

## UNIT – I

### **Objective:**

- To familiarize with the concepts of Prestressing and IS Code Provisions

### **Syllabus**

**Introduction:** Historic development- General principles of prestressing, pretensioning and post tensioning- Advantages and limitations of prestressed concrete- Materials- High strength concrete and high strength steel their characteristics. I.S. Code provisions, Methods and Systems of Prestressing- Pre-tensioning and post tensioning methods – Different systems of prestressing like Hoyer System, Magnel System, Freyssinet system and Gifford – Udall System.

### **Learning Outcomes:**

Students will be able to

- understand apply the principles of prestressing
- illustrate the advantages and disadvantages of prestressed concrete
- explain the characteristics of high strength steel and concrete
- apply the Concepts and methods for prestressing systems

## **Learning Material**

### **UNIT-I Introduction**

#### **1.1 Introduction**

This section covers the following topics.

- [Basic Concept](#)
- [Early Attempts of Prestressing](#)
- [Brief History](#)
- [Development of Building Materials](#)

##### **1.1.1 Basic Concept**

A prestressed concrete structure is different from a conventional reinforced concrete structure due to the application of an initial load on the structure prior to its use.

The initial load or ‘prestress’ is applied to enable the structure to counteract the stresses arising during its service period.

The prestressing of a structure is not the only instance of prestressing. The concept of prestressing existed before the applications in concrete. Two examples of prestressing before the development of prestressed concrete are provided. Force-fitting of metal bands on wooden

barrels the metal bands induce a state of initial hoop compression, to counteract the hoop tension caused by filling of liquid in the barrels. Pre-tensioning the spokes in a bicycle wheel the pre-tension of a spoke in a bicycle wheel is applied to such an extent that there will always be a residual tension in the spoke. For concrete, internal stresses are induced (usually, by means of tensioned steel) for the following reasons.

The tensile strength of concrete is only about 8% to 14% of its compressive strength. Cracks tend to develop at early stages of loading in flexural members such as beams and slabs. To prevent such cracks, compressive force can be suitably applied in the perpendicular direction. Prestressing enhances the bending, shear and torsional capacities of the flexural members. In pipes and liquid storage tanks, the hoop tensile stresses can be effectively counteracted by circular prestressing.

### **1.1.2 Early Attempts of Prestressing**

Prestressing of structures was introduced in late nineteenth century Mild steel rods are stretched and concrete is poured around them. After hardening of concrete, the tension in the rods is released. The rods will try to regain their original length, but this is prevented by the surrounding concrete to which the steel is bonded. Thus, the concrete is now effectively in a state of pre-compression. It is capable of counteracting tensile stress, such as arising from the load shown in the following sketch. But, the early attempts of prestressing were not completely successful. It was observed that the effect of prestress reduced with time. The load resisting capacities of the members were limited. Under sustained loads, the members were found to fail. This was due to the following reason.

Concrete shrinks with time. Moreover under sustained load, the strain in concrete increases with increase in time. This is known as creep strain. The reduction in length due to creep and shrinkage is also applicable to the embedded steel, resulting in significant loss in the tensile strain. In the early applications, the strength of the mild steel and the strain during prestressing were less. The residual strain and hence, the residual prestress was only about 10% of the initial value.

The residual strain in steel = original tensile strain in steel – compressive strains corresponding to short-term and long-term losses. The total loss in strain due to elastic shortening, creep and shrinkage was also close to 0.0007. Thus, the residual strain was negligible. The solution to increase the residual strain and the effective prestress are as follows. Adopt high strength steel with much higher original strain. This leads to the scope of high prestressing force. Adopt high strength concrete to withstand the high prestressing force.

### 1.1.3 Brief History

Before the development of prestressed concrete, two significant developments of reinforced concrete are the invention of Portland cement and introduction of steel in concrete. These are also mentioned as the part of the history. The key developments are mentioned next to the corresponding year.

<b>1824 Aspdin, J., (England)</b>	Obtained a patent for the manufacture of Portland cement.
<b>1857 Monier, J., (France)</b>	Introduced steel wires in concrete to make flower pots, pipes, arches and slabs.

The following events were significant in the development of prestressed concrete.

<b>1886 Jackson, P. H., (USA)</b>	Introduced the concept of tightening steel tie rods in artificial stone and concrete arches.
<b>1888 Dohring, C. E. W., (Germany)</b>	Manufactured concrete slabs and small beams with embedded tensioned steel.
<b>1908 Stainer, C. R., (USA)</b>	Recognized losses due to shrinkage and creep, and suggested retightening the rods to recover lost prestress.
<b>1923 Emperger, F., (Austria)</b>	Developed a method of winding and pre-tensioning high tensile steel wires around concrete pipes.
<b>1924 Hewett, W. H., (USA)</b>	Introduced hoop-stressed horizontal reinforcement around walls of concrete tanks through the use of turnbuckles.

Thousands of liquid storage tanks and concrete pipes were built in the two decades to follow.

<b>1925 Dill, R. H., (USA)</b>	Used high strength unbonded steel rods. The rods were tensioned and anchored after hardening of the concrete
<b>1926 Eugene Freyssinet (France)</b>	Used high tensile steel wires, with ultimate strength as high as 1725 MPa and yield stress over 1240 MPa. He is often referred to as the Father of Prestressed concrete.
<b>1938 Hoyer, E., (Germany)</b>	Developed 'long line' pre-tensioning method.
<b>1940 Magnel, G., (Belgium)</b>	Developed an anchoring system for post-tensioning, using flat wedges.

During the Second World War, applications of prestressed and precast concrete increased rapidly. The names of a few persons involved in developing prestressed concrete are

mentioned. Guyon, Y., (France) built numerous prestressed concrete bridges in western and central Europe. Abeles, P. W., (England) introduced the concept of partial prestressing. Leonhardt, F., (Germany), Mikhailov, V., (Russia) and Lin, T. Y., (USA) are famous in the field of prestressed concrete.

The International Federation for Prestressing (FIP), a professional organisation in Europe was established in 1952. The Precast/Prestressed Concrete Institute (PCI) was established in USA in 1954. Prestressed concrete was started to be used in building frames, parking structures, stadiums, railway sleepers, transmission line poles and other types of structures and elements. In India, the applications of prestressed concrete diversified over the years. The first prestressed concrete bridge was built in 1948 under the Assam Rail Link Project. Among bridges, the Pamban Road Bridge at Rameshwaram, Tamilnadu, remains a classic example of the use of prestressed concrete girders.

#### **1.1.4 Development of Building Materials**

The development of prestressed concrete can be studied in the perspective of traditional building materials. In the ancient period, stones and bricks were extensively used. These materials are strong in compression, but weak in tension. For tension, bamboos and coir ropes were used in bridges. Subsequently iron and steel bars were used to resist tension. These members tend to buckle under compression. Wood and structural steel members were effective both in tension and compression.

In reinforced concrete, concrete and steel are combined such that concrete resists compression and steel resists tension. This is a passive combination of the two materials. In prestressed concrete high strength concrete and high strength steel are combined such that the full section is effective in resisting tension and compression. This is an active combination of the two materials. The following sketch shows the use of the different materials with the progress of time.

### **1.2 BASIC PRINCIPLE OF PRESTRESSING**

Since the tensile strength of concrete is low, a homogeneous concrete beam has very little flexural strength. To offset this deficiency, steel reinforcement is provided near the bottom of simple beams to carry the tensile stresses. However, a substantial area of concrete below the neutral axis merely retains the reinforcement in position, but its tensile strength is neglected in the computation for the flexural strength in case of reinforced concrete beams.

If the tensile reinforcement of beam is subjected to tensile stresses before applying the external loads, then compressive stresses are induced in the concrete of the beam (and this is done by

prestressing). Usually the tensile stresses in the concrete caused by the external loads are completely absorbed or counteracted by the compressive stresses in concrete, resulting from prestressing the reinforcement. The concrete, therefore, is being used effectively in resisting tensile stresses produced by external loads rather than being neglected as in case of RCC.

### **1.3 APPLICATIONS OF PRESTRESSED CONCRETE**

Prestressed concrete can be applied to almost all concrete constructions where ordinary reinforced concrete is used. But due to high cost of prestressing and better quality materials used, its use is made under special condition, particularly for precast members. In addition to structural precast members, viz., joists, beams, slabs, columns, girders, etc. Prestressed concrete is used for the framed multi-storied buildings. A large variety of industrial structures such as silos, roof trusses, water tanks, piles, pipes, nuclear power stations, factories, steel plants, electric sub-stations, etc. can also be built in prestressed concrete.

### **1.4 Advantages of Prestressed Concrete**

The prestressing of concrete has several advantages as compared to traditional reinforced concrete (RC) without prestressing. A fully prestressed concrete member is usually subjected to compression during service life. This rectifies several deficiencies of concrete. The following text broadly mentions the advantages of a pre-stressed concrete member with an equivalent RC member. For each effect, the benefits are listed.

#### **A) Section remains un-cracked under service loads**

1. Reduction of steel corrosion
  - Increase in durability.
2. Full section is utilized
  - Higher moment of inertia (higher stiffness)
3. Less deformations (improved serviceability).
4. Increase in shear capacity
5. Suitable for use in pressure vessels, liquid retaining structures.
  - Improved performance (resilience) under dynamic and fatigue loading.

#### **B) High span-to-depth ratios**

1. Larger spans possible with prestressing  
(Bridges, buildings with large column-free spaces)
1. Typical values of span-to-depth ratios in slabs are given below.

For the same span, less depth compared to RC member.

- Reduction in self-weight
- More esthetic appeal due to slender sections

- More economical sections.

C) Suitable for precast construction the advantages of precast construction are as follows.

- Rapid construction
- Better quality control
- Reduced maintenance
- Suitable for repetitive construction
- Multiple use of formwork

### **1.5 Limitations of Prestressing**

Although prestressing has advantages, some aspects need to be carefully addressed.

- Prestressing needs skilled technology. Hence, it is not as common as reinforced concrete.
- The use of high strength materials is costly.
- There is additional cost in auxiliary equipment's.
- There is need for quality control and inspection.

### **1.6 Disadvantages of prestressed concrete**

1. The following are among the advantages of using prestressed concrete.
2. The unit cost of high strength materials being used is higher.
3. Extra initial cost is incurred due to use of prestressing equipment and its installation.
4. Extra labor cost for prestressing is also there.
5. Prestressing is uneconomical for short spans and light loads.

### **1.7 Applications of High-Strength Concrete**

High-strength concrete is required in engineering projects that have concrete components that must resist high compressive loads. High-strength concrete is typically used in the erection of high-rise structures. It has been used in components such as columns (especially on lower floors where the loads will be greatest), shear walls, and foundations. High strengths are also occasionally used in bridge applications as well.

In high-rise structures, high-strength concrete has been successfully used in many U.S. cities. A high-rise structure suitable for high-strength concrete use is considered to be a structure over 30 stories. Not only has special concrete made such projects feasible due to load capacity, it has also allowed for the reduction of column and beam dimensions. Lower dead loads result, reducing the loads associated with foundation design. Also, owners benefit economically since the amount of rentable floor space, primarily on the lower floors, increases as the space occupied by the columns decreases. It is estimated that a 50-story structure with 4-foot diameter columns using 4000 psi concrete can reduce column diameters by approximately 33% by using 8000 psi concrete (Peterman).

High –strength concrete is occasionally used in the construction of highway bridges. High-strength concrete permits reinforced or prestressed concrete girders to span greater lengths than normal strength concrete girders.

Also, the greater individual girder capacities may enable a decrease in the number of girders required. Thus, an economical advantage is created for concrete producers in that concrete is promoted for use in a particular bridge project as opposed to steel.

### **1.8 Applications of High-Strength Steel**

In prestressed member, High strength steel is used because of the following reasons:-

1. Magnitude of pre stressing force is very high which can be increased either by increasing areas of steel or by increasing initial stress. Area can't be increased much as there will be problem in concreting so using high grade steel is good.
2. They are various losses present in steel so if we use low grade steel so effective stress will reduce leading to poor performance and high grade concrete:
  - a. To develop greater bond stress in pre tensioned member.
  - b. To resist high bearing and bending stresses
  - c. Reduce losses due to elastic shortening.

### **1.9 IS code Provisions:**

Provisions for prestressing as per IS 1343-2012 against the following conditions:

- a) **Tensioning apparatus:** The applied force should be measured with in an error not exceeding 5%. The releasing device should ensure that tension in the pre-stressing element is fully maintained during the period between tensioning and release.
- b) **Anchorage:** The anchorage should hold without more than normal slip the prestressing tendon subjected to a load midway between the proposed initial prestressing load and the ultimate strength of prestressing tendon.
- c) **Tensioning:** when two or more prestressing tendons are to be tensioned simultaneously, care shall be taken to ensure that all such tendons are of same length from grip to grip, particularly of tendons are of smaller length than 7.5m.
- d) Measurement of prestressing force
- e) **Transfer of prestressing force:** It should be carried out gradually avoiding the severe eccentricities of prestressing force and then sudden application of stress to be concrete.
- f) Grouting requirements: The requirements of the grout are fluidity and low sedimentation in the plastic state. The compressive strength of 100mm cubes of the grout shall not be less than  $1700/\text{mm}^2$  at 7 days.

g) **Mixing:** water shall be measured and added to the mixer first and the mixing shall continue for at least 2min after all ingredients are added.

h) **Duct preparation:** duct shall be kept clean at all times, grout vents shall be provided at all crests and at intervals of 20m to 30m.

### **1.10 Methods of Precompression impart to concrete:**

The various methods by which pre-compression are imparted to concrete are classified as follows:

1. Generation of compressive force between the structural elements and its abutments using flat jack.
2. Development of hoop compression in cylindrically shaped structures by circumferential wire binding.
3. Use of longitudinally tensioned steel embedded in concrete or housed in ducts.
4. Use of principle of distortion of a statically indeterminate structure either by displacement or by rotation of one part relative to the remainder.
5. Use of deflected structural steel sections embedded in concrete until the hardening of the latter.
6. Development of limited tension in steel and compression in concrete by using expanding cements.

The most widely used method for prestressing of structural concrete elements is longitudinal tensioning of steel by different tensioning devices. Prestressing by the application of direct forces between abutments is generally used for arches and pavements, while flat jacks are invariably used to impart the desired forces.

### **Tensioning Devices:**

The various types devices used for tensioning steel are grouped under four principal categories, viz.

**1. Mechanical devices:** The mechanical devices generally used include weights with or without lever transmission, geared transmission in conjunction with pulley blocks, screw jacks with or without gear devices and wire-winding machines. These devices are employed mainly for prestressing structural concrete components produced on a mass scale in factory.

**2. Hydraulic devices:** These are simplest means for producing large prestressing force, extensively used as tensioning devices.

**3. Electrical devices:** The wires are electrically heated and anchored before placing concrete in the mould. This method is often referred to as thermo-prestressing and used for tensioning of steel wires and deformed bars.



**4. Chemical devices:** Expanding cements are used and the degree of expansion is controlled by varying the curing condition. Since the expansive action of cement while setting is restrained, it induces tensile forces in tendons and compressive stresses in concrete.

### **1.11 Methods of Prestressing:**

a) External prestressing

b) Internal prestressing

i) Linear and circular prestressing

ii) Pre-tensioning and post tensioning

iii) End anchored and non anchored tendons

iv) Bonded and unbounded tendons

v) Precast, cast in situ and composite construction

vi) Full and partial prestressing

#### **a) External prestressing:**

When the prestressing is achieved by elements located outside the concrete, it is called external prestressing. The tendons can lie outside the member (for example in I-girders or walls) or inside the hollow space of a box girder. This technique is adopted in bridges and strengthening of buildings.

#### **b) Internal prestressing**

When the prestressing is achieved by elements located inside the concrete member (commonly, by embedded tendons), it is called internal prestressing. Most of the applications of prestressing are internal prestressing.

##### **i) Linear and circular prestressing**

When the prestressed members are straight or flat, in the direction of prestressing, the prestressing is called linear prestressing. For example, prestressing of beams, piles, poles and slabs. The profile of the prestressing tendon may be curved.

When the prestressed members are curved, in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing of tanks, silos, pipes and similar structures.

##### **ii) Pre-tensioning and post tensioning**

**Pre-tensioning:** The tension is applied to the tendons before casting of the concrete. The precompression is transmitted from steel to concrete through bond over the transmission length near the ends.

Post-tensioning: The tension is applied to the tendons (located in a duct) after hardening of the concrete. The pre-compression is transmitted from steel to concrete by the anchorage device (at the end blocks).

### **iii) End anchored and non anchored tendons**

In case of post tensioned system the tendons are anchored at their ends

### **iv) Bonded and unbonded tendons**

Bonded tendons are bonded throughout their length to surrounding concrete.

### **v) Precast, cast in situ and composite construction**

In case of precasting concreting is usually done somewhere near to the site, cast in situ requires more form of form work and scaffolding

### **vi) Full and partial prestressing**

When the level of prestressing is such that no tensile stress is allowed in concrete under service loads, it is called Full Prestressing

When the level of prestressing is such that the tensile stress under service loads is within the cracking stress of concrete, it is called Limited Prestressing.

When the level of prestressing is such that under tensile stresses due to service loads, the crack width is within the allowable limit, it is called Partial Prestressing.

### **Uniaxial, Biaxial or Multiaxial Prestressing**

When the prestressing tendons are parallel to one axis, it is called Uniaxial Prestressing. For example, longitudinal prestressing of beams.

When there are prestressing tendons parallel to two axes, it is called Biaxial Prestressing.

When the prestressing tendons are parallel to more than two axes, it is called Multiaxial Prestressing. For example, prestressing of domes.

## **1.12 systems of prestressing:**

### **a) Pretensioning system:**

In the pretensioning system, the tendons are first tensioned between rigid anchor-blocks cast on the ground or in a column or unit-mould-type pretensioning bed, prior to the casting of concrete in the moulds. A typical column-type tensioning bed is shown in Fig. The tendons comprising individual wires or strands are stretched with constant eccentricity as shown in (a) or variable eccentricity as shown in (b) with tendon anchorage at one end and jacks at the other. With the forms in place, the concrete is cast around the stressed tendon

Fig: Methods of pretensioning

High early-strength concrete is often used in a factory to facilitate early stripping and reuse of moulds. When the concrete attains sufficient strength, the jacking pressure is released. The high-tensile wires tend to shorten but are checked by the bond between concrete and steel. In this way the prestress is transferred to the concrete by bond, mostly near the ends of the beam, and no special anchorages are required in pretensioned members \_

For mass production of pretensioned elements, the long-line process developed by Hoyer is generally used in a factory. In this method the tendons are stretched between two bulk heads several hundred meters apart so that a number of similar units may be cast along the same group of tensioned wires as shown in Fig. The tension is applied by hydraulic jacks or by a moveable stressing machine. The wires or strands when tensioned singly or in groups are generally anchored to the abutments by steel wedges.

Fig: Hoyer's long line system of prestressing

The transfer of prestress to concrete is usually achieved by large hydraulic or screw jacks by which all the wires are simultaneously released after the concrete attains the requisite compressive strength. Generally, strands of up to 18 mm diameter and high-tensile wires of up to 7 mm diameter anchor themselves satisfactorily with the help of the surface bond and the interlocking of the surrounding matrix in the micro indentations on the wires. The bond of prestressing wires may be considerably improved by forming surface indentations and by helical crimping of the wires. Strands have considerably better bond characteristics than plain wires of equal cross-sectional area. Supplementary anchoring devices are required when single wires of larger diameter (exceeding 7 mm) are used in the pretensioned units.

## **2. Post-tensioned system:**

In post-tensioning the concrete unit are first cast by incorporating ducts or grooves to house the tendons. When the concrete attains sufficient strength, the high-tensile wires are tensioned by means of jack bearing on the end of the face of the member and anchored by wedge or nuts. The forces are transmitted to the concrete by means of end anchorage and, when the cable is curved, through the radial pressure between the cable and the duct. The space between the tendons and the duct is generally grouted after the tensioning operation. Most of the commercially patented prestressing systems are based on the following principle of anchoring the tendons:

1. Wedge action producing a frictional grip on the wire.
2. Direct bearing from the rivet or bolt heads formed at the end of the wire.
3. Looping the wire around the concrete.

**Methods:**

- a) Freyssinet system
- b) Gifford-Udall system
- c) Magnel -Blaton system

**a) Freyssinet system:**

Freyssinet anchorage system, which is widely used in Europe and India, Consists of a cylinder with a conical interior through which the. High-tensile wires pass and against the walls of which the wires are wedged by a conical plug lined longitudinally with grooves to house the wires as shown in Fig.

The main advantage of the Freyssinet system is that large number of wires or strands can lie simultaneously tensioned using the double. - Acting hydraulic jack as. Shown. In Fig

Fig: Freyssinet system

**b) Gifford-Udall system:**

The Gifford-Udall system developed in UK, consists of steel split cone cylindrical female-cone anchorages to house the high- tensile wires bearing against steel plates as shown in Fig. Each wire is tensioned separately and anchored by forcing a sleeve wedge into a cylindrical grip resting against a bearing plate. The ducts are generally formed by metal sheaths cast into the concrete member.

Fig: Gifford-Udall system

**c) Magnel -Blaton system:**

The Magnel-Blaton post-tensioning system adopts metallic sandwich plates, flat wedges, and a distribution plate for anchoring the wires. Each sandwich plate can house up to four pairs of wires. The distribution plate may be cast into the member at the desired location. The number of wires in the Magnel cable varies from 2 to The Magnel-Blaton anchorage as shown in Fig,

Fig: Magnel -Blaton system

**1.13 Comparative Study: Pretension Vs Post-tensioned Member:**

<b>Pre-tensioned member</b>	<b>Post-tensioned member</b>
1. In pretensioned prestress concrete, steel is tensioned prior to that of concrete. It is released once the concrete is placed and hardened. The stresses are transferred all along the wire by means of <b>bond</b> .	1. Concreting is done first then wires are tensioned and anchored at ends. The stress transfer is by <b>end bearing</b> not by bond.

<p>2. Suitable for short span and precast products like sleepers, electric poles on mass production.</p>	<p>2. Suitable for long span bridges</p>
<p>3. In pretensioning the cables are basically straight and horizontal. Placing them in curved or inclined position is difficult. However the wires can be kept with eccentricity. Since cables cannot be aligned similar to B.M.D. structural advantages are less compare to that of post-tensioned.</p>	<p>3. The post tensioning cables can be aligned in any manner to suit the B.M.D due to external load system. Therefore it is more economical particularly for long span bridges. The curved or inclined cables can have vertical component at ends. These components will reduce the design shear force. Hence post-tensioned beams are superior to pretensioned beams both from flexural and shear resistances point.</p>
<p>4. Prestress losses are more compare to that of Post-tensioned concrete.</p>	<p>4. Losses are less compare to pre-tensioned concrete</p>

## UNIT 2

### Analysis of Prestress Member

#### Basic assumption

1. Concrete is a homogenous material.
2. Within the range of working stress, both concrete & steel behave elastically, notwithstanding the small amount of creep, which occurs in both the materials under the sustained loading.
3. A plane section before bending is assumed to remain plane even after bending, which implies a linear strain distribution across the depth of the member.

#### Analysis of prestress member

The stress due to prestressing alone are generally combined stresses due to the action of direct load bending from an eccentrically applied load. The following notations and sign conventions are used for the analysis of prestress members.

$P$  = Pre-stressing force (Positive when compressive)

$e$  = Eccentricity of prestressing force

$M = Pe$  = Moment

$A$  = Cross-sectional area of the concrete member

$I$  = Second moment of area of the section about its centroid

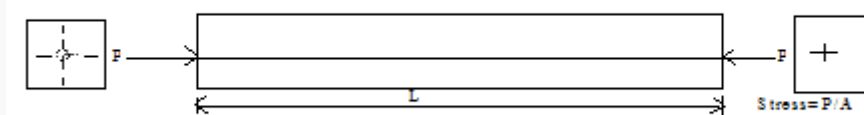
$Z_t, Z_b$  Section modulus of the top & bottom fiber respectively

$f_{top}, f_{bot}$  Prestress in concrete developed at the top & bottom fibers

$y_t, y_b$  Distance of the top & bottom fiber from the centroid of the section

$r$  Radius of gyration.

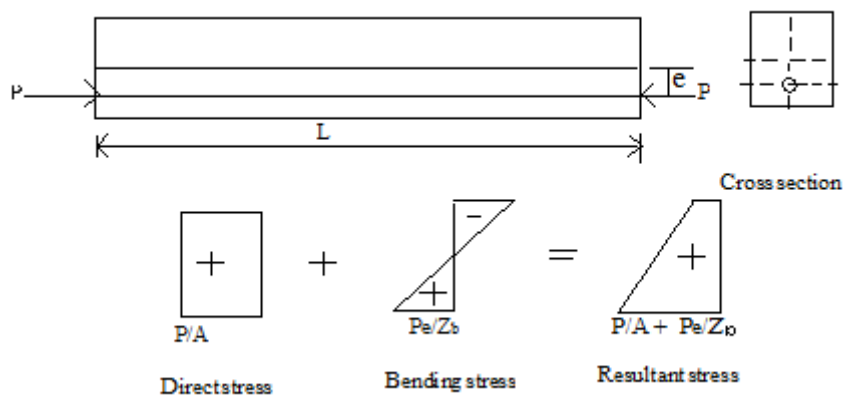
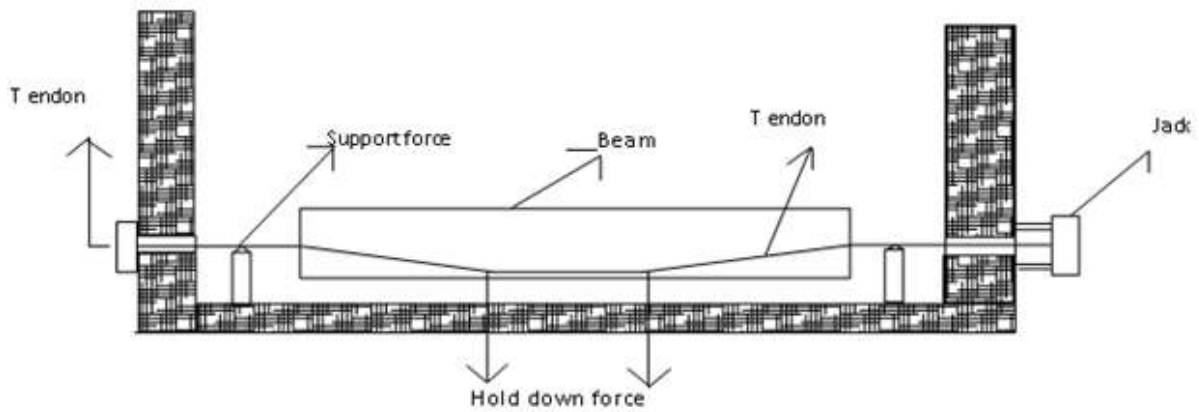
#### (i) Concentric tendon



Concentric prestressing

In this case, the load is applied concentrically and a compressive stress of magnitude  $(P/A)$  will act throughout the section. Thus the stress will generate in the section as shown in the figure below.

**(ii) Eccentric Tendon**



**Stress Distribution for Eccentric Tendon**

## UNIT 3

### Losses of Prestress

#### Objective:

- To familiarize with the different types of losses in pre-stress.

#### Syllabus

Loss of prestress in pre-tensioned and post-tensioned members due to various causes like elastic shortening of concrete, shrinkage of concrete, creep of concrete, relaxation of steel, slip in anchorage bending of member and frictional losses. Total losses allowed for design.

#### Learning Outcomes:

Students will be able to

- Understand the losses of pre-stress in pre-tensioned and post-tensioned members
- Understand the analysis of sections for flexure for different conditions like straight, concentric, eccentric, bent and parabolic tendons.

#### INTRODUCTION

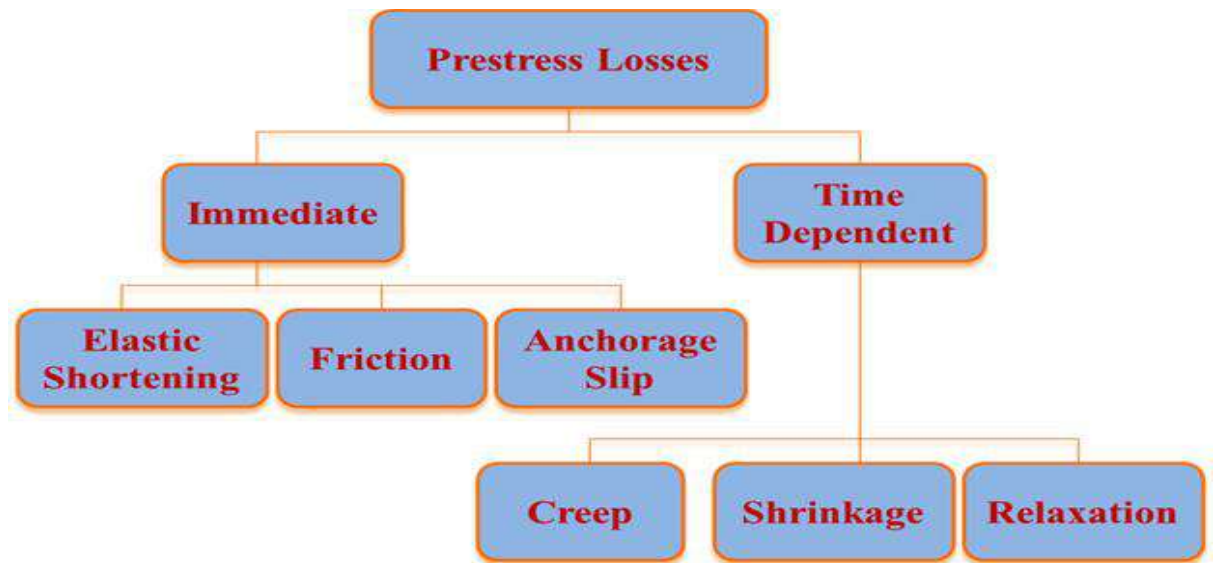
1. In pre-stressed concrete applications, most important variable is the prestress. Prestress does not remain constant (reduces) with time. Even during pre-stressing of tendons, and transfer of prestress, there is a drop of prestress from the initially applied stress. Reduction of prestress is nothing but the loss in prestress.

Loss of prestress is classified into two types:

2. Short-Term or Immediate Losses
  - Immediate losses occur during prestressing of tendons, and transfer of prestress to concrete member.
3. Long-Term or Time Dependent Losses
  - Time dependent losses occur during service life of structure.
4. **Immediate Losses includes**
  - Elastic Shortening of Concrete
  - Slip at anchorages immediately after prestressing and
  - Friction between tendon and tendon duct, and wobble Effect
5. **Time Dependent Losses includes**
  1. Creep and Shrinkage of concrete and



## 2. Relaxation of prestressing steel .



### Losses in Various Prestressing Systems

Type of Loss	Pre-tensioning	Post-tensioning
1. Elastic Shortening	Yes	No, if all the cables are simultaneously tensioned. If the wires are tensioned in stages loss will exist.
2. Anchorage Slip	No	Yes
3. Friction Loss	No	Yes
4. Creep and Shrinkage of Concrete	Yes	Yes
5. Relaxation of Steel	Yes	Yes

### Immediate Losses

- **Elastic Shortening of Concrete**

1. In pre-tensioned concrete, when the prestress is transferred to concrete, the member shortens and the prestressing steel also shortens in it. Hence there is a loss of prestress.
2. In case of post-tensioning, if all the cables are tensioned simultaneously there is no loss since the applied stress is recorded after the elastic shortening has completely occurred.

3. If the cables are tensioned sequentially, there is loss in a tendon during subsequent stretching of other tendons.
4. Loss of prestress mainly depends on modular ratio and average stress in concrete at the level of steel.
5. Loss due to elastic shortening is quantified by drop in prestress ( $\Delta f_p$ ) in a tendon due to change in strain in tendon ( $\Delta \epsilon_p$ ).
6. The change in strain in tendon is equal to the strain in concrete ( $\epsilon_c$ ) at the level of tendon due to prestressing force.
7. This assumption is due to strain compatibility between concrete and steel.
8. Strain in concrete at the level of tendon is calculated from the stress in concrete ( $f_c$ ) at the same level due to prestressing force.

The loss of prestress due to deformation of concrete depends on the modular ratio & the average stress in concrete at the level of steel as per IS 1343

If

$f_c$  Prestress in concrete at the level of steel

$E_s$  Modulus of elasticity of steel

$E_c$  Modulus of elasticity of concrete

$\alpha_e$  Modular ratio

Therefore, Loss of stress in steel =  $\alpha_e f_c$

If the initial stress in steel is known, the percentage loss of stress in steel due to elastic deformation of concrete can be computed.

### **Loss due to shrinkage of concrete**

#### Factors affecting the shrinkage in concrete

1. The loss due to shrinkage of concrete results in shortening of tensioned wires & hence contributes to the loss of stress.
2. The shrinkage of concrete is influenced by the type of cement, aggregate & the method of curing used.
3. Use of high strength concrete with low water cement ratio results in reduction in shrinkage and consequent loss of prestress
4. The primary cause of drying shrinkage is the progressive loss of water from concrete.
5. The rate of shrinkage is higher at the surface of the member.
6. The differential shrinkage between the interior surfaces of large member may result in strain gradients leading to surface cracking.

Hence, proper curing is essential to prevent cracks due to shrinkage in prestress members. In the case of pretensioned members, generally moist curing is restored in order to prevent shrinkage until the time of transfer. Consequently, the total residual shrinkage strain will be larger in pretensioned members after transfer of prestress in comparison with post-tensioned members, where a portion of shrinkage will have already taken place by the time of transfer of stress. This aspect has been considered in the recommendation made by the code (IS: 1343) for the loss of prestress due to shrinkage of concrete and is obtained below:

If

$$\begin{aligned} \epsilon_{cs} &= \text{Total residual shrinkage strain} = 300 \times 10^{-6} \text{ for pre-tensioning member} \\ &= 200 \times 10^{-6} \div \log_{10}(t+2) \text{ for post-tensioning member} \end{aligned}$$

$t$  = age of concrete at transfer

Then, the loss of stress =  $\epsilon_{cs} E_s$

### Loss due to creep of concrete

The sustained prestress in the concrete of a prestress member results in creep of concrete which is effectively reduces the stress in high tensile steel. The loss of stress in steel due to creep of concrete can be estimated if the magnitude of ultimate creep strain or creep- coefficient is know.

#### 1. Ultimate Creep strain method

The loss of stress in steel due to creep of concrete =  $\epsilon_{cc} f_c E_s$

Where,  $\epsilon_{cc}$  = Ultimate creep strain for a sustained unit stress

$f_c$  = compressive stress in concrete at the level of steel

### Loss due to relaxation of stress in steel

Most of the codes provide for the loss of stress due to relaxation of steel as a percentage of initial stress in steel. The BIS recommends a value varying from 0 to 90 N/mm<sup>2</sup> for stress in wires varying from  $0.5 f_{pu}$  to  $0.8 f_{pu}$

### Loss of stress due to friction

The magnitude of loss of stress due to friction is of following types: -

- Loss due to curvature effect, which depends upon the tendon form or alignment, which generally follows a curved profile along the length of the beam.
- Loss of stress due to wobble effect, which depends upon the local deviations in the alignment of the cable.

$$P_x = P_0 e^{-(\mu\alpha + kx)}$$

$P_0$  = The Prestressing force at the jacking end.

$\mu$  = coefficient of friction

$\alpha$  = the cumulative angle in radians

$k$  = friction coefficient for wave effect

### **Loss due to Anchorage slip**

The magnitude of loss of stress due to the slip in anchorage is computed as follows:

If

$\Delta$  = slip of anchorage in mm

$L$  = length of the cable in mm

$A$  = cross-sectional area of the cable in  $\text{mm}^2$

$P$  = prestressing force

Hence, loss of stress due to anchorage slip =  $E_s\Delta/L$

## UNIT - 4

### DESIGN OF SECTION FOR FLEXURE AND SHEAR

#### Objective:

- To familiarize with design of Flexure and shear of the section.

#### Syllabus

Design of section for flexure and shear: Allowable stress, design criteria as per I.S. code-elastic design of simple rectangular and I-section for flexure, shear, and principal stresses-design for shear in beams- kern –lines, cable profile.

#### Learning Outcomes:

Students will be able to

- design a section for flexure
- design a section for shear

#### DESIGN OF SECTION FOR SHEAR

We do not use the principles of compatibility of strains or the constitutive relationships for the stress-strain behavior of concrete or steel. We use an equilibrium equation that is based on the ultimate strength of the concrete under flexural shear. The total shear capacity of the member is the summation of the capacity of concrete and that of steel.

The ultimate resistance of a section, which is represented as  $V_uR$  consists of a concrete contribution  $V_c$  and the stirrup contribution  $V_s$ . Thus,  $V_uR = V_c + V_s$ . Here,  $V_c$  includes  $V_{cz}$  which is the contribution from uncracked concrete,  $V_a$  which is the aggregate interlock, and  $V_d$  which is the dowel action. Last time, when we studied the components of shear resistance, we found that for a prestressed concrete section, the components of shear resistance are  $V_{cz}$ ,  $V_a$ ,  $V_d$ , and  $V_s$ . If the tendon is inclined, then we get a vertical component of the prestressing force, which is represented as  $V_p$ . Three of these components are grouped together under the contribution of concrete, which is represented as  $V_c$ . Thus, now we are having three components: one is  $V_c$ , another is  $V_s$ , and when the tendon is inclined we include  $V_p$ .

The value of  $V_c$  depends on whether the section is cracked due to flexure or not. Section 22.4 of IS: 1343 - 1980, gives two expressions of  $V_c$ : one for cracked section, and the other for uncracked section. Last time, we had studied that when the concrete member is loaded, first the flexural cracks are initiated. Next, the flexural cracks tend to become flexure–shear cracks. As we move away from the mid-span, we may find some flexure–shear cracks developing.

The shear strength of concrete depends on whether the section has cracked or not. The code says that we need to calculate the shear capacity of concrete from two expressions. One expression is given for the cracked section, and the other expression is given for an uncracked section. Both the expressions need to be evaluated at a particular section, and the lower value obtained from the two expressions is selected. Usually, the expression for the uncracked section will govern near the support. The expression for the cracked section will govern near the mid-span.

Usually, for a beam under a uniformly distributed load, cracks generate near mid-span and in that region the expression for the uncracked section will govern. For the portions near the support, cracks may not develop. Usually, the expression for uncracked section will govern in that region.

Next, we are moving on to the two expressions for  $V_c$ . For uncracked section,  $V_c = V_{c0}$ , where  $V_{c0}$  is expressed as  $0.67bD \sqrt{f_t^2 + 0.8 f_{cp} f_t}$ .  $V_{c0}$  is the shear causing web shear crack at CGC. Thus, the capacity of concrete for an uncracked section is the value of shear which causes a web shear crack near the support. This expression can be derived from the concept of Mohr's circle.

In this expression,  $b$  is equal to breadth of the section, which is equal to  $b_w$ , the breadth of the web for flanged sections;  $D$  is the total depth of the section which we have represented as  $h$  in earlier occasions;  $f_t$  is the direct tensile strength of concrete which is  $0.24\sqrt{f_{ck}}$ ;  $f_{cp}$  is the compressive stress in concrete at CGC due to the prestress, which is equal to  $P_e/A$ . We are considering the numeric value of  $f_{cp}$ ; actually  $f_{cp}$  is compressive, but we are taking only the numeric value of  $f_{cp}$  in the expression.

The previous equation can be derived based on the expression of the principal tensile stress  $s_1$  at CGC. If we take an element at the CGC, then there is a shear stress ( $v$ ) and a compressive stress ( $f_{cp}$ ) at the vertical face. This state of two dimensional stresses can be transformed to a state of principal stresses, where  $s_1$  is the principal tensile stress and  $s_2$  is the principal compressive stress. If we plot the Mohr's circle, the point corresponding to the vertical face of the element has coordinates  $(-f_{cp}, v)$ . The point corresponding to the horizontal face is on the vertical axis, with no normal stress, but with a shear stress of  $v$ . We observe that the value of  $s_1$  is much smaller than the magnitude of  $s_2$ . This is due to the pre-stressing force. The value of  $s_1$  can be calculated if we are able to locate the origin of the circle. The origin of the circle is at  $(-f_{cp}/2, 0)$ . The distance of  $s_1$  from the origin is the radius of the circle, which is equal to  $\sqrt{((-f_{cp}/2)^2 + v^2)}$ . Thus,

$$s_1 = -f_{cp}/2 + \sqrt{(f_{cp}^2/4 + v^2)}$$

At the generation of web shear crack at the CGC, the principal tensile stress ( $s_1$ ) is equated to the direct tensile strength of concrete ( $f_t$ ). Also,  $v$  is expressed as  $v = Vc_0Q/Ib$ , which is derived in mechanics of materials. In the previous equation,  $I$  is the gross moment of inertia;  $Q$  is the product of the area above the CGC times the vertical distance of the centroid of the area from CGC, that is  $Q = A\bar{y}$ . We can transpose the terms to get the expression of the shear force, which initiates a web shear crack at the CGC.

Transposing the terms,  $Vc_0 = Ib/Q \sqrt{(f_t^2 + f_{cp}^2)}$ . The expression of  $Vc_0$  is in terms of the sectional properties of the section, which are  $I$ ,  $b$  and  $Q$ , the tensile strength of concrete  $f_t$ , and the amount of prestress  $f_{cp}$ . Here, we can note that if  $f_{cp}$  is increased, that means, if the prestress is increased, the shear corresponding to the initiation of web shear crack also increases. Thus, we are utilizing the benefit of the prestress in increasing the shear capacity of the section. The above expression is substituted by a simpler expression  $Vc_0 = 0.67bD \sqrt{(f_t^2 + 0.8f_{cp}^2)}$ . The term  $0.67bD$  represents  $Ib/Q$  for a section. It is an exact substitution for a rectangular section, but it is a conservative substitution for other types of sections. Also,  $f_{cp}$  is substituted by  $0.8f_{cp}$ . The reason is, to be conservative, only 80% of the prestress is considered in increasing the shear capacity.

The first term is an empirical expression. The notations are as follows:  $f_{pe}$  is the effective prestress in the tendon after all losses and it should be less than or equal to  $0.6 f_{pk}$ ;  $t_c$  is the ultimate stress capacity of concrete which is obtained from Table 6 of IS: 1343 – 1980.

The values of  $t_c$  are plotted in this graph. It is given in terms of the amount of prestressing steel. The graphs are given for two grades of concrete M30 and M40. We observe that as the amount of the prestressing steel is increasing, the strength of the concrete is also increasing; but, it gradually tapers off to a constant value. There is not much difference between the values for the two grades of concrete. From this we can infer that the strength of the concrete under shear even for a cracked section, increases with the amount of the prestressing steel. Even for the cracked region, since the prestressing steel applies a compression, the growth of crack is reduced, the depth of concrete under compression is increased, the aggregate interlock is increased, and the bond between the longitudinal steel and concrete is retained to generate dowel action. Hence, the strength of concrete under shear for a cracked section increases with the amount of prestressing steel.

In the expression,  $b$  is the breadth of the section, which is equal to  $b_w$  the breadth of the web for a flanged section, and  $d$  is the distance from the extreme compression fiber to the centroid of the tendons at the section considered.

The second term  $M_0 (V/M) = V(M_0/M)$  is the shear corresponding to the moment  $M_0$  that decompresses, or nullifies the effect of prestress at the tension face, and initiates a flexural crack. In this expression,  $M_0/M$  is a ratio of moments. Hence, it is an expression without any units.  $V$  is the shear corresponding to  $M$ , the moment at the section of investigation due to external loading. The expression of  $M_0$  is given as  $0.8f_{pt} I/y$ .

In the previous expression,  $f_{pt}$  is the compressive stress in concrete at the level of CGS due to prestress only. An equal amount of tensile stress is required to decompress the level of CGS.

In the expression of  $M_0$ ,  $y$  is the depth of the CGS from CGC. The expression is actually 80% of  $f_{pt} I/y$ , the moment that causes decompression of the concrete at the level of CGS. To recollect,  $M_0$  is the moment corresponding to the decompression of the tension face, which occurs earlier. Hence,  $M_0$  is taken as 80% of that moment which causes decompression at the level of CGS.

Since the concrete is cracked and the inclination of tendon is small away from the supports, any vertical component of the prestressing force is not added to  $V_{cr}$ . Earlier we had seen, that the code allows us to incorporate  $V_p$  along with  $V_{c0}$ , which the shear is causing web shear cracking. But here, we find that the code does not allow us to include  $V_p$  with  $V_{cr}$ , which is the shear corresponding to flexure–shear cracking.

## **DESIGN OF SECTION FOR FLEXURE**

When pre-stressed concrete members are subjected to bending loads, different types of flexural failures are possible at critical sections, depending upon the principal controlling parameters, such as the percentage of reinforcement in the section, degree of bond between tendons and concrete, compressive strength of concrete and the ultimate tensile strength of the tendons.

## **CODE PROCEDURES**

### **Indian Code Provisions**

The Indian standard code method (IS:1343) for computing the flexural strength of rectangular sections or T- sections in which neutral axis lies within the flange is based on the rectangular and parabolic stress block

The moment of resistance of rectangular sections or T-sections in which neutral axis lies within the flange may be obtained as follows:

$$M_u = f_{pb} A_{ps} (d - 0.42 x_u)$$

Where  $M_u$  = moment of resistance of the section,

$f_{pb}$  = tensile stress in the tendon at failure,

$f_{pe}$  = effective prestress in tendon,

$A_{ps}$  = area of pre-tensioning tendons in the tension zone,



$d$  = effective depth to the centroid of the steel area, and

$x_u$  = neutral axis depth.

For pretensioned members and for post-tensioned members with effective bond between the concrete and tendons, values of  $f_{pb}$  and  $x_u$  are given in Table 11. It shall be ensured that the effective prestress,  $f_{pe}$  after all losses is not less than  $0.45 f_{pu}$ , where  $f_{pu}$  is the characteristic tensile strength of tendon. Prestressing tendons in the compression zone should be ignored in the strength calculations when using this method.

The value of  $f_{pb}$  depends upon the effective reinforcement ratio

$$\left( \frac{P_a p}{b d f_{ck}} \right)$$

### **SHEAR STRESS**

The shear stress distribution in an un-cracked structural concrete member for which the deformation is assumed to be linear is function of shear force and the properties of the cross section of the member.

The shear stress at a point is expressed as

$$\tau_{\max} = V(A_y)/Ib$$

The effect the shear stress is to induce principle tensile stress on diagonal planes, local failures first appear in the form of diagonal tension cracks in regions of high stress shear.

Where the max & min principle stresses are given by

## UNIT V: Deflections of Prestressed Concrete

### Essential Reasons for Control of the Deflections

- Excessive, sagging of principal structural members is not only unsightly, but at times, also renders the floor unsuitable for the intended use.
- Large deflections under dynamic effects and under the influence of variable loads may cause discomfort to the users.
- Excessive deflections are unlikely to cause damage to finishes, partitions and associated structures.

### Factors Influencing Deflections

The deflections of prestressed concrete members are influenced by the following salient factors:

1. Imposed load and self-weight
2. Magnitude of the prestressing force.
3. Cable profile
4. Second moment of area of cross section
5. Modulus of elasticity of concrete
6. Shrinkage, creep and relaxation of steel stress
7. Span of the member
8. Fixity conditions.

In the precracking stage, the whole cross section is effective and the deflections in this stage are computed by using the second moment of area of the gross concrete section. The computation of short-term or instantaneous deflections, which occur immediately after transfer of prestress and on application of loads, is conveniently done by using Mohr's theorems.

In the post-cracking stage, a prestressed concrete beam behaves in a manner similar to that of a reinforced concrete beam and the computation of deflections in this stage is made by considering moment-curvature relationships which involve the section, properties of the cracked beam.

In both cases, the effect of creep and shrinkage of concrete is to increase the long-term deflections under sustained loads, which is estimated by using empirical methods that involve the use of effective (long term) modulus of elasticity or by multiplying short-term deflections by suitable factors.

### **Derivation for Short Term Deflection of Uncracked members using Mohr's Theorems.**

Short-term or instantaneous deflections of prestressed members are governed by the bending moment distribution along the span and the flexural rigidity of the members. Mohr's moment area theorems are readily applicable for the estimation of deflections due to the prestressing force, self-weight and imposed loads.

Consider the beam AB is subjected to a bending moment distribution due to the prestressing force or self-weight or imposed loads. ACB is the centre line of the deformed structure under the system of given loads.

Let

$\Theta$  = slope of the elastic curve at A

AD = intercept between the tangent at C and the vertical at A

a = deflection at the centre for symmetrically loaded, simply supported beam (since the tangent is horizontal for such cases)

A = area of the BMD between A and C

x = distance of the centroid of the BMD between A and C from the left support.

EI = flexural rigidity of the beam.

Then, according to Mohr's first theorem,

$$\text{Slope} = \frac{\text{area of BMD}}{\text{Flexural rigidity}}$$

$$\Theta = \frac{A}{EI}$$

Mohr's second theorem states that

Intercept,  $a = \frac{\text{moment of the area of B.M.D.}}{\text{Flexural rigidity}}$

$$= \frac{Ax}{EI}$$

### Derivations for deflections due to Pre-stressed Tendons

**a) Straight Tendons:** a beam with a straight tendon at a uniform eccentricity below the centroidal axis.

If upward deflections are considered as negative and

$P$  = effective prestressing force

$e$  = eccentricity

$L$  = Length of the beam

$$a = - \frac{(PeL)(L/4)}{EI} = - \frac{PeL^2}{8EI}$$

**b) Trapezoidal Tendons:** A draped tendon with a trapezoidal profile is shown in figure. Considering the BMD, the deflection at the centre of the beam is obtained by taking the moment of area of the BMD over one-half of the span. Thus,

$$a = - \frac{Pe}{EI} [(l_2 (l_1 + l_2/2) + (l_1/2) (2/3l_1))] = \frac{Pe}{6EI} [2l_1^2 + 6l_1l_2 + 3l_2^2]$$

**c) Parabolic Tendons (Central Anchors):** the deflection of a beam with parabolic tendons having an eccentricity  $e$  at the centre and zero at the support is given by,

$$a = - \frac{Pe}{EI} \left[ \frac{2}{3} L, \frac{5}{8} L \right] = - \frac{5 PeL^2}{48 EI}$$

**d) Parabolic Tendons (Eccentric Anchors):** Figure shows a beam, with a parabolic tendon having an eccentricity  $e_1$  at the centre of span and  $e_2$  at the support sections. The resultant

deflection at the centre is obtained as the sum of the upward deflection of a beam with a parabolic tendon of eccentricity ( $e_1 + e_2$ ) at the centre and zero at the supports and the downward deflection of a beam subjected to a uniform sagging bending moment of intensity  $Pe_2$  throughout the length. Consequently, the resultant deflection becomes,

$$a = \frac{-5 PL^2}{48 EI} (e_1 + e_2) + \frac{Pe_2 L^2}{8EI}$$

$$a = \frac{PL^2}{48EI} (-5e_1 + e_2)$$

**e) Sloping Tendons (Eccentric Anchors):** from the figure, the deflection is

$$a = \frac{PL^2}{12 EI} (e_1 + e_2) + \frac{Pe_2 L^2}{8EI} = \frac{PL^2}{24EI} (-2e_1 + e_2)$$

**f) Parabolic and Straight Tendons:** Referring to figure, the deflection at the centre of the beam is obtained as,

$$a = -\frac{Pe}{EI} \left[ \left(\frac{2}{3}\right) l_1 + \left(\frac{5}{8}\right) l_1 + l_2 \left( l_1 + \frac{l_2}{2} \right) \right]$$

$$a = -\frac{Pe}{EI} \left[ (5l_1^2 + 12l_1 l_2 + 6l_2^2) \right]$$

**g) Parabolic and Straight Tendons (Eccentric Anchors):** The maximum central deflection is obtained From figure, it is seen that,

$$a = - \frac{P(e_1 + e_2)}{12 EI} [ (5l_1^2 + 12l_1l_2 + 6l_2^2) ] + \frac{Pe_2L_2}{8 EI}$$

### **Deflections due to Self-weight and Imposed Loads:**

At the time of transfer of prestress, the beam hogs up due to the effect of prestressing. At this stage, the self-weight of the beam induces downward deflections, which further increase due to the effect of imposed loads on the beam.

If  $g$  = self-weight of the beam/m

$q$  = imposed load/m (uniformly distributed),

The downward deflection is computed as,

$$a = \frac{5(g + q)L^4}{384 EI}$$

Deflections due to concentrated live loads can be directly computed by using Mohr's theorems.

## UNIT VI

### **Transmission Length:**

The length required at the ends of a pretensioned member for the build-up of stress in concrete is of great importance, particularly in short pretensioned units, since it controls the working bending moment and the shear force allowable on the section. The transmission length depends mainly on the diameter and surface characteristics of the wire, the elastic properties of steel and concrete, and the coefficient of friction between steel and concrete.

Hoyer has developed an expression for computing the transmission length, which is given by,

$$L_t = \frac{\phi (1 + \nu_c) \alpha_e - f_{pi}}{2\mu \nu_s E_c} \frac{f_{pe}}{2f_{pi} - f_{pe}}$$

Where  $L_t$  = transmission length

$\phi$  = wire diameter

$\mu$  = coefficient of friction between steel and concrete

$\nu_c$  = Poisson's ratio for concrete

$\nu_s$  = Poisson's ratio for steel

$\alpha_e$  = modular ratio ( $E_s/E_c$ )

$E_c$  = modulus of elasticity of concrete

$F_{pi}$  = initial stress in steel

$F_{pe}$  = effective stress in steel

Marshall and Krishna Murthy empirical formulae

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$$L_t = \sqrt{\frac{f_{cu} \times 10^3}{\beta}}$$

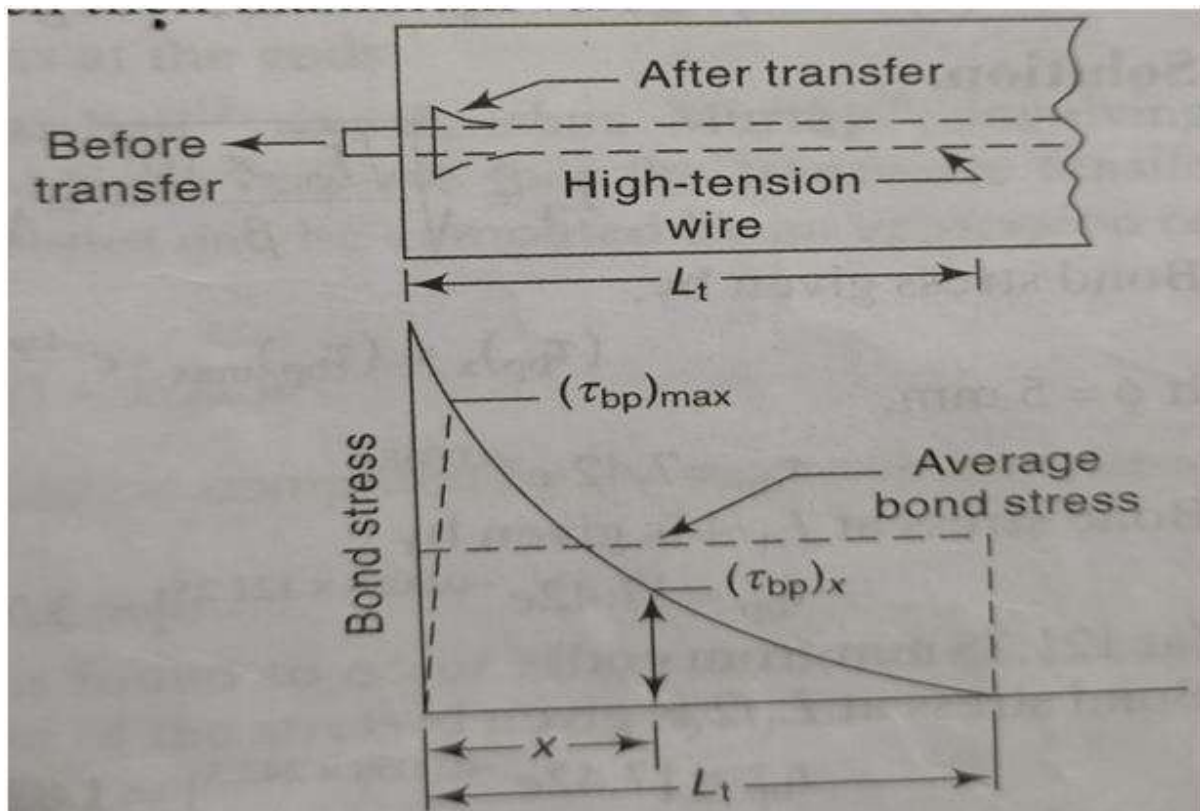
$f_{cu}$  = cube strength of concrete at transfer, expressed in  $N/mm^2$

$L_t$  = transmission length in mm

$\beta$  = constant, depending upon the details of strand and wire.

## BOND STRESSES

The magnitude of bond stresses developed between concrete and steel and its variation in the transfer zone of pretensioned beams is shown in figure. The bond stress is zero at the ends but builds up rapidly to a maximum over a very short length. This value decreases as the stress in the wire builds up. At a distance equal to the transmission length, the bond stress is almost zero while the stress in steel and concrete reach their maximum values.



If  $(\tau_{bp})_{max}$  = maximum value of bond stress

$(\tau_{bp})_x$  = bond stress at a distance  $x$  from the free end

$\phi$  = diameter of the wire

$f_s$  = stress in steel at distance  $x$  from the free end

$f_{sc}$  = effective stress in steel at the end of the transfer zone

Based on tests conducted at the University of Leeds, the following relations have been proposed by Marshall.



$$(T_{bp})_x = (T_{bp})_{\max} e^{-4\psi x/\phi}$$

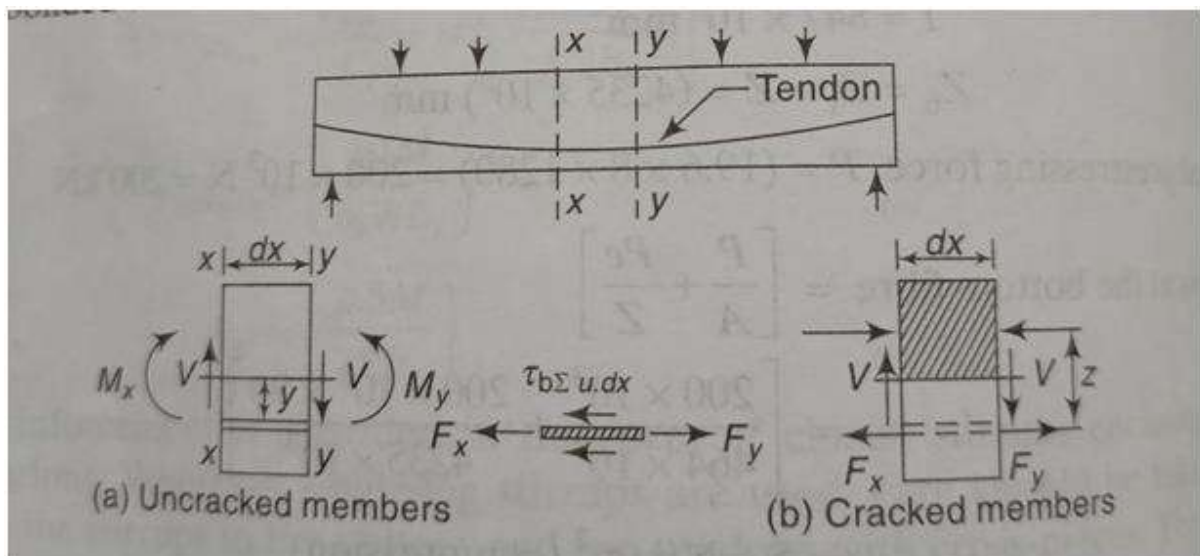
$$f_s = f_{se} (1 - e^{-4\psi x/\phi})$$

Where  $\psi = \text{constant}$ , as the ratio of change in bond stress to steel stress

$x = \text{distance measured from the free end, expressed in mm.}$

### FLEXURAL-BOND STRESSES

Pretensioned or post-tensioned beams with bonded tendons develop bond stresses between steel and concrete when the sections are subjected to transverse shear forces due to the rate of change of moment along the beam length. In the case of uncracked members, bond stresses are computed by considering the complete section, which is effective. Figure shows a beam with bonded tendons subjected to transverse loads.



Where  $T_b = \text{bond stress between steel and concrete}$

$V = \text{shear force}$

$M_x$  and  $M_y = \text{moments at sections } xx \text{ and } yy$

$E_u = \text{total perimeter of the tendons}$

$y = \text{distance of the tendon from the centroidal axis}$

$I = \text{second moment of area of the section}$

$\alpha_e = \text{modular ratio} = (E_s/E_c)$

$A_s = \text{area of steel}$

$f_x$  and  $f_y$  = bending stress in concrete at the level of steel at sections xx and yy.

From Fig. 9.5 (a), considering the forces and moments,

$$(M_y - M_x) = Vdx = \left[ \left( \frac{f_y I}{y} \right) \times \left( \frac{f_x I}{y} \right) \right]$$

$$\therefore Vdx = \left( \frac{I}{\alpha_c A_s y} \right) (\alpha_c A_s f_y - \alpha_c A_s f_x) = \frac{I}{\alpha_c A_s y} (F_y - F_x)$$

$$Vdx = \left( \frac{I}{\alpha_c A_s y} \right) (\tau_b \Sigma u dx)$$

$$\therefore \tau_b = \left( \frac{\alpha_c A_s y V}{I \Sigma u} \right)$$

If round wires are used,

$$\left( \frac{A_s}{\Sigma u} \right) = \Phi/4$$

where  $\Phi$  = diameter of the wires,

then 
$$\tau_b = \left( \frac{Vy\alpha_c\Phi}{4I} \right)$$