in a rate of return which is less than MARR. In addition the present worth of the incremental investment associated with alternative-3 **(A3)** over alternative-1 **(A1)** at MARR i.e. 20% is less than zero i.e. –Rs.66150 <0.

Now the values of equivalent present worth of the total cash flow of the cost alternatives at MARR (20%) are found to be -Rs.11280643, -Rs.11165528, -Rs.11098700 and -Rs.11032550 for alternatives A2, A4, A3 and A1 respectively. Thus alternative A1 (the best alternative) exhibits lowest negative equivalent present worth as compared to other cost alternatives.

6.Benefit-costanalysis:-

The benefit-cost analysis method is mainly used for economic evaluation of public projects which are mostly funded by government organizations. In addition this method can also used for economic evaluation of alternatives for private projects. The main objective of this method is used to find out desirability of public projects as far as the expected benefits on the capital investment are concerned. As the name indicates, this method involves the calculation of ratio of benefits to the costs involved in a project.

In benefit-cost analysis method, a project is considered to be desirable, when the net benefit (total benefit less disbenefits) associated with it exceeds its cost. Thus it becomes imperative to list out separately the costs, benefits and disbenefits associated with a public project. Costs are the expenditures namely initial capital investment, annual operating cost, annual maintenance cost etc. to be incurred by the owner of the project and salvage value if any is subtracted from the costs. Benefits are the gains or advantages whereas disbenefits are the losses, both of which are experienced by the owner in the project. In case of public projects which are funded by the government organizations, owner is the government. However this fund is generally taxpayers‟ money i.e. tax collected by government from general public, thereby the actual owners of public projects are the general public. Thus in case of public projects, the cost is incurred by the government whereas the benefits and disbenefits are mostly experienced by the general public.

In order to know the costs, benefits and disbenefits associated with a public project, consider that a public sector organization is planning to set up a thermal power plant at a particular location. The costs to be incurred by the public sector organization are cost of purchasing the land required for the thermal power plant, cost of construction of various facilities, cost of purchase and installation of various equipments, annual operating and maintenance cost, and other recurring costs etc. The benefits associated with the project are generation of electric power that will cater to the need of the public, generation of revenue by supplying the electricity to the customers, job opportunity for local residents, development other infrastructure in the nearby areas etc. The disbenefits associated with project are loss of land of the local residents on which the thermal power plant will come up. If it is agricultural land, then the framers will lose their valuable land along with the annual revenue generated from farming, even though they get money for their land from the public sector organization at the beginning. The other disbenefits to the local residents are greater likelihood of air pollution in the region because of the thermal power plant, chances of contamination of water in the nearby water-bodies etc.

In benefit-cost analysis method, the time value of money is taken in to account for calculating the equivalent worth of the costs and benefits associated with a project. The benefit-cost ratio of a project is calculated by taking the ratio of the equivalent worth of benefits to that of the costs associated with that project. Either of present worth, annual worth or future worth methods can be used to find out the equivalent worth of costs and benefits associated with the project.

The benefit-cost ratio of projects is determined in different forms namely conventional benefit-cost ratio and modified benefit-cost ratio. The benefit-cost ratio is generally designated as **B/Cratio**.

Conventional B/Cratio

The conventional benefit-cost ratio of a project is mentioned as follows;

$$\text{Conventional } B/C \text{ ratio} = \frac{\text{Equivalentworth of Benefits} - \text{Equivalentworth of Disbenefits}}{\text{Equivalentworth of total cost - Equivalent worth of salvage value}}$$

The disbenefits associated with the project are subtracted from the benefits in the numerator of the ratio to obtain the net benefit associated with the project. Similarly the equivalent worth of salvage value of the initial investment is subtracted from equivalent worth of cost in the denominator of the ratio. The total cost mainly consists of initial cost (initial capital investment) plus the operating and maintenance cost.

As already stated the equivalent worth may be calculated either by present worth method, annual worth method or future worth method. Thus the expression for conventional benefit-cost ratio (B/C ratio) is mentioned as follows;

$$\text{Conventional } B/C \text{ ratio} = \frac{PW \text{ of Benefits} - PW \text{ of Disbenefits}}{\text{Initialcost} + PW \text{ of operating and maintenance cost - } PW \text{ of salvage value}}$$

Or

$$\text{Conventional } B/C \text{ ratio} = \frac{AW \text{ of Benefits} - AW \text{ of Disbenefits}}{AW \text{ of initialcost} + AW \text{ of operating and maintenancecost - } AW \text{ of salvage value}}$$

Or

$$\text{Conventional } B/C \text{ ratio} = \frac{FW \text{ of Benefits} - FW \text{ of Disbenefits}}{FW \text{ of initialcost} + FW \text{ of operating and maintenance cost - Salvage value}}$$

In the above expressions, PW, AW, and FW refer to equivalent present worth, annual worth and future worth respectively.

Modified B/C ratio

In the modified benefit-cost ratio method, the operating and maintenance cost is subtracted from the benefits in the numerator of the ratio. In other words, operating and maintenance cost is considered similar to the disbenefits. The expression for modified benefit-cost ratio using PW, AW or FW is given as follows

$$\text{Modified } B/C \text{ ratio} = \frac{PW \text{ of Benefits} - PW \text{ of Disbenefits - } PW \text{ of operating and maintenancecost}}{\text{Initialcost - } PW \text{ of salvage value}}$$

Or

$$\text{Modified } B/C \text{ ratio} = \frac{AW \text{ of Benefits} - AW \text{ of Disbenefits - } AW \text{ of operating and maintenance cost}}{AW \text{ of initialcost - } AW \text{ of salvage value}}$$

Or

$$\text{Modified } B/C \text{ ratio} = \frac{FW \text{ of Benefits} - FW \text{ of Disbenefits - } FW \text{ of operating and maintenance cost}}{FW \text{ of initialcost - Salvage value}}$$

A project is considered to be acceptable when the conventional or modified B/C ratio is greater than or equal to 1.0. The illustration of conventional and modified B/C ratio methods is described in the followingexample.

Example -10

The cash flow details of a public project is as follows

Initial cost = Rs.21000000

Annual operating cost = Rs.1600000

Worth of annual benefits = Rs.5000000

Worth of annual disbenefits = Rs.1100000

Salvage value = Rs.4000000

Interest rate per year = 8% and useful lie = 30 Years

Using benefit-cost ratio method (both conventional and modified), find out the economical acceptability of the public project. Use PW, AW and FW methods to find out the equivalent worth of costs, benefits and disbenefits.

Solution:

First the conventional benefit-cost ratio (B/C ratio) of the project is computed.

Conventional B/C ratio using Present worth:

The conventional benefit-cost ratio of the public project is calculated as follows;

$$Conventional B/C ratio = \frac{PW\ of\ benefits - PW of\ \ disbenefits}{Initial\ cost + PW\ of\ operating\ cost - PW\ of\ salvage value}$$

$$Conventional B/C ratio = \frac{[5000000(P/A,i,n)] - [1100000(P/A,i,n)]}{21000000 + 1600000(P/A,i,n) - [4000000(P/F,i,n)]}$$

$$Conventional B/C ratio = \frac{[5000000(P/A,8\%,30)] - [1100000(P/A,8\%,30)]}{21000000 + 1600000(P/A,8\%,30) - [4000000(P/F,8\%,30)]}$$

$$Conventional B/C ratio = \frac{5000000 \times 11.2578 - 1100000 \times 11.2578}{21000000 + 1600000 \times 11.2578 - 4000000 \times 0.0994}$$

Conventional B/C ratio = 1.137

<u>Conventional B/C ratio using Annual worth:</u>

$$Conventional\,B/C\,ratio = \frac{AW\text{ of benefits} - AW\text{ of disbenefits}}{AW\text{ of initial cost} + AW\text{ of operating cost} - AW\text{ of salvage value}}$$

$$Conventional\,B/C\,ratio = \frac{5000000 - 1100000}{21000000(A/P,i,n)+1600000-4000000(A/F,i,n)}$$

$$Conventional\,B/C\,ratio = \frac{5000000 - 1100000}{21000000(A/P,8\%,30)+1600000-4000000(A/F,8\%,30)}$$

$$Conventional\,B/C\,ratio = \frac{5000000 - 1100000}{21000000(0.0888)+1600000-4000000(0.0088)}$$

Conventional B/C ratio = 1.137

<u>Conventional B/C ratio using Future worth:</u>

$$Conventional\,B/C\,ratio = \frac{FW\text{ of benefits} - FW\text{ of disbenefits}}{FW\text{ of initial cost} + FW\text{ of operating cost} - Salvage\text{ value}}$$

$$Conventional\,B/C\,ratio = \frac{5000000(F/A,i,n) - 1100000(F/A,i,n)}{21000000(F/P,i,n)+1600000(F/A,i,n)-4000000}$$

$$Conventional\,B/C\,ratio = \frac{5000000(F/A,8\%,30) - 1100000(F/A,8\%,30)}{21000000(F/P,8\%,30)+1600000(F/A,8\%,30)-4000000}$$

$$Conventional\,B/C\,ratio = \frac{5000000 \times 113.2832 - 1100000 \times 113.2832}{21000000 \times 10.0627+1600000 \times 113.2832-4000000}$$

Conventional B/C ratio = 1.137

As calculated above, the conventional benefit-cost ratio is found to be same by using any of the equivalent worth methods i.e. PW method, AW method or FW method. As the benefit-cost ratio of the public project is 1.137 (i.e. greater than 1.0), the project is acceptable.

Now the modified benefit-cost ratio (B/C ratio) of the project is calculated.

Modified B/C ratio using Present worth:

The modified benefit-cost ratio of the public project is calculated as follows;

$$\text{Modified B/C ratio} = \frac{\text{PW of benefits} - \text{PW of disbenefits} - \text{PW of operating cost}}{\text{Initial cost} - \text{PW of salvage value}}$$

$$\text{Modified B/C ratio} = \frac{5000000(P/A,i,n) - 1100000(P/A,i,n) - 1600000(P/A,i,n)}{21000000 - 4000000(P/F,i,n)}$$

$$\text{Modified B/C ratio} = \frac{5000000(P/A,8\%,30) - 1100000(P/A,8\%,30) - 1600000(P/A,8\%,30)}{21000000 - 4000000(P/F,8\%,30)}$$

$$\text{Modified B/C ratio} = \frac{5000000 \times 11.2578 - 1100000 \times 11.2578 - 1600000 \times 11.2578}{21000000 - 4000000 \times 0.0994}$$

Modified B/C ratio = 1.257

Modified B/C ratio using Annual worth:

$$\text{Modified B/C ratio} = \frac{\text{AW of benefits} - \text{AW of disbenefits} - \text{AW of operating cost}}{\text{AW of initial cost} - \text{AW of salvage value}}$$

$$\text{Modified B/C ratio} = \frac{5000000 - 1100000 - 1600000}{21000000(A/P,i,n) - 4000000(A/F,i,n)}$$

$$\text{Modified B/C ratio} = \frac{5000000 - 1100000 - 1600000}{21000000(A/P,8\%,30) - 4000000(A/F,8\%,30)}$$

$$\text{Modified B/C ratio} = \frac{5000000 - 1100000 - 1600000}{21000000 \times 0.0888 - 4000000 \times 0.0088}$$

Conventional B/C ratio = 1.257

Modified B/C ratio using Future worth:

$$\text{Modified B/C ratio} = \frac{\text{FW of benefits} - \text{FW of disbenefits} - \text{FW of operating cost}}{}$$

$$FW \text{ of initial cost } - \text{ Salvagevalue}$$

$$Modified\ B/C\ ratio = \frac{5000000(F/A,i,n) - 1100000(F/A,i,n) - 1600000(F/A,i,n)}{21000000(F/P,i,n) - 4000000}$$

$$Modified\ B/C\ ratio = \frac{5000000(F/A,8\%,30) - 1100000(F/A,8\%,30) - 1600000(F/A,8\%,30)}{21000000(F/P,8\%,30) - 4000000}$$

$$Modified\ B/C\ ratio = \frac{5000000 \times 113.2832 - 1100000 \times 113.2832 - 1600000 \times 113.2832}{21000000 \times 10.0627 - 4000000}$$

Modified B/C ratio = 1.257

The modified benefit-cost ratio of the public project is found to be 1.257.

As observed from above calculations, the B/C ratio of the project from both methods (conventional and modified) is greater than 1.0, although the value is different. It may be noted here that, although the magnitude of benefit-cost ratio differs between two methods i.e. conventional B/C ratio and modified B/C ratio, but the decision to select or not a project is not changed by use of any of the two methods.

7. **Breakeven analysis**

The breakeven analysis is used to calculate the value of a factor (or variable) at which the expenditures and revenues of a project or alternative are equal. This value of the variable is known as the breakeven point. Corresponding to the breakeven point, profit or loss can be determined if the expected value of the variable is higher or lower than the breakeven value. In this regard the breakeven point governs the economic acceptability of the project or the alternative. The breakeven analysis is also used for comparing two alternatives by determining the breakeven point i.e. the quantity of a factor (common to both the alternatives) at which the total equivalent worth of both alternatives are equal. The examples of some of the factors which are used in the breakeven analysis are quantities produced per year, hours of operation per year, rate of return per year and useful life etc. and the breakeven value of these factors are calculated to find out the economical acceptability of a single alternative or to select the best one between the alternatives. The breakeven point between expenditure and revenue for a single alternative is shown in Fig. 2.32. Here „x" is the factor that mainly affects theexpenditure and revenue of thealternative.

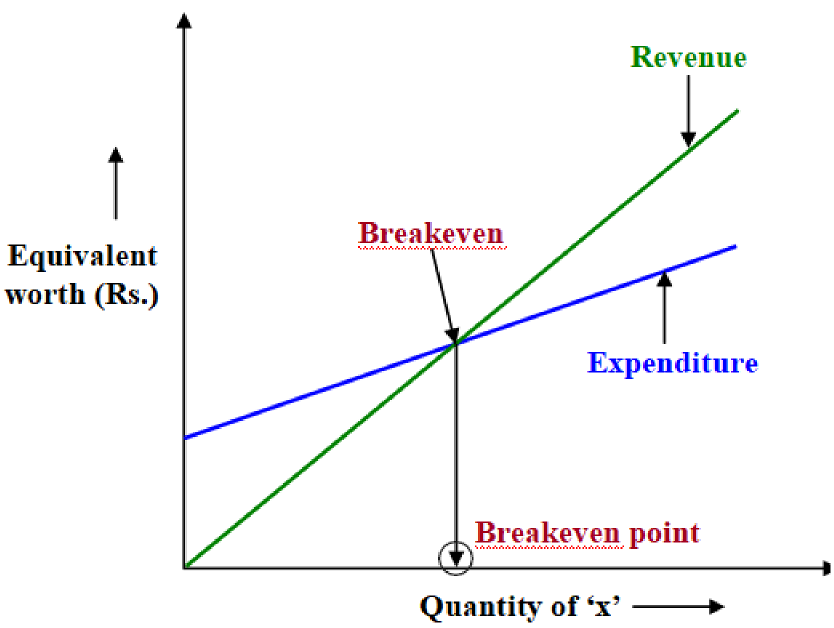


Fig. 2.32 Schematic diagram for breakeven point of a single alternative

In Fig. 2.32, the equivalent worth of expenditure and revenue are plotted as functions of the quantity of factor „x". The breakeven point corresponds to that value of the factor „x" at which the equivalent worth of expenditure and revenue of the alternative are equal i.e. the relationships representing the expenditure and revenue as functions of „x‟ intersect each other (shown in Fig. 2.32).

The breakeven point between two alternatives is shown in Fig. 2.33.

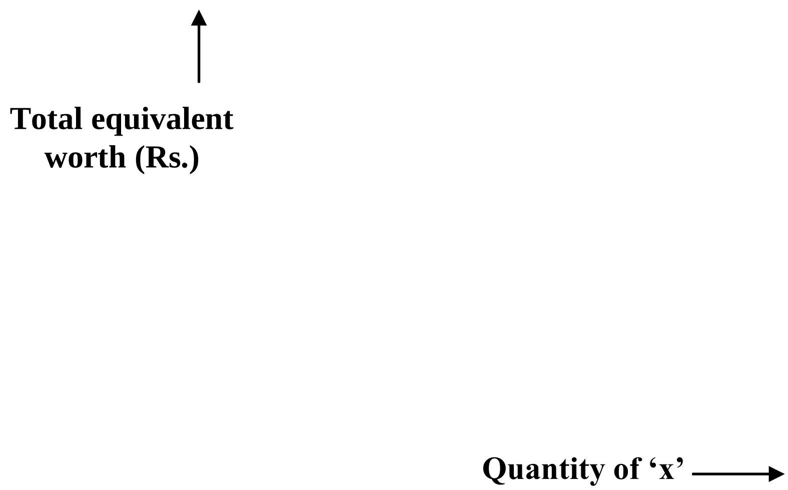


Fig. 2.33 Schematic diagram of breakeven point between two alternatives

In this figure the total equivalent worth i.e. equivalent worth of net cash flow (i.e. expenditures and revenues) of the alternatives are plotted at various values of the common factor „x‟. The intersection of the total equivalent worth of two alternatives gives the breakeven point i.e. the value of the common factor „x‟ at which the values of total equivalent worth of the two alternatives are equal. If the expected value of „x‟ isless than the breakeven value, Alternative-1 is selected as its total equivalent worth (assuming it as negative cash flow i.e. cost greater than revenue) is less than that of Alternative-2 as evident from Fig. 2.33. Similarly when the expected value of „x‟ is greater than the breakeven value, Alternative-2 is selected as it shows lower equivalent worth (i.e. lower cost) compared to Alternative-1. In Fig. 2.32 the variations of equivalent worth of expenditure and revenue of the single alternative and in Fig. 2.33 the variations of total equivalent worth of two alternatives are considered as linear functions of the value of factor „x‟. Sometimes these relationships may also be non-linear. In thebreakeven analysis, the equivalent worth of expenditures and revenues can be calculated either by present worth method, future worth method or annual worth method by taking into account the time value of money. The annual worth method is normally used when the quantities of the variable (on which the expenditures and the revenues mostly depend) are expressed on annual basis.

The following example will illustrate the breakeven analysis for a single alternative.

Example -11

A concrete mixer has the following cash flow details;

Initial purchase price = Rs.750000,

Annual operating and maintenance cost = Rs.45000

Salvage value = Rs.210000,

Useful life = 10 years

In addition one operator is required to operate the concrete mixer at cost of Rs.30 per hour. The production (preparation) rate of concrete of the mixer is 0.1m^3 per hour. The revenue to be generated from production of 1m^3 of concrete is Rs.1000. The interest rate is 11% per year. How many „m^3‟ of concrete need to be produced per year so that the revenue generated breakevens with the expenditure?

Solution:

In order to find out the breakeven value of the concrete volume (in „m^3‟) per year, the equivalent uniform annual worth of expenditure will be equated to that of revenue.

Let „x‟ m^3 is the volume of concrete produced by the concrete mixer per year.

The operator cost is Rs.30 per hour.

The operator cost (Rs.) per year is given by;

$$\frac{Rs.30}{1hour} \times \frac{1hour}{0.1m^3} \times \frac{x\ m^3}{year} = Rs.300x$$

Now the equivalent uniform annual worth (Rs.) of expenditure is given by;

$$AW_e = 750000(A/P,i,n) + 45000 + 300x$$

$$AW_e = 750000(A/P,11\%,10) + 45000 + 300x$$

$$AW_e = 750000 \times 0.1698 + 45000 + 300x = 172350 + 300x$$

The equivalent uniform annual worth (Rs.) of revenue is calculated as follows;

$$AW_r = 210000(A/F,i,n) + 1000x$$

$$AW_r = 210000(A/F,11\%,10) + 1000x$$

$$AW_r = 210000 \times 0.0598 + 1000x = 12558 + 1000x$$

Now equating equivalent uniform annual worth of expenditure with that of revenue;

$$AW_e = AW_r$$

$$172350 + 300x = 12558 + 1000x$$

$$x = 228.274\ m^3$$

Thus the volume of concrete to be produced by the concrete mixer per year i.e. the breakeven quantity at which the expenditure incurred is equal to the revenue generated is 228.274 m^3. If the volume of concrete produced per year is different from the breakeven value, then there will change in the net cash flow as shown below;

If x is equal to 200 m^3 (i.e. less than breakeven value), the equivalent uniform annual worth of expenditure and revenue are given by;

<u>Expenditure</u>

$$AW_e = 750000(A/P,11\%,10) + 45000 + 300x$$

$$AW_e = 750000 \times 0.1698 + 45000 + 300 \times 200 = Rs.232350$$

<u>Revenue</u>

$$AW_r = 210000(A/F,11\%,10) + 1000x$$

$$AW_r = 210000 \times 0.0598 + 1000 \times 200 = Rs.212558$$

$AW_e > AW_r$

If x is equal to 250 m^3 (i.e. greater than breakeven value), the equivalent uniform annual worth of expenditure and revenue are given by;

<u>Expenditure</u>

$$AW_e = 750000(A/P,11\%,10) + 45000 + 300x$$

$$AW_e = 750000 \times 0.1698 + 45000 + 300 \times 250 = Rs.247350$$

<u>Revenue</u>

$$AW_r = 210000(A/F,11\%,10) + 1000x$$

$$AW_r = 210000 \times 0.0598 + 1000 \times 250 = Rs.262558$$

$AW_r > AW_e$

Thus from above calculations it is observed that, equivalent annual worth of revenue is less than that of expenditure, when the volume of concrete produced per year is less than the breakeven value and on the other hand, equivalent annual worth of revenue is more than that of expenditure, when the volume of concrete produced per year is greater than the breakeven value.

The breakeven point is also graphically shown in Fig. 2.34.

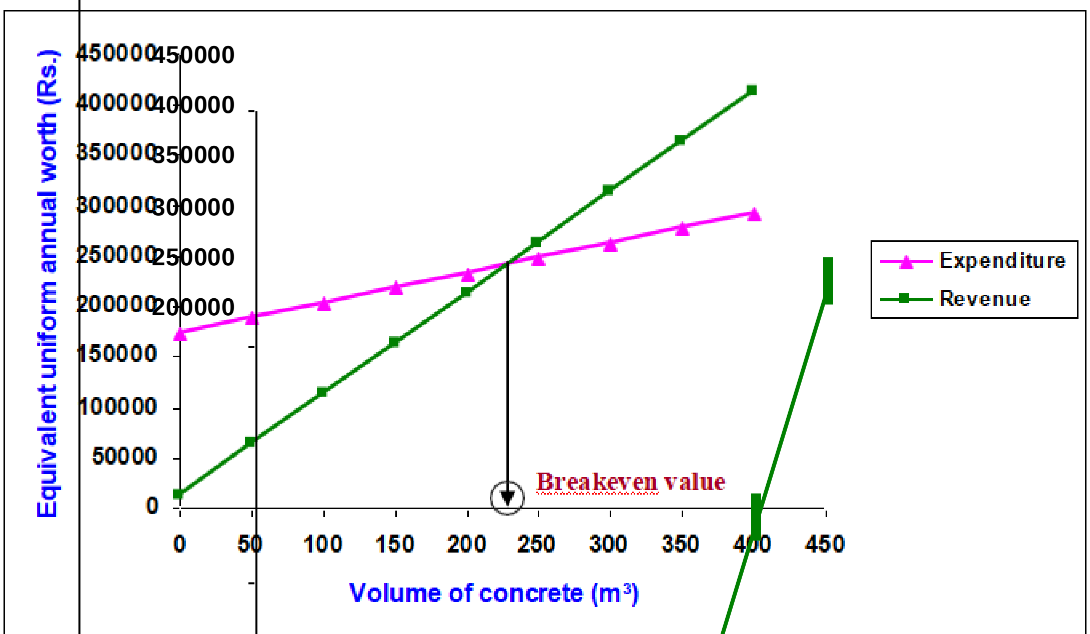


Fig. 2.34 Equivalent uniform annual worth of expenditure and revenue and breakeven point

The equivalent uniform annual worth of expenditure and revenue are calculated at different values of volume of concrete produced per year using the respective expressions as mentioned earlier and are shown in the above figure.

The breakeven point can also be calculated by equating the equivalent present worth of expenditures to that of revenues as shown below.

<u>Present worth of expenditure:</u>

$$PW_e = 750000 + 45000(P/A,i,n) + 300x(P/A,i,n)$$

$$PW_e = 750000 + 45000(P/A,11\%,10) + 300x(P/A,11\%,10)$$

$PW_e = 750000 + 45000 \times 5.8892 + 5.8892 \times 300x = 1015014 + 1766.76x$

<u>Present worth of revenue:</u>

$PW_r = 210000(P/F,i,n) + 1000x(P/A,i,n)$

$PW_r = 210000(P/F,11\%,10) + 1000x(P/A,11\%,10)$

$PW_r = 210000(0.3522) + (5.8892)1000x = 73962 + 5889.2x$

Now equating equivalent present worth of expenditure with that of revenue;

$PW_e = PW_r$

$1015014 + 1766.76x = 73962 + 5889.2x$

$x = 228.275m^3$

Thus the breakeven value of volume of concrete to be produced by the concrete mixer per year is 228.275 m^3 which is same as the value obtained by annual worth method stated earlier.

Questions

1. The different cost elements and other parameters to be considered while making the _______________
   a. economic comparison
   b. costs
   c. procurement assessment
   d. ethics of company
   Ans, a
2. Economic comparison of the alternatives are
   a. annual operating and maintenance cost
   b. annual income or receipts
   c. expected salvage value
   d. income tax benefit
   e. All the above
   Ans. E

3. Net present worth for an alternative is equal to _____________.
   a. Present worth (benefits)
   b. Present worth (cost)
   c. Present worth (benefits) - Present worth (cost)
   d. Present worth (cost) - Present worth (benefits)
   Ans. C

4. When the alternatives have identical cost, as per present worth analysis technique the focus should be on ____________
   a. Maximizing present worth (benefit)
   b. Maximizing present worth (cost)
   c. Maximizing present worth (cost - benefits)
   d. None of the above
   Ans. A

5. A company must install one of two production machines that have identical costs.
   What criterion should be selected to determine which machine to install as per present worth analysis?

   (A) Choose the machine from the closest vendor.
   (B) Choose the machine with the higher PW (benefits).
   (C) Choose the machine with the lower PW (costs).
   (D) Choose either machine
   Ans. B

6. Assuming annual interest of i%, If initial present sum is P and required end-of-period withdrawal is of A forever without diminishing initial sum P, then P and A are related as ..
   (A) $P = Ai$
   (B) $A = Pi$
   (C) $P = A(i+1)$
   (D) $A = P (i+1)$

Unit – IV Cash Flows

TIME VALUE OF MONEY

If an investor invests a sum of Rs. 100 in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of every year will be as shown in Table 3.1.

Table 3.1 Compound Amounts

(amount of deposit = Rs. 100.00)

Year end	*Interest* (Rs.)	*Compound amount* (Rs.)
0		100.00
1	15.00	115.00
2	17.25	132.25
3	19.84	152.09
4	22.81	174.90
5	26.24	201.14

The formula to find the future worth in the third column is

$$F = P \times (1 + i)^n$$

where

P = principal amount invested at time 0,
F = future amount,
i = interest rate compounded annually,
n = period of deposit.

The maturity value at the end of the fifth year is Rs. 201.14. This means that the amount Rs. 201.14 at the end of the fifth year is equivalent to Rs. 100.00 at time 0 (i.e. at present). This is diagrammatically shown in Fig. 3.1. This explanation assumes that the inflation is at zero percentage.

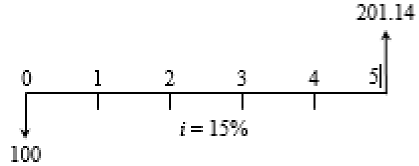


Fig. 3.1 Time value of money.

Alternatively, the above concept may be discussed as follows: If we want Rs. 100.00 at the end of the *n*th year, what is the amount that we should deposit now at a given interest rate, say 15%? A detailed working is shown in Table 3**.2.**

End of year (n)	Present worth	Compound amount after n year(s)
0		100
1	86.96	100
2	75.61	100
3	65.75	100
4	57.18	100
5	49.72	100
6	43.29	100
7	37.59	100
8	32.69	100
9	28.43	100
10	24.72	100

Table 3.2 Present Worth Amounts

The formula to find the present worth in the second column is

$$P = F/(1 + i)^n$$

From Table 3.2, it is clear that if we want Rs. 100 at the end of the fifth year, we should now deposit an amount of Rs. 49.72. Similarly, if we want Rs. 100.00 at the end of the 10th year, we should now deposit an amount of Rs. 24.72.

Also, this concept can be stated as follows:

A person has received a prize from a finance company during the recent festival contest. But the prize will be given in either of the following two modes:

1. Spot payment of Rs. 24.72 or
2. Rs. 100 after 10 years from now (this is based on 15% interest rate compounded annually).

Cash flow diagram:

Cash flows are the amounts of money estimated for future projects or observed for project events that have taken place. All cash flows occur during specific time periods, such as 1 month, every 6 months, or 1 year. Annual is the most common time period.

Cash inflows are the receipts, revenues, incomes, and savings generated by project and business activity. A plus sign indicates a cash inflow

Cash outflows are costs, disbursements, expenses, and taxes caused by projects and business activity. A negative or minus sign indicates a cash outflow. When a project involves only costs, the minus sign may be omitted for some techniques, such as benefit/cost analysis.

Net cash flow= cash inflows -cash outflows

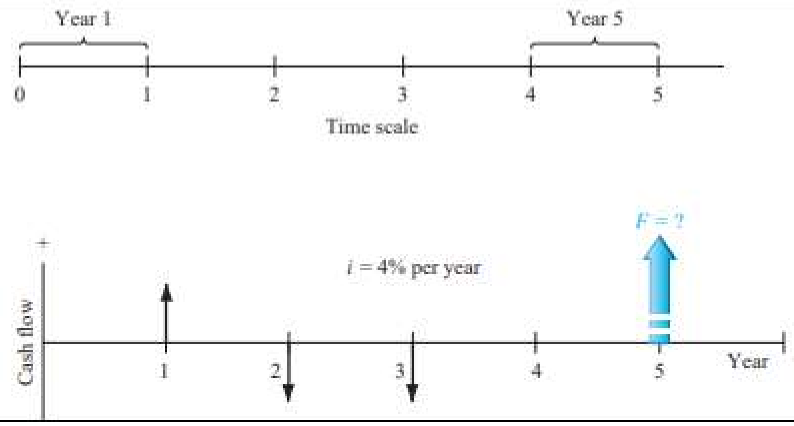


Figure 1-4 A typical cash flow time scale for 5 years.

Figure 1-5 Example of positive and negative cash flows.

INTEREST FORMULAS

Interest rate can be classified into *simple interest rate* and *compound interest rate*.

Simple interest rate

In simple interest, the interest is calculated, based on the initial deposit for every interest period. In this case, calculation of interest on interest is not applicable

Compound interest rate

In compound interest, the interest for the current period is computed based on the amount (principal plus interest up to the end of the previous period) at the beginning of the current period

The notations which are used in various interest formulae are as follows:

P = principal amount

n = No. of interest periods

i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F = future amount at the end of year n

A = equal amount deposited at the end of every interest period

G = uniform amount which will be added/subtracted period after period to/ from the amount of deposit A1 at the end of period 1

Nominal and effective interest continuous interest

Effective interest rate:

Let i be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year. For example, compounding may be monthly, quarterly, or semi-annually. Compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is a year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is

Effective interest rate, R = (1 + i/C)C – 1

i = the nominal interest rate
C = the number of interest periods in a year.

Single Payment Compound Amount Factor(P/F,F/P) –

Here, the objective is to find the single future sum (*F*) of the initial payment (*P*) made at time 0 after *n* periods at an interest rate *i* compounded every period. The cash flow diagram of this situation is shown in Fig. 3.2.

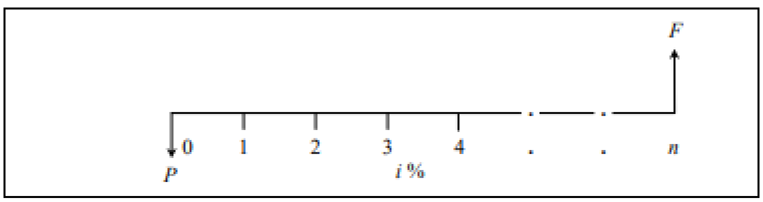


Fig. 3.2 Cash flow diagram of single-payment compound amount.

The formula to obtain the single-payment compound amount is

$$F = P(1 + i)^n = P(F/P, i, n)$$

where
(*F*/*P*, *i*, *n*) is called as single-payment compound amount factor.

1. **A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.**

Solution

P = Rs. 20,000

i = 18% compounded annually

n = 10 years

$F = P(1 + i)^n = P(F/P, i, n)$

= 20,000 (F/P, 18%, 10)

= 20,000 × 5.234 = Rs. 1,04,680

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount (P) of a single future sum

(F) which will be received after n periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.3.

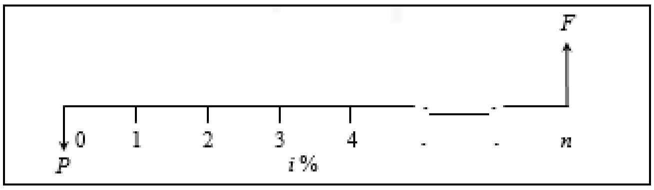


Fig .Cash flow diagram of single-payment present worth amount.

Where

(P/F, i, n) is termed as single-payment present worth factor.

2. **A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.**

Solution

F = Rs. 1,00,000

i = 15%, compounded annually

n = 10 years

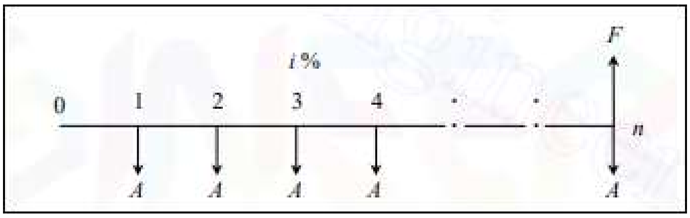
P = F/(1 + i) n = F(P/F, i, n) = 1,00,000 (P/F, 15%, 10)

= 1,00,000 0.2472 = Rs. 24,720

The person has to invest Rs. 24,720 now so that he will get a sum

Uniform series of Payments (F/A, A/F, F/P, A/P)

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the nth interest period at an interest rate of i compounded at the end of each interest period.



The corresponding cash flow diagram is shown in Fig. 3.4.

Fig. 3.4 Cash flow diagram of equal-payment series compound amount.

In Fig. 3.4, A = equal amount deposited at the end of each interest period

n = No. of interest periods

i = rate of interest

F = single future amount

The formula to get F is F = A (1 1 +) i i n − = A(F/A, i, n) where (F/A, i, n) is termed as equal-payment series compound amount factor.

1. A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

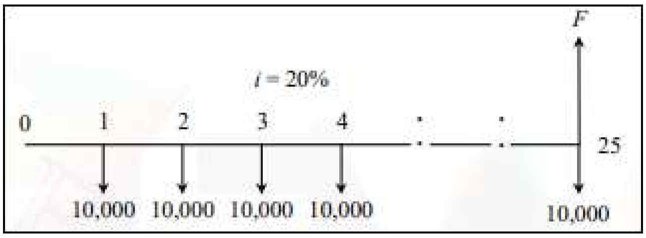
Solution A = Rs. 10,000

n = 25 years

i = 20%

F = ?

The corresponding cash flow diagram is shown in Fig. 3.5



$$F = A\,\frac{(1+i)^n - 1}{i}$$
$$= A(F/A,\ i,\ n)$$

= 10,000(F/A, 20%, 25)

= 10,000 X 471.981

= Rs. 47, 19,810

The future sum of the annual equal payments after 25 years is equal to Rs. 47, 19,810

Uniform-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i.

The corresponding cash flow diagram is shown in Fig. 3.6

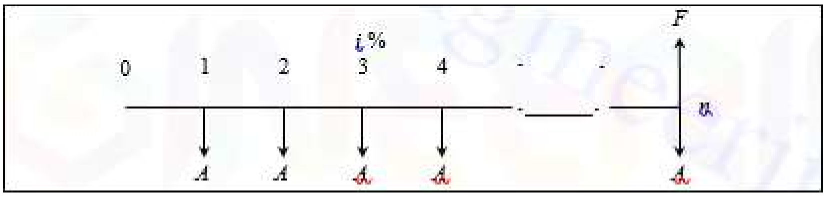


Fig. 3.6 Cash flow diagram of equal-payment series sinking fund.

A = equal amount to be deposited at the end of each interest period

n = No. of interest periods

i = rate of interest

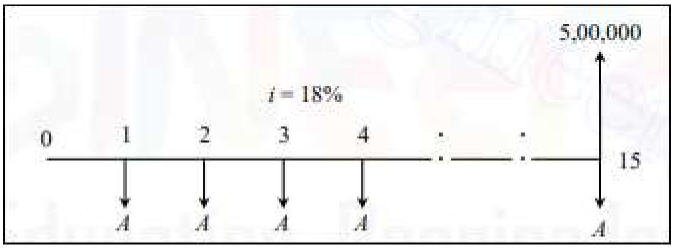
F = single future amount at the end of the nth period

The formula to get F is

$$\boxed{A = F\frac{i}{(1+i)^n - 1} = F(A/F,\, i,\, n)}$$

Where, (A/F, i, n) is called as equal-payment series sinking fund factor

1. **A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.**
   Solution F = Rs. 5,00,000
   n = 15 years
   i = 18%
   A =?
   The corresponding cash flow diagram is shown in Fig. 3.7.



Cash flow diagram of equal-payment series sinking fund.

$$\boxed{A = F\frac{i}{(1+i)^n - 1} = F(A/F,\, i,\, n)}$$

= 5,00,000(A/F, 18%, 15)
= 5,00,000 X 0.0164
= Rs. 8,200
The annual equal amount which must be deposited for 15 years is Rs. 8,200

Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal payment made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.8.

Here, P = present worth

A = annual equivalent payment

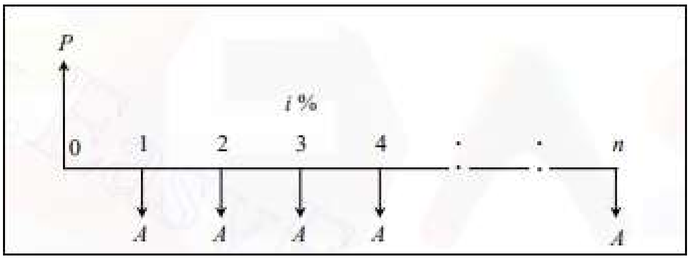
i = interest rate

n = No. of interest periods

The formula to compute P is

$$P = A\,\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A,\ i,\ n)$$

= A(P/A, i, n) where (P/A, i, n) is called equal-payment series present worth factor



1. A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

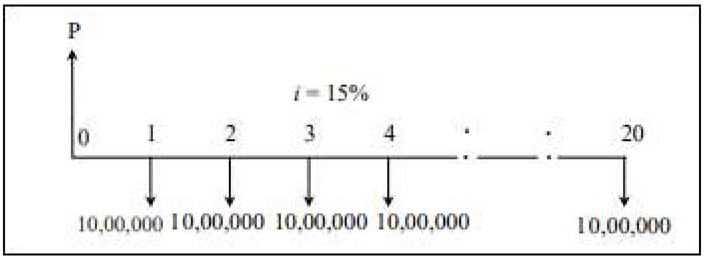
Solution A = Rs. 10,00,000

i = 15%

n = 20 years

P = ?

The corresponding cash flow diagram is illustrated in Fig. 3.9.



Cash flow diagram of equal-payment series present worth amount.

$$P = A\,\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A,\ i,\ n)$$

= 10, 00,000 (P/A, 15%, 20)

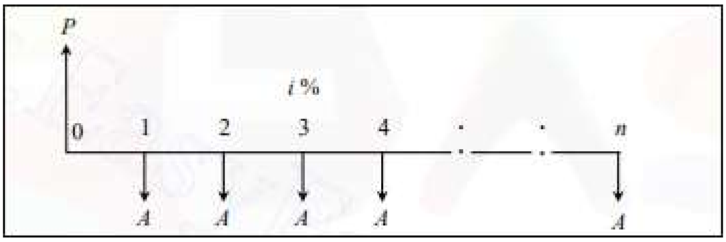
= 10, 00,000 X 6.2593

= Rs. 62, 59,300

The amount of reserve which must be set-up now is equal to Rs. 62, 59,300

Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period (see Fig. 3.10).



P = present worth (loan amount)

A = annual equivalent payment (recovery amount)

i = interest rate

n = No. of interest periods

The formula to compute P is as follows:

$$A = P\,\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P,\ i,\ n)$$

= P(A/P, i, n)

where, (A/P, i, n) is called equal-payment series capital recovery factor

1. A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

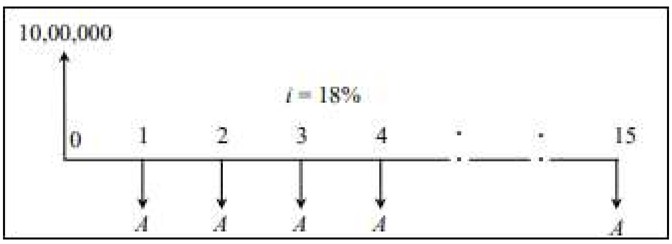
Solution

P = Rs. 10,00,000

i = 18%

n = 15 years

A = ?



$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n)$$

= 10, 00,000 (A/P, 18%, 15)

= 10, 00,000 X (0.1964)

= Rs. 1, 96,400

The annual equivalent installment to be paid by the company to the bank is Rs. 1, 96,400

Arithmetic Gradient(G)

An arithmetic gradient series is a cash fl ow series that either increases or decreases by a constant amount each period. The amount of change is called the gradient. The objective of this mode of investment is to find the annual equivalent amount of a series with an amount A1 at the end of the first year and with an equal increment (G) at the end of each of the following n – 1 years with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.12 annual equivalent amount.

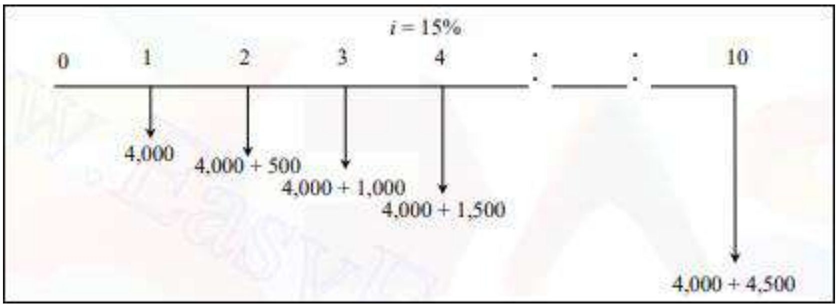
The formula to compute

$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

= A1 + G (A/G, i, n) where (A/G, i, n) is called uniform gradient series factor.

1. A **person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series**.

   Solution Here, A1 = Rs. 4,000
   G = Rs. 500
   i = 15%
   n = 10 years
   A = ? & F = ?



$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$= A1 + G(A/G, i, n)$

$= 4,000 + 500(A/G, 15\%, 10)$

$= 4,000 + 500 \;\; 3.3832$

$=$ Rs. 5,691.60

This is equivalent to paying an equivalent amount of Rs. 5,691.60 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$F = A(F/A, i, n)$

$= A(F/A, 15\%, 10)$

$= 5,691.60(20.304)$

$=$ Rs. 1,15,562.25

At the end of the 10th year, the compound amount of all his payments will be Rs. 1,15,562.25.

Geometric Gradient Series Factors

- It is common for annual revenues and annual costs such as maintenance, operations, and labor to go up or down by a constant percentage, for example, 5% or ———— 3% per year. This change occurs every year on top of a starting amount in the fi rst year of the project.
- A geometric gradient series is a cash fl ow series that either increases or decreases by a constant percentage each period. The uniform change is called the rate of change.

  g -constant rate of change, in decimal form, by which cash fl ow values increase or decrease

  from one period to the next. The gradient g can be or ——————————

  .

  A1 -initial cash fl ow in year 1 of the geometric series

  P g - present worth of the entire geometric gradient series, including the initial amount A1

➢ Note that the initial cash fl ow A1 is not considered separately when working with geometric gradients.

- Figure shows increasing and decreasing geometric gradients starting at an amount A1 in time period 1 with present worth P g located at time 0. The relation to determine the total present worth Pg for the entire cash fl ow series may be derived by multiplying each cash flow in Figure a by the PF factor 1(1 i) n

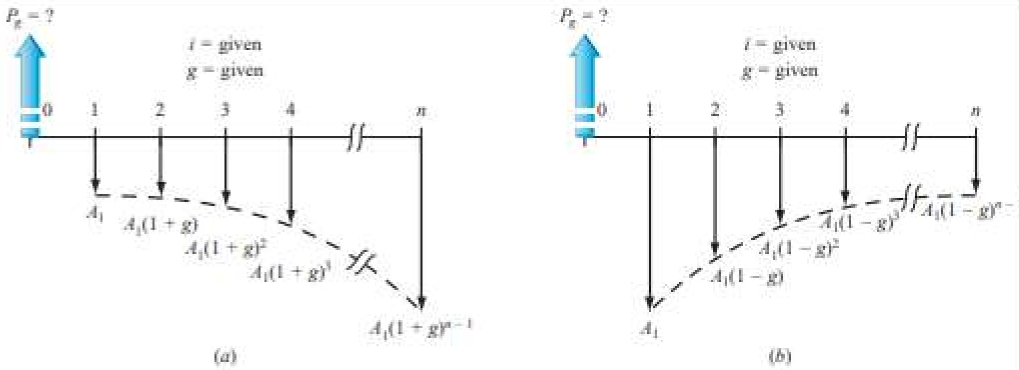


Figure 2–21

Cash flow diagram of (*a*) increasing and (*b*) decreasing geometric gradient series and present worth P_g.

$$P_g = \frac{A_1}{(1+i)^1} + \frac{A_1(1+g)}{(1+i)^2} + \frac{A_1(1+g)^2}{(1+i)^3} + \cdots + \frac{A_1(1+g)^{n-1}}{(1+i)^n}$$

$$= A_1\left[\frac{1}{1+i} + \frac{1+g}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \cdots + \frac{(1+g)^{n-1}}{(1+i)^n}\right] \qquad [2.31]$$

Multiply both sides by $(1+g)/(1+i)$, subtract Equation [2.31] from the result, factor out P_g, and obtain

$$P_g\left(\frac{1+g}{1+i} - 1\right) = A_1\left[\frac{(1+g)^n}{(1+i)^{n+1}} - \frac{1}{1+i}\right]$$

Solve for P_g and simplify.

$$P_g = A_1\left[\frac{1-\left(\frac{1+g}{1+i}\right)^n}{i-g}\right] \qquad g \neq i \qquad [2.32]$$

The term in brackets in Equation [2.32] is the $(P/A,g,i,n)$ or *geometric gradient series present worth factor* for values of g not equal to the interest rate i. When $g = i$, substitute i for g in Equation [2.31] and observe that the term $1/(1+i)$ appears n times.

$$P_g = A_1\left(\frac{1}{(1+i)} + \frac{1}{(1+i)} + \frac{1}{(1+i)} + \cdots + \frac{1}{(1+i)}\right)$$

$$P_g = \frac{nA_1}{(1+i)} \qquad [2.33]$$

The $(P/A,g,i,n)$ factor calculates P_g in period $t = 0$ for a geometric gradient series **starting in period 1** in the amount A_1 and increasing by a constant rate of g each period.

The equation for P_g and the $(P/A,g,i,n)$ factor formula are

$$P_g = A_1(P/A,g,i,n) \qquad [2.34]$$

$$(P/A,g,i,n) = \begin{cases} \dfrac{1-\left(\frac{1+g}{1+i}\right)^n}{i-g} & g \neq i \\[2ex] \dfrac{n}{1+i} & g = i \end{cases} \qquad [2.35]$$

It is possible to derive factors for the equivalent A and F values; however, it is easier to determine the P_g amount and then multiply by the A/P or F/P factor.

As with the arithmetic gradient series, there are no direct spreadsheet functions for geometric gradient series. Once the cash flows are entered, P and A are determined using the NPV and PMT functions, respectively.

UNIT – V

RISK MANAGEMENT

WHAT IS RISK?

The definition of the term *risk* merits some discussion. It is often assumed that the word *risk* implies a negative outcome. For example, if someone said to me, ‘‘That is a very risky assumption,’’ I would take it to mean that she thinks that my assumption is likely to be wrong and, consequently, something bad will happen as a result. The fact of the matter is that risk represents an uncertain outcome. Risks may have either positive or negative outcomes. A negative risk is defined as a *threat* while a positive risk is defined as an *opportunity*. Therefore, something that is properly defined as *risky* does not necessarily mean that it is a bad thing, only that it is an uncertain thing.

WHAT IS RISK MANAGEMENT?

A project manager responsible for the delivery of a new office building identifies a permitting concern that could delay the approval of her project. A structural engineer is assessing the quality of the data of a geotechnical report that was performed and fears that the abutments of the bridge he is designing could experience differential settlement. A school district superintendent is concerned that the environmental document could be delayed by public comment. A general contractor fears that the recent volatility in the price of steel could turn a profitable project into a money loser.

All of these scenarios are everyday occurrences within the design and construction industry; however, the manner in which these risks are addressed will have a large impact on project outcomes. The practice of risk management can certainly play an important role in ensuring that the outcomes will be positive ones. However, a lack of risk management will likely result in increases to a project’s cost and schedule.

Another definition of risk management provided by the International Organization for Standardization (ISO) identifies the following principles of risk management:

Risk management should:

■ create value
■ be an integral part of organizational processes
■ be part of decision making
■ explicitly address uncertainty
■ be systematic and structured
■ be based on the best available information
■ be tailored
■ take into account human factors
■ be transparent and inclusive
■ be dynamic, iterative, and responsive to change
■ be capable of continual improvement and enhancement

WHY RISK MANAGEMENT?

Research has shown that historically the majority of construction projects experience cost and/or schedule overruns. A cost overrun is defined as the difference between the low bid and the actual incurred costs at the time of construction completion.

A study focused on analyzing the costs of public works projects in Europe and North America found that the incidence and severity of cost overruns was significantly higher than indicated by the previous source. This same study found that cost overruns were found in 86 percent of the 258 projects that were sampled. Further, actual costs were, on average, 28 percent higher than estimated costs. The following factors were the primary culprits in cost overruns:

■ Lack of proper risk analysis in developing estimates
■ Poorly defined scope at the time initial project budgets were developed
■ Larger public projects are prone to intentional underestimation due to political pressure (In other words, there was a deliberate misrepresentation of project costs and/or schedule in order to further political agendas.

TYPES OF CONSTRUCTION RISKS

For proper construction risk management, you need to know the types of risks inherent in construction projects. These can be financial, contractual, operational, and environmental and can be caused by both internal and external sources.

Common risks include:

- Safety hazards that lead to worker accidents and injuries
- Managing change orders
- Incomplete drawings and poorly defined scope
- Unknown site conditions
- Poorly written contracts
- Unexpected increases in material costs
- Labor shortages
- Damage or theft to equipment and tools
- Natural disasters
- Issues with subcontractors and suppliers
- Availability of building materials
- Poor project management

When risks come to fruition, they can have a serious impact on costs, schedules, and performance of your project which will lead to delays and disputes down the road. The good news is most of these risks can be managed and mitigated with proper planning and good project management

RISK IDENTIFICATION TECHNIQUES

Information Gathering Techniques

The given techniques are similar to the techniques used to collect requirements. Let's look at a few of them:

Brainstorming

Brainstorming is done with a group of people who focus on identification of risk for the project.

Delphi Technique

A team of experts is consulted anonymously. A list of required information is sent to experts, responses are compiled, and results are sent back to them for further review until a consensus is reached.

Interviewing

An interview is conducted with project participants, stakeholders, experts, etc to identify risks.

Root Cause Analysis

Root causes are determined for the identified risks. These root causes are further used to identify additional risks.

Swot Analysis (STRENGTH, Weakness, Opportunities and Threats)

Strengths and weaknesses are identified for the project and thus, risks are determined.

Checklist Analysis

The checklist of risk categories is used to come up with additional risks for the project.

Assumption Analysis

Identification of different assumptions of the project and determining their validity further helps in identifying risks for the project.

Outputs to Identify Risks

This process of Risk Identification results in creation of Risk Register.

Risk Register

A Risk Register is a living document that is updated regularly throughout the life cycle of the project. It becomes a part of project documents and is included in the historical records that are used for future projects. The risk register includes:

- List of Risks
- List of Potential Responses
- Root Causes of Risks
- Updated Risk Categories

MOST COMMON PROJECT RISKS

- **Cost risk**, typically escalation of project costs due to poor cost estimating accuracy and scope creep.
- **Schedule risk**, the risk that activities will take longer than expected. Slippages in schedule typically increase costs and, also, delay the receipt of project benefits, with a possible loss of competitive advantage.
- **Performance risk**, the risk that the project will fail to produce results consistent with project specifications.

OTHER TYPES OF RISKS

There are many other types of risks of concern to projects. These risks can result in cost, schedule, or performance problems and create other types of adverse consequences for the organization. For example:

- **Governance risk** relates to board and management performance with regard to ethics, community stewardship, and company reputation.
- **Strategic risks** result from errors in strategy, such as choosing a technology that can't be made to work.
- **Operational risk** includes risks from poor implementation and process problems such as procurement, production, and distribution.
- **Market risks** include competition, foreign exchange, commodity markets, and interest rate risk, as well as liquidity and credit risks.
- **Legal risks** arise from legal and regulatory obligations, including contract risks and litigation brought against the organization.
- **Risks associated with external hazards**, including storms, floods, earthquakes, terrorism, labor strikes and civil unrest.

As indicated by these examples, project risks include both internal risks associated with successfully completing each stage of the project, plus risks that are beyond the control of the project team. These latter types include external risks that arise from outside the organization but affect the ultimate value to be derived from the project. In all cases, the seriousness of the risk depends on the nature and magnitude of the possible end consequences and their probabilities.

In addition to project risk, project deferral risk can be important. Project deferral risk refers to the risks associated with failing to do a project. Like project risk, project deferral risk can arise from any of the risk sources listed above. Project deferral risk can also occur if there is only a limited window of opportunity for conducting a project—if the project is not conducted now, there may be a risk that it might never be possible to effectively do it later.

RISK MANAGEMENT PROCESS

The risk management process is a framework for the actions that need to be taken. There are five basic steps that are taken to manage risk; these steps are referred to as the risk management process. It begins with identifying risks, goes on to analyze risks, then the risk is prioritized, a solution is implemented, and finally, the risk is monitored. In manual systems, each step involves a lot of documentation and administration.

Step 1: Identify the Risk

The first step is to identify the risks that the business is exposed to in its operating environment. There are many different types of risks – legal risks, environmental risks, market risks, regulatory risks, and much more. It is important to identify as many of these risk factors as possible. In a manual environment, these risks are noted down manually. If the organization has a risk management solution employed all this information is inserted directly into the system. The advantage of this approach is that these risks are now visible to every stakeholder in the organization with access to the system. Instead of this vital information being locked away in a report which has to be requested via email, anyone who wants to see which risks have been identified can access the information in the risk management system.

Step 2: Analyze the Risk

Once a risk has been identified it needs to be analyzed. The scope of the risk must be determined. It is also important to understand the link between the risk and different factors within the organization. To determine the severity and seriousness of the risk it is necessary to see how many business functions the risk affects. There are risks that can bring the whole business to a standstill if actualized, while there are risks that will only be minor inconveniences in the analysis. In a manual risk management environment, this analysis must be done manually. When a risk management solution is implemented one of the most important basic steps is to map risks to different documents, policies, procedures, and business processes. This means that the system will already have a mapped risk framework that will evaluate risks and let you know the far-reaching effects of each risk.

Step 3: Evaluate the Risk

Risks need to be ranked and prioritized. Most risk management solutions have different categories of risks, depending on the severity of the risk. A risk that may cause some inconvenience is rated lowly, risks that can result in catastrophic loss are rated the highest. It is important to rank risks because it allows the organization to gain a holistic view of the risk exposure of the whole organization. The business may be vulnerable to several low-level risks, but it may not require upper management intervention. On the other hand, just one of the highest-rated risks is enough to require immediate intervention.

Step 4: Treat the Risk

Every risk needs to be eliminated or contained as much as possible. This is done by connecting with the experts of the field to which the risk belongs. In a manual environment, this entails contacting each and every stakeholder and then setting up meetings so everyone can talk and discuss the issues. The problem is that the discussion is broken into many different email threads, across different documents and spreadsheets, and many different phone calls. In a risk management solution, all the relevant stakeholders can be sent notifications from within the system. The discussion regarding the risk and its possible solution can take place from within the system. Upper management can also keep a close eye on the solutions being suggested and the progress being made within the system. Instead of everyone contacting each other to get updates, everyone can get updates directly from within the risk management solution.

Step 5: Monitor and Review the Risk

Not all risks can be eliminated – some risks are always present. Market risks and environmental risks are just two examples of risks that always need to be monitored. Under manual systems monitoring happens through diligent employees. These professionals must make sure that they keep a close watch on all risk factors. Under a digital environment, the risk management system monitors the entire risk framework of the organization. If any factor or risk changes, it is immediately visible to everyone. Computers are also much better at continuously monitoring risks than people. Monitoring risks also allows your business to ensure continuity.

DIFFERENT RISKS THAT MAY HAVE TO BE CONSIDERED IN ANY TYPICAL INFRASTRUCTURE CONSTRUCTION PROJECT

All construction projects carry some level of risk. Being able to identify and manage risks requires skill, careful planning, and being able to make good decisions quickly. When risks become reality, they can be detrimental to the successful completion of your project. Properly managed risks can lead to higher profits, stronger relationships with clients and the ability to grow and expand your business.

Here are some common risk factors to watch out for on construction projects along with tips on how to properly manage them and prevent them from derailing your project.

Labor Shortages & Productivity Issues

Not having enough workers available to complete a project or hit productivity goals is a huge risk when taking on new projects. Without the manpower to perform the work, the project can suffer from longer construction schedules and potential delays in delivering the project on time to the owner.

Issues with labor shortages have been plaguing the construction industry since recovery from the last recession began. Construction companies have been struggling to fill positions to keep up with the growing demand for their services.

The construction industry lost over a million jobs from February to March 2020 due to the COVID-19 pandemic but has since gained back 857,000 as of December 2020. Despite the gains, the unemployment rate in construction has increased as of December 2020 to 9.6% According to the latest Job Openings and Labor Turnover Summary (JOLTS) report from the Bureau of Labor Statistics, in November 2020 there were 236,000 openings in construction in June.

To fill vacancies, many construction firms have been taking on employees with little or no previous construction experience. This isn't necessarily a bad thing, but there are additional risks that come with having a less experienced crew. These workers won't have the same skill sets as experienced workers, meaning they will be less productive and will probably need closer supervision when they are first starting out

Safety is also a construction project risk factor to consider when working with new employees. They lack the training and experience to know all the rules or be able to identify hazardous situations on the jobsite. Safety training is just as, if not more, important as skills training and should be a top priority with new hires.

To combat labor shortages, offer competitive wages and benefits and develop a strong company culture that values employees and rewards hard work and dedication. This requires time and money to invest in training and development of your workforce.

To retain workers, provide opportunities for training, mentoring, and continuing education courses available to both your new and existing employees. Establish advancement opportunities and career paths for workers to move up within your organization.

Health & Safety Hazards

Keeping workers safe should be the top priority on every jobsite. Site conditions can change rapidly, and unexpected hazards can crop up at any time creating unexpected project risks. Major accidents can result in serious injuries or fatalities to your employees. Your goal on every project should be to be accident-free and ensure every worker goes home safe to their family.

In addition to the potential harm to workers, a serious accident can cause work to be stopped or delayed and lead to a decrease in productivity due to low morale among your workers. This can put your project, and your company, in huge financial risk due to all the costs associated with dealing with an accident.

Subcontractor Default

Dealing with a subcontractor that fails to perform on a project is a major risk factor for general contractors on construction projects. A defaulting subcontractor that isn't meeting its contractual obligations can completely wreck your project schedule and destroy your profit margin. Schedule delays can also impact other subcontractors and can result in costly rework.

No subcontractor starts a job with the intention of defaulting on the work. Subcontractors must front a substantial portion of the costs on a project before they start getting paid. This can quickly lead to cash flow problems if they overextend themselves by taking on too much work or payments on other projects are being delayed.

Be proactive in monitoring your subcontractors if you suspect things might be getting off track. Common signs to watch for include a sudden decrease in the subcontractor's workforce on the jobsite, delayed materials deliveries, and failure to pay their subcontractors or suppliers on time.

Replacing a terminated subcontractor or supplementing their uncompleted work can kill a project and hurt your company's reputation. You might be better off working with them to solve any issues to complete the project rather than letting them go.

If a subcontractor is experiencing difficulties, they might not be upfront with you about the problem. Address any red flags with your subcontractor regardless of their performance. If you wait too long to confront a struggling subcontractor, you might not be able to recover.

Change Orders

Change orders are an inevitable part of construction and can be a major risk factor when not managed properly. A change order is simply an addendum or amendment to the original construction contract or the scope of work. They can be initiated by the owner, general contractor, or subcontractors. They typically require performing additional work for reasons such as omissions or errors in the original scope of work or ambiguous construction drawings.

Increased project costs, delays in hitting contract milestones, interruptions of workflow, and not completing a project on time are some of the issues caused by poorly handled change orders. Managing change orders takes preparation, understanding, and lots of communication with all parties involved with the project.

Other Construction Project Risks

Other risk factors common on construction projects include incomplete drawings and poorly defined scope, design errors, unknown site conditions, poorly written contracts, unexpected increases in material costs, and poor project management. Properly identifying and managing construction risks are key to completing successful and profitable projects.

Definition of Risk

In the ordinary sense, the risk is the outcome of an action taken or not taken, in a particular situation which may result in loss or gain. It is termed as a chance or loss or exposure to danger, arising out of internal or external factors that can be minimised through preventive measures.

In the financial glossary, the meaning of risk is not much different. It implies the uncertainty regarding the expected returns on the investments made i.e. the probability of actual returns may not be equal to the expected returns. Such a risk may include the probability of losing the part or whole investment. Although the higher the risk, the higher is the expectation of returns, because investors are paid off for the additional risk they take on their investments. The major elements of risk are defined as below:

- **Systematic Risk**: Interest Risk, Inflation Risk, Market Risk, etc.
- **Unsystematic Risk**: Business Risk and Financial Risk.

Definition of Uncertainty

By the term uncertainty, we mean the absence of certainty or something which is not known. It refers to a situation where there are multiple alternatives resulting in a specific outcome, but the probability of the outcome is not certain. This is because of insufficient information or knowledge about the present condition. Hence, it is hard to define or predict the future outcome or events.

Uncertainty cannot be measured in quantitative terms through past models. Therefore, probabilities cannot be applied to the potential outcomes, because the probabilities are unknown.

Key Differences between Risk and Uncertainty

The difference between risk and uncertainty can be drawn clearly on the following grounds:

1. The risk is defined as the situation of winning or losing something worthy. Uncertainty is a condition where there is no knowledge about the future events.
2. Risk can be measured and quantified, through theoretical models. Conversely, it is not possible to measure uncertainty in quantitative terms, as the future events are unpredictable.
3. The potential outcomes are known in risk, whereas in the case of uncertainty, the outcomes are unknown.
4. Risk can be controlled if proper measures are taken to control it. On the other hand, uncertainty is beyond the control of the person or enterprise, as the future is uncertain.
5. Minimization of risk can be done, by taking necessary precautions. As opposed to the uncertainty that cannot be minimised.
6. In risk, probabilities are assigned to a set of circumstances which is not possible in case of uncertainty.

FINANCIAL MANAGEMENT

WHAT IS FINANCIAL MANAGEMENT

Financial management is the use of a company's financial resources. This includes the use of cash and other assets— such as equipment. Many everyday decisions affect a company's financial future. For example, the decision to bid on a large project can have a great impact on the finances of a company

WHY IS CONSTRUCTION FINANCIAL MANAGEMENT DIFFERENT?

Construction companies are different from most other companies and are faced with many unique challenges and problems not faced by companies in other industries. Although the construction industry is producing a product—as do manufacturing plants—the construction of buildings, roads, and other structures is different from the manufacturing of most other products. Because of these unique characteristics, the financial management principles applied to other product-producing industries often need to be modified before they are applied to the construction industry, otherwise they are useless.

PROJECT FINANCING AND ITS DIFFERENT SOURCES

Project financing is a means of obtaining funds for industrial projects, long-term infrastructure, and public services. Many businesses use this funding method to take care of major projects using a non-recourse or limited financial structure. There are several ways to secure project finance, such as investor, loans, private finance, equity, funds, grants, etc. The repayment is managed from the cash-flow generated off the project.

It is a secured form of lending, accepting the project's rights, assets, and interests as collateral. Project loans are useful in more than one way. It can help expand the manufacturing capacity, rent a workstation, upgrade technology, handle unexpected expenses, experimentation for a new service or a product, create a cash pool, etc.

Below, are different sources from where one can obtain project financing.

1. **Business Angels**

Business Angels have a vast experience in the industry they operate in. Private investors may invest in a company for a capital gain. The investment is for a place on board or an equity stake.

2. **Venture Capital**

Venture capitalists invest in a project for a non-executive position on the board. They provide capital in exchange of an equity share or a position at a strategic level. Once the value of shares increase, they may sell those for a profit.

3. **Loan for Business**

Apart from secured lending, a company can choose unsecured **business loan** that comes for a fixed tenure with a repayment plan. The cost of loan is determined by estimating the returns from the project. The interest payment is tax deductible in some cases. An agreement is made between the financial institution and the borrower for a specific loan amount and tenure.

4. **Overdrafts**

Overdrafts are ideal for a short-term finance. The period of overdraft facility is for a year or less. The interest is only charged on the amount spent from the person's account. Such financing can be arranged quickly like business loans.

5. **Share Capital**

The shareholders get profits from dividend. This share of profit is derived from ordinary shares (owned by business owners who can share profits of an organization from dividends) or preference shares (does not belong to company owners but a third-party). Capital gain is expected from selling the shares in future. It is the company shareholders who raise the Share Capital.

6. **Debentures**

Debenture loans come with a fixed or a floating rate and provided against an organization's assets. The debenture holders receive payment of interest before the shareholders receive their dividend payment. If the business fails, then these holders are liable as preferential creditors.

A project loan offers a great opportunity to fund-providers and investors to be a part of the company's growth process and share its profits. The above-mentioned sources for project financing are crucial for new companies. Apart from these sources, a few others to mention are project grants and government funding.

IMPORTANCE OF COST CONTROL IN CONSTRUCTION PROJECTS

The main objective of cost control of a project is to gain the maximum profit within the designated period within the budget. To monitor and control actual expenditure against the estimated project budget.

The cost control is a process that should be continued through the construction period to ensure that the cost of the building is kept within the agreed cost limits. The cost control can be divided into major areas: the control of cost during design stages and the control of cost by the contractors once the construction project has started.

According to Nunnaly (1998), cost control of projects involves the measuring and collecting the cost record of a project and the work progress. It involves the comparison of actual progress with the planning.

The main aims of the cost control:

a) To gain the maximum profit within the designated period within the budget.

b) To keep the total expenditure within the amount agreed by client, frequently based on approximate estimate of cost prepared by the quantity surveyor in the early stages of the design process. There is a need for strict cost discipline thought all stages of design and execution to ensure that the initial estimate, tender figure and final account sum all are closely related.

c) To give the building client good value for money i.e., a building which is soundly constructed, of satisfactory appearance and well suited to perform the functions for which it is required, combined with economical construction and layout.

d) To achieve a balanced and logical distribution of the available funds between the various parts of the building.

The cost controlling system we can use as a tool for estimating the new projects based on previous experience. To win a new project

TECHNIQUES AND WAYS FOR CONTROLLING COSTS

The most important of all the cost control techniques is to appoint a small team of qualified and experienced people well versed in the financial management team to manage the daily finances of the company in a very professional and systematic manner.

The cost control software can be helpful in doing the work in comparatively less time and with more accuracy.

A. COST VALUE RECONCILATION

The cost and value of the project must be reconciled and monitored regularly. This allows a contractor to manage and control actual against estimated expenditure.

This will also allow the contractor to monitor costs and value movement and importantly also monitor profit levels.

For the purpose of cost control, it is not sufficient to consider only the past records of costs and revenues incurred in a project. Good project commercial managers should focus upon future revenues, future costs and technical problems. For this purpose, traditional financial accounting schemes are not adequate to reflect the dynamic nature of building project.

B. CONTROL OF PROJECT CASH FLOW

A positive cash flow is critical to any business. On building projects cash flow is typically generated from the periodic payment for works completed, i.e. interim payments.

Cash flow forecasts are unusually prepared and based on the program of works for internal purposes and for use by the client.

Sample cash flow forecast

C. BREAK - EVEN ANALYSIS

A break even analysis determines the point at which one method becomes superior to another method of accomplishing some task or objective. Break even analysis is a common and important part of cost control.

D. BUDGETARY CONTROL

Budgets are used for planning and controlling the income and expenditure in many different organizations. It is through the budget that a company has plans and objectives can be converted into quantitative and monetary terms. Without these a company has little control. The budget may represent a total sum divided among a number of subheadings or work packages. It is important that the various sub headings include a timescale, since the expenditure by both the contractor and the client needs to be matched against income or the availability of funds.

E. CONTRACTORS COST CONTROL

The contractor, having priced successfully enough to win the contract through tendering, must now ensure that the work can be completed for the estimated costs. One of the duties of the contractor as quantity surveyors is to monitor the expenditure and advice site management of action that should be taken. This process also includes the costs of subcontractors, since these forms a part of the main contractor has total expenditure. The contractor has surveyors also comment on the profitability of different site operations. Where loss-making situations are encountered, decisions need to be taken to reverse this position if at all possible.

F. COST COMPARISION

In practice it is always difficult make to comparison between costs and valuations, since either the full items of expenditure are unavailable or the valuation has only been approximately prepared. However, the contractor does need to determine which contracts are profitable and which are not, and also to determine which operations gain or lose money. The information which is the generated may be used to form the basis of contractual claims or to assist in future tendering and the contractor as selection of projects for which to tender.

G. SCHEDULE CONTROL

In addition to cost control, commercial managers must also give considerable attention to monitoring schedules. (E.g. variation schedules) Construction typically involves a deadline for work completion, so contractual agreements will force attention to schedules. More generally, delays in construction represent additional costs to the client due to late facility occupancy or other factors. Just as costs incurred are compared to budgeted costs, actual activity durations may be compared to expected durations. In this process, forecasting the time to complete particular activities may be required.

COST ESTIMATION

Costs Associated with Constructed Facilities

The costs of a constructed facility to the owner include both the initial capital cost and the subsequent operation and maintenance costs. Each of these major cost categories consists of a number of cost components.

The capital cost for a construction project includes the expenses related to the inital establishment of the facility:

- Land acquisition, including assembly, holding and improvement
- Planning and feasibility studies
- Architectural and engineering design
- Construction, including materials, equipment and labor
- Field supervision of construction
- Construction financing
- Insurance and taxes during construction
- Owner's general office overhead
- Equipment and furnishings not included in construction
- Inspection and testing

The operation and maintenance cost in subsequent years over the project life cycle includes the following expenses:

- Land rent, if applicable
- Operating staff
- Labor and material for maintenance and repairs
- Periodic renovations
- Insurance and taxes
- Financing costs
- Utilities
- Owner's other expenses

The magnitude of each of these cost components depends on the nature, size and location of the project as well as the management organization, among many considerations. The owner is interested in achieving the lowest possible overall project cost that is consistent with its investment objectives.

APPROACHES TO COST ESTIMATION

Cost estimating is one of the most important steps in project management. A cost estimate establishes the base line of the project cost at different stages of development of the project. A cost estimate at a given stage of project development represents a prediction provided by the cost engineer or estimator on the basis of available data. According to the American Association of

Cost Engineers, cost engineering is defined as that area of engineering practice where engineering judgment and experience are utilized in the application of scientific principles and techniques to the problem of cost estimation, cost control and profitability.

Virtually all cost estimation is performed according to one or some combination of the following basic approaches:

Production function. In microeconomics, the relationship between the output of a process and the necessary resources is referred to as the production function. In construction, the production function may be expressed by the relationship between the volume of construction and a factor of production such as labor or capital. A production function relates the amount or volume of output to the various inputs of labor, material and equipment

Empirical cost inference. Empirical estimation of cost functions requires statistical techniques which relate the cost of constructing or operating a facility to a few important characteristics or attributes of the system. The role of statistical inference is to estimate the best parameter values or constants in an assumed cost function. Usually, this is accomplished by means of regression analysis techniques.

Unit costs for bill of quantities. A unit cost is assigned to each of the facility components or tasks as represented by the bill of quantities. The total cost is the summation of the products of the quantities multiplied by the corresponding unit costs. The unit cost method is straightforward in principle but quite laborious in application. The initial step is to break down or disaggregate a process into a number of tasks. Collectively, these tasks must be completed for the construction of a facility. Once these tasks are defined and quantities representing these tasks are assessed, a unit cost is assigned to each and then the total cost is determined by summing the costs incurred in each task. The level of detail in decomposing into tasks will vary considerably from one estimate to another.

Allocation of joint costs. Allocations of cost from existing accounts may be used to develop a cost function of an operation. The basic idea in this method is that each expenditure item can be assigned to particular characteristics of the operation. Ideally, the allocation of joint costs should be causally related to the category of basic costs in an allocation process. In many instances, however, a causal relationship between the allocation factor and the cost item cannot be identified or may not exist. For example, in construction projects, the accounts for basic costs may be classified according to (1) labor, (2) material, (3) construction equipment, (4) construction supervision, and (5) general office overhead. These basic costs may then be allocated proportionally to various tasks which are subdivisions of a project.

TYPES OF CONSTRUCTION COST ESTIMATES

Construction cost constitutes only a fraction, though a substantial fraction, of the total project cost. However, it is the part of the cost under the control of the construction project manager. The required levels of accuracy of construction cost estimates vary at different stages of project development, ranging from ball park figures in the early stage to fairly reliable figures for budget control prior to construction. Since design decisions made at the beginning stage of a project life cycle are more tentative than those made at a later stage, the cost estimates made at the earlier stage are expected to be less accurate. Generally, the accuracy of a cost estimate will reflect the information available at the time of estimation.

Construction cost estimates may be viewed from different perspectives because of different institutional requirements. In spite of the many types of cost estimates used at different stages of a project, cost estimates can best be classified into three major categories according to their functions. A construction cost estimate serves one of the three basic functions: design, bid and control. For establishing the financing of a project, either a design estimate or a bid estimate is used.

1. **Design Estimates.** For the owner or its designated design professionals, the types of cost estimates encountered run parallel with the planning and design as follows:
    - Screening estimates (or order of magnitude estimates)
    - Preliminary estimates (or conceptual estimates)
    - Detailed estimates (or definitive estimates)
    - Engineer's estimates based on plans and specifications

    For each of these different estimates, the amount of design information available typically increases.

2. **Bid Estimates.** For the contractor, a bid estimate submitted to the owner either for competitive bidding or negotiation consists of direct construction cost including field supervision, plus a markup to cover general overhead and profits. The direct cost of construction for bid estimates is usually derived from a combination of the following approaches.
    - Subcontractor quotations
    - Quantity takeoffs
    - Construction procedures.
3. **Control Estimates.** For monitoring the project during construction, a control estimate is derived from available information to establish:
    - Budget estimate for financing
    - Budgeted cost after contracting but prior to construction
    - Estimated cost to completion during the progress of construction.

Design Estimates

In the planning and design stages of a project, various design estimates reflect the progress of the design. At the very early stage, the screening estimate or order of magnitude estimate is usually made before the facility is designed, and must therefore rely on the cost data of similar facilities built in the past. A preliminary estimate or conceptual estimate is based on the conceptual design of the facility at the state when the basic technologies for the design are known. The detailed estimate or definitive estimate is made when the scope of work is clearly defined and the detailed design is in progress so that the essential features of the facility are identifiable. The engineer's estimate is based on the completed plans and specifications when they are ready for the owner to solicit bids from construction contractors. In preparing these estimates, the design professional will include expected amounts for contractors' overhead and profits.

The costs associated with a facility may be decomposed into a hierarchy of levels that are appropriate for the purpose of cost estimation. The level of detail in decomposing the facility into tasks depends on the type of cost estimate to be prepared. For conceptual estimates, for example, the level of detail in defining tasks is quite coarse; for detailed estimates, the level of detail can be quite fine.

As an example, consider the cost estimates for a proposed bridge across a river. A screening estimate is made for each of the potential alternatives, such as a tied arch bridge or a cantilever truss bridge. As the bridge type is selected, e.g. the technology is chosen to be a tied arch bridge instead of some new bridge form, a preliminary estimate is made on the basis of the layout of the selected bridge form on the basis of the preliminary or conceptual design. When the detailed design has progressed to a point when the essential details are known, a detailed estimate is made on the basis of the well-defined scope of the project. When the detailed plans and specifications are completed, an engineer's estimate can be made on the basis of items and quantities of work.

Bid Estimates

The contractor's bid estimates often reflect the desire of the contractor to secure the job as well as the estimating tools at its disposal. Some contractors have well established cost estimating procedures while others do not. Since only the lowest bidder will be the winner of the contract in most bidding contests, any effort devoted to cost estimating is a loss to the contractor who is not a successful bidder. Consequently, the contractor may put in the least amount of possible effort for making a cost estimate if it believes that its chance of success is not high.

If a general contractor intends to use subcontractors in the construction of a facility, it may solicit price quotations for various tasks to be subcontracted to specialty subcontractors. Thus, the general subcontractor will shift the burden of cost estimating to subcontractors. If all or part of the construction is to be undertaken by the general contractor, a bid estimate may be prepared on the basis of the quantity take-offs from the plans provided by the owner or on the basis of the construction procedures devised by the contractor for implementing the project. For example, the cost of a footing of a certain type and size may be found in commercial publications on cost data which can be used to facilitate cost estimates from quantity take-offs. However, the contractor may want to assess the actual cost of construction by considering the actual construction procedures to be used and the associated costs if the project is deemed to be different from typical designs. Hence, items such as labor, material and equipment needed to perform various tasks may be used as parameters for the cost estimates.

Control Estimates

Both the owner and the contractor must adopt some base line for cost control during the construction. For the owner, a budget estimate must be adopted early enough for planning long term financing of the facility. Consequently, the detailed estimate is often used as the budget estimate since it is sufficient definitive to reflect the project scope and is available long before the engineer's estimate. As the work progresses, the budgeted cost must be revised periodically to reflect the estimated cost to completion. A revised estimated cost is necessary either because of change orders initiated by the owner or due to unexpected cost overruns or savings.

For the contractor, the bid estimate is usually regarded as the budget estimate, which will be used for control purposes as well as for planning construction financing. The budgeted cost should also be updated periodically to reflect the estimated cost to completion as well as to insure adequate cash flows for the completion of the project.

ACCOUNTING SYSTEM

An accounting system is the system used to manage the income, expenses, and other financial activities of a business.

An accounting system allows a business to keep track of all types of financial transactions, including purchases (expenses), sales (invoices and income), liabilities (funding, accounts payable), etc. and is capable of generating comprehensive statistical reports that provide management or interested parties with a clear set of data to aid in the decision-making process.

1. <u>Expenses</u>: The amount of cash that flows out of the company in exchange for goods or services from another person or company are the expenses. In older accounting software or with a manual system such as Excel, it is necessary to manually enter, balance, and categorise each expense. An automatic accounting system allows quick entry, categorisation and automatic balance of expenses.
2. <u>Invoices</u>: Creating a professional looking invoice is an important part of developing a positive brand image and building confidence with customers. Today, some accounting systems such as Debitoor allow for instant invoice creation with the ability to customise and automatically keep track of paid invoices and income.
3. <u>Funding</u>: All the business liabilities, whether accounts payable, bank loans taken to support the business, or mortgages, etc. An accounting system keeps track of these liabilities as payable values and automatically updates the balances as soon a payment is made and accounts are settled.