UNIT-1

Learning Material

Introduction:

Design of reinforced concrete structures started in the beginning of this century following purely empirical approach. Thereafter came the so called rigorous elastic theory where it is assumed that concrete is elastic and reinforcing steel bars and concrete act together elastically. The load-deflection relation is linear and both concrete and steel obey Hooke's law. The method is designated as working stress method as the loads for the design of structures are the service loads or the working loads. The failure of the structure will occur at a much higher load. The ratio of the Ultimate loads to the working loads is the factor of safety. Accordingly, the stresses of concrete and steel in a structure designed by the working stress method are not allowed to exceed some specified values of stresses known as permissible stresses. The permissible stresses are determined dividing the characteristic strength f_{ck} of the material by the respective factor of safety. The values of the factor of safety depend on the grade of the material and the type of stress.

Materials of reinforced concrete

a) **Cement :**

Type of Cement	**IS. No.**	**Purpose**
OPC	IS 269-1976	General construction
Low heat cement	IS 269-1976	Massive construction
Rapid hardening cement	IS 8041-1990	For quick removal of form work
Pozzolana cement	IS 1489-1991	Chemical resistance
High strength cement	IS 8112-1989	Prestressed concrete
Hydrophobic cement	IS 8043-1991	Water-proof concstruction

b) **Grades of cements:**

Grades of cement is based on crushing strength of cement motar cube of size 70.71mm(surfaced area of 50 cm^2) cured and tested at 28 days. They basically differ in terms of fineness of cement which in turn is expressed as specific surface area.

Specific surface: is the surface area of the particles in 1 gram of cement. Chemically all the three grades of cement, i.e garde 33, grade 43, grade 53 are almost similar.

Their characteristics are listed below:

Gr 33- specific surface area is minimum 2250 cm^2/gram(IS: 269)
Gr 43- specific surface area is minimum 3400 cm^2/gram(IS: 8112-1989)
Gr 53- specific surface area is much greater than 3400 cm^2/gram(IS: 12269-1987)

Grade 53 cements have more shrinkage compared to other grades, but having higher early strength.

c) **Aggregates:** As per IS: 383-1970 generally Hard Blasted granite chips(HBG)

(i) Coarse aggregates:

- Nominal maximum size of coarse aggregate for RCC is 20mm.
- In no case greater than one-fourth of minimum thickness of member
- In heavily reinforced members 5 mm less than the minimum clear distance between the main bars or 5 mm less than the minimum cover to the reinforcement which ever is smaller

(ii) Fine aggregate: Generally medium sand, Zone II sand as per IS: 456

d) **Reinforcement:**

i) Mild steel amd medium tensile steel bars-IS: 432
ii) Hot rolled deformed bars- IS: 1139
iii) Cold twisted bars-IS: 1786
iv) Hard drawn steel wire fabric- IS: 1566

e) **Minimum grade of concrete for various structures:**

Type of construction	**Minimum grade of concrete**
1. Lean concrete bases	M5 and M7.5
2. Plain concrete	M10
3. R.C.C	M20
4. Water tanks, domes, folded plates, shell roofs	M20
5. R.C.C in sea water	M30 for RCC and M20 for PCC
6. Post-tensioned pre-stressed concrete	M30
7. Pre-tensioned pre-stressed concrete	M40

Permissible Stresses in Concrete

- The permissible stress of concrete in direct tension is denoted by σ_{td}. The values of σ_{td} for member in direct tension for different grades of concrete are given in cl. B-2.1.1 of IS 456.
- The permissible stresses of concrete in bending compression σ_{cbc}, in direct compression σ_{cc} and the average bond for plain bars in tension τ_{bd} are given in Table 21 of IS 456 for different grades of concrete.
- For plain bars in compression, the values of average bond stress are obtained by increasing the respective value in tension by 25 percent, as

given in the note of Table 21 of IS 456.

- For deformed bars, the values of Table 21 are to be increased by sixty per cent, as stipulated in cl. B-2.1.2 of IS 456.

Grade of Concrete	**Direct tension σ_{td} (N/mm^2)**	**Bending compression σ_{cbc} (N/mm^2)**	**Direct compression σ_{cc} (N/mm^2)**	**Average bond τ_{bd} for plain bars in tension(N/mm^2)**
M 20	2.8	7.0	5.0	0.8
M 25	3.2	8.5	6.0	0.9
M 30	3.6	10.0	8.0	1.0
M 35	4.0	11.5	9.0	1.1
M 40	4.4	13.0	10.0	1.2

Permissible Stresses in Steel Reinforcement

- Permissible stresses in steel reinforcement for different grades of steel, diameters of bars and the types of stress in steel reinforcement are given in Table 22 of IS 456.
- Selective values of permissible stresses of steel of grade Fe 250 (mild steel) and Fe 415 (high yield strength deformed bars) in tension (σ_{st} and σ_{sh}) and compression in column (σ_{sc}) are furnished in Table below as a ready reference. It may be noted from the values of Table 13.2 that the factor of safety in steel for these stresses is about 1.8, much lower than concrete due to high quality control during the production of steel in the industry in comparison to preparing of concrete.

Type of stress in steel Reinforcement	**Mild steel bars, Fe 250, (N/mm^2)**	**High yield strength deformed bars, Fe 415, (N/mm^2)**
Tension σ_{st} or σ_{ss}		
(a) up to and including 20 mm diameter	140	230
(b) over 20 mm diameter	130	230
Compression in column bars σ_{sc}	130	190

Permissible Shear Stress in Concrete τ_c

- Permissible shear stress in concrete in beams without any shear reinforcement depends on the grade of concrete and the percentage of main tensile reinforcement in beams.
- Table 23 of IS 456 furnishes the values of τ_c for wide range of percentage of tensile steel reinforcement for different grades concrete.
- Other relevant clauses regarding the permissible shear stress of concrete are given in cls.B-5.2.1.1, B-5.2.2 and B-5.2.3 of IS 456.

Increase in Permissible Stresses

Clause B-2.3 of IS 456 recommends the increase of permissible stresses of concrete and steel given in Tables 21 to 23 up to a limit of 33.33 per cent, where stresses due to wind (or earthquake), temperature and shrinkage effects are combined with those due to dead, live and impact loads.

Assumptions for Design of Members by Working Stress Method

As mentioned earlier, the working stress method is based on elastic theory, where the following assumptions are made, as specified in cl. B-1.3 of IS 456.

1. Plane sections before bending remain plane after bending.
2. Normally, concrete is not considered for taking the tensile stresses except otherwise specifically permitted. Therefore, all tensile stresses are taken up by reinforcement only.
3. The stress-strain relationship of steel and concrete is a straight line under working loads.
4. The modular ratio m has the value of $280/3\sigma_{cbc}$, where σ_{cbc} is the permissible compressive stress in concrete due to bending in N/mm^2. The values of σ_{cbc} are given in Table 21 of IS 456. The modular ratio is explained in the next section.

Modular Ratio (*m*)

- In the elastic theory, structures having different materials are made equivalent to one common material.

- In the reinforced concrete structure, concrete and reinforcing steel are, therefore, converted into one material.
- This is done by transformation using the modular ratio m which is the ratio of modulus of elasticity of steel and concrete.
- Thus, $m = E_s/E_c$. where E_s is the modulus of elasticity of steel which is 200000 N/mm^2. However, concrete has different moduli, as it is not a perfectly elastic material.

Flexural Members – Singly Reinforced Sections

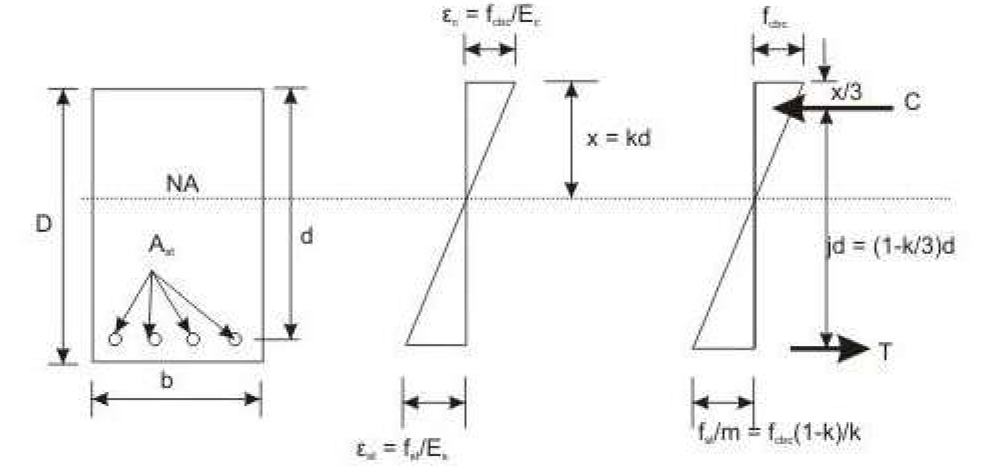


Fig. 1.1.1(a): Section Fig. 1.1.1(b): Strain profile Fig. 1.1.1(c): Stress distribution

Fig. 1.1.1 : Singly reinforced rectangular beam

A simply supported beam subjected to two point loads shall have pure moment and no shear in the middle-third zone, as shown in Fig.1, the cross-sections of the beam in this zone are under pure flexure. Figures1(a), (b) and (c) show the cross-section of a singly-reinforced beam, strain profile and stress distribution across the depth of the beam, respectively due to the loads applied on the beam.

$x = kd$ = depth of the neutral axis, where k is neutral axis depth factor,

f_{cbc} = actual stress of concrete in bending compression at the top fibre which should not exceed the respective permissible stress of concrete in bending compression σ_{cbc},

f_{st} = actual stress of steel at the level of centroid of steel which should not exceed the respective permissible stress of steel in tension σ_{st},

jd = *d(1-k/3)* = lever arm i.e., the distance between lines of action of total compressive and tensile forces *C* and *T* , respectively.

Figures1(b) and (c) show linear strain profile and stress distribution, respectively. However, the value of the stress at the level of centroid of steel of Fig.1 (c) is f_{st}/m due to the transformation of steel into equivalent concrete of area mA_{st}.

Balanced Section – Singly-Reinforced

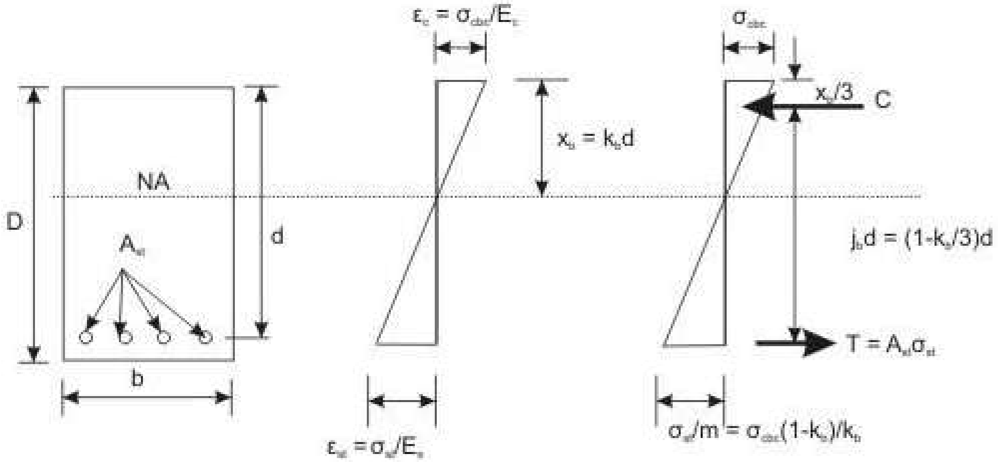


Fig. 2(a): Section Fig. 2(b): Strain profile Fig. 2(c): Stress distribution

Fig. 2 : Singly-reinforced balanced section

In a balanced cross-section both f_{cbc} and f_{st} reach their respective permissible values of σ_{cbc} and σ_{st} at the same time as shown in Fig. 2c. The depth of neutral axis is $x_b = k_b d$. From the stress distribution of Fig.2c, we have

An expression of k_b is obtained by substituting the expression of *m* as

(1.2)

into Eq. 1.1. This gives

$k_b = 93.33 \ / \ (\sigma_{st} + 93.33)$ (1.3)

Equation 1.3 shows that the value of k_b for balanced section depends only on σ_{st} . It is independent of σ_{cbc}.

The lever arm, $j_b\, d = (1-)\, d$ (1.4)

The expressions of total compressive and tensile forces, C and T are:

$$C = (1/2)\ \sigma_{cbc}\, b\, x_b = (1/2)\ \sigma_{cbc}\, b\, k_b\, d \qquad (1.5)$$

$$T = A_{st}\ \sigma_{st} \qquad (1.6)$$

The total compressive force is acting at a depth of $x_b/3$ from the top fibre of the section. The moment of resistance of the balanced cross-section M_b is obtained by taking moment of the total compressive force C about the centroid of steel or moment of the tensile force T about the line of action of the total compressive force C. Thus,

$M_b = C(j_b d) = (1/2)\ \sigma_{cbc}\, k_b\, j_b (bd^2)$

(1.7)

Or, (1.8)

As

(1.9)

where $p_{t,bal}$ = balanced percentage of steel

From Eqs.1.7 and 1.8, we can write

$$M_b = R_b\, bd^2 \qquad (1.10)$$

where $R_b = (1/2)\ \sigma_{cbc}\, k_b\, j_b = (p_{t,bal}/100)\ \sigma_{st}\, j_b$ (1.11)

and $j_b = 1 - (k_b/3)$ (1.12)

The expression of the balanced percentage of steel pt,bal is obtained by equating the total compressive force C to the tensile force T from Eqs. 1.5 and 1.6. This gives,

$A_{st}\, \sigma_{st} = (\sigma_{cbc}/2)\ b\, k_b\, d,$

which gives:

or $p_{t,bal} = 50\ k_b(\sigma_{cbc}/\sigma_{st})$

(1.13)

It is always desirable, though may not be possible in most cases, to design the beam as balanced since the actual stresses of concrete f_{cbc} at the top compression fiber and steel at the centroid of steel should reach their respective permissible stresses σ_{cbc} and σ_{st} in this case only. The procedure of the design is given below.

Treating the design moment as the balanced moment of resistance M_b and assuming the width, b of the beam as 250 mm, 300 mm or 350 mm, the effective depth d is obtained from Eq.1.10. The required balanced area of steel A_{st} is then obtained from Eq. 1.9 getting the values of k_b from Eq. 1.3 and then $p_{t,bal}$ from Eq. 1.13.

The values of R_b, the moment of resistance factor M_b/bd^2 are obtained from Eq. 1.11 for different values of σ_{cbc} and σ_{st} (different grades of concrete and steel) and are presented. Similarly, the values of balanced percentage of tensile reinforcement, $p_{t,\, bal}$ obtained from Eq. 1.13 for different grades of concrete and steel.

Moment of resistance factor R_b in N/mm² for balanced rectangular section.

σcbc	σ_{st} (N/mm²)		
(N/mm²)	140	230	275
7.0	1.21	0.91	0.81
8.5	1.47	1.11	0.99
10.0	1.73	1.30	1.16

Percentage of tensile reinforcement $p_{t,bal}$ for singly-reinforced balanced section.

σcbc	σ_{st} (N/mm²)		
(N/mm²)	140	230	275
7.0	1.0	0.44	0.32
8.5	1.21	0.53	0.39
10.0	1.43	0.63	0.46

Values of R_b and $p_{t.bal}$ and reveal the following:

For given values of width and effective depth, b and d of a rectangular section, the balanced moment of resistance M_b increases with higher grade of concrete for a particular grade of steel. However, the balanced moment of resistance decreases with higher grade of steel for a particular grade of concrete.

The balanced percentage of steel, $p_{t.bal}$ increases with the increase of grade of concrete for a particular grade of steel for given values of width and effective depth of a rectangular section. On the other hand, the balanced percentage of steel, $p_{t.bal}$ decreases with the increased grade of steel for a particular grade of concrete.

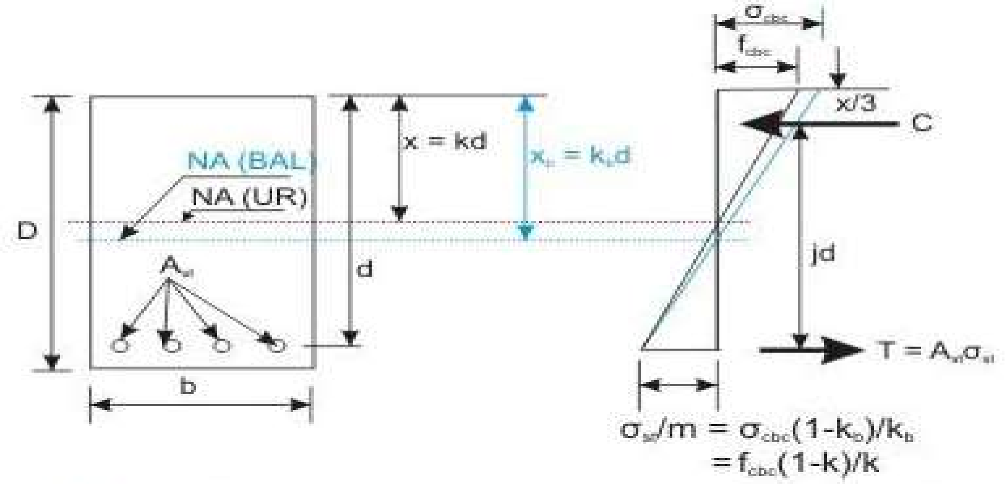


Fig. 3(a): Section

Fig. 3(b): Stress distribution

Fig. 3: Under-reinforced beam

- As mentioned earlier, it may not be possible to design a balanced section since the area of steel required for the balanced condition is difficult to satisfy with available bar diameters.
- In such cases, it is essential that the beam should be provided with the steel less than the balanced steel so that the actual stress of steel in tension reaches the permissible value σ_{st} and the actual stress of concrete f_{cbc} is less than the permissible value.
- Such sections are designated as under-reinforced sections and moment of resistance shall be governed by the tensile stress of steel σ_{st}, which is known.
- The depth of the neutral axis will be less than the balanced depth of the neutral axis, as shown in Fig.3b.
- The relevant equations for the design of under-reinforced section are established in the next section.

Under-reinforced Section -- Singly Reinforced

Figure 3a shows the cross-section where x (= kd) is the depth of the neutral axis.

The depth of the neutral axis is determined by taking moment of the area of concrete in compression (= bx) and the transformed area of steel (= mA_{st}) about the neutral axis, which gives

$b\,() = m\,()\,(d-kd)$

or $\quad k^2 + () - () = 0$ (1.14)

Equation 1.14 has two roots of k given by

$k = -\,() \pm \{()^2 + ()\}^{1/2}$ (1.15)

Since k cannot be negative, we have the positive root to be considered as
$k = -\,() \pm \{()^2 + ()\}^{1/2}$ (1.16)

The moment of resistance of the under-reinforced section is obtained from

$M = T\,(\text{lever arm}) = A_{st}\,\sigma_{st}\;d(1 -) = ()\,\sigma_{st}\;d\,(1 - \;)$

Therefore, we have:

$M = ()\sigma_{st}\,(1 -)\,bd^2$ (1.17)

which can also be expressed as
$M = R\,bd^2$ (1.18)
where
$R = ()\,\sigma_{st}\,(1 -)$ (1.19)
is the moment of resistance factor M/bd^2. The values of R are obtained for given values of p_t for different grades of steel and concrete. Tables 68 to 71 of SP-16 furnish the values of R for four grades concrete and five values of σ_{st}.
The actual stress of concrete at the top fibre f_{cbc} shall not reach σ_{cbc} in under-reinforced sections. The actual stress f_{cbc} is determined from the equation

C=T as explained below:

With reference to Fig.3c, the compressive force C of concrete and tensile for T of steel are:

$C = (1/2)\ f_{cbc}\ b\ kd$

(1.20)

$T = \sigma_{st}\ A_{st}$

For the tensile force, the actual stress of steel f_{st} shall reach the value of σ_{st}. So, we are using Eq. 1.6, the same equation as for the balanced section.

Equating C and T from Eqs. 1.20 and 1.6, we get

$(1/2)f_{cbc}\ b\ kd = \sigma_{st}\ A_{st}$

or, $f_{cbc} =$ (1.21)

Expressing $A_{st} =$ (1.22)

and using Eq. 1.22 in Eq. 13.21, we get

$f_{cbc} =$ (1.23)

The two types of problems: (i) Analysis type and (ii) Design .

Analysis Type of Problems

For the purpose of analyzing a singly-reinforced beam where the working loads, area of steel, b and d of the cross section are given, the actual stresses of concrete at the top fibre and steel at the centroid of steel are to be determined in the following manner.

Step 1: To determine the depth of the neutral axis kd

Step 2: The beam is under-reinforced, balanced or over-reinforced, if k is less than, equal to or greater than k_b, to be obtained.

Step 3: The actual compressive stress of concrete f_{cbc} and tensile stress of steel at the centroid of steel f_{st} are determined in the following manner for the three cases of Step 2.

Case (i) When $k < k_b$ (under-reinforced section)

From the moment equation, we have

$M= A_{st}\ f_{st}\ d(1\ -\ )$

or $f_{st} =\}$

where M is obtained from the given load, A_{st} and d are given, and k is determined in Step 1.

Equating $C = T$, we have: $(1/2)\ f_{cbc}\ b(kd) = A_{st}\, f_{st}$

or $\quad f_{cbc} =$

where A_{st}, b and d are given and k and f_{st} are determined in steps 1 and 2, respectively.

Case (ii) When $k = k_b$ (balanced section)

In the balanced section $f_{cbc} = \sigma_{cbc}$ and $f_{st} = \sigma_{st}$.

Case (iii) When $k > k_b$ (over-reinforced section)

Such beams are not to be used as in this case f_{cbc} shall reach σ_{cbc} while f_{st} shall not reach σst. These sections are to be redesigned either by increasing the depth of the beam or by providing compression reinforcement. Beams with compression and tension reinforcement are known as doubly-reinforced beam and is taken up in sec.

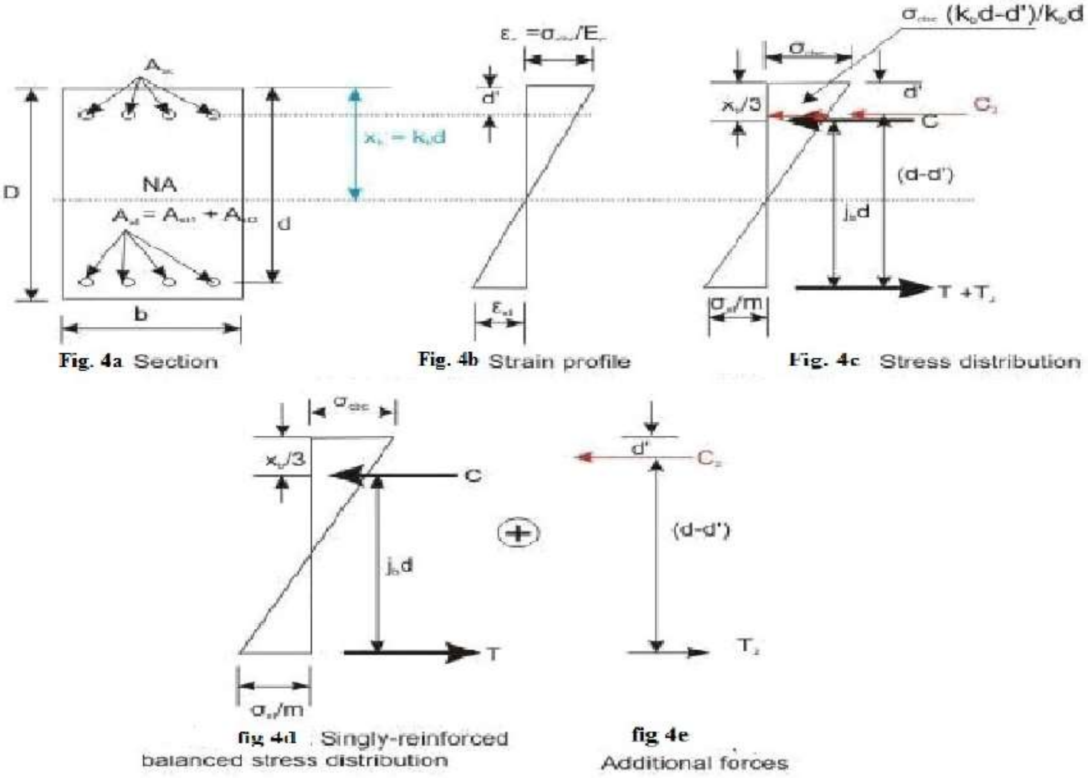


Fig. 4a Section Fig. 4b Strain profile Fig. 4c Stress distribution

fig 4d Singly-reinforced balanced stress distribution fig 4e Additional forces

Fig. 4 Doubly-reinforced beam

Figures 4a to c show the cross-section, strain profile and stress distribution of a doubly-reinforced section. Since, the design moment is more than the balanced moment of resistance of the section, we have

$M = M_b + M'$ (1.26)

The additional moment M' is resisted by providing compression reinforcement A_{sc} (= $p_c bd/100$) and additional tensile reinforcement A_{st2}. The modular ratio of the compression steel is taken as 1.5 m, where m is the modular ratio as explained in sec.

Figure 4c shows that the stress of concrete at the level of compression steel is $\sigma_{cbc} (k_b d - d') / k_d d$. Accordingly, the stress in the compression steel reinforcement is $1.5m\ \sigma_{cbc} (k_b d - d') / k_b d$.

Figure4d and e present separate stress distribution for the balanced beam (shown in Fig2c) and the compressive and tensile forces of compressive and tensile reinforcing bars C_2 and T_2, respectively. The expression of the additional moment M' is obtained by multiplying C_2 and T_2 with the lever arm ($d - d'$), where d' is the distance of the centroid of compression steel from the top fibre. We have, therefore,

$C_2 =$ (1.27)

$T_2 = (p_t - p_{t,\,bal})\ ()\ \sigma_{st}$ (1.28)

$M' = C_2\ (d - d') = (p_c\ )\ (1.5m - 1)\ \sigma_{cbc}$

or, $M' = ()(1.5m - 1)\ \sigma_{cbc}\ (1 -)(1 -)bd^2$ (1.29)

also, $M' = T_2\ (d - d'\ )\ d = (p_t - p_{t,\,bal})\ ()\sigma_{st}\ (d - d'\ )$

or, $M' = (p_t - p_{t,\,bal})\ /\ 100\ \sigma_{st}\ (1 -)\ bd^2$ (1.30)

Equating $T_2 = C_2$ from Eqs. 1.28 and 1.27, we have

$(p_t - p_{t,\,bal})\ \sigma_{st} = p_c\ (1.5m - 1)\ \sigma_{cbc}\ (1 -)$ (1.31)

The total moment M is obtained by adding M_{bal} and M', as given below:

$M = M_{bal} + (p_t - p_{t,\,bal})\ /\ 100\ \sigma_{st}\ (1 -)\ bd^2$ (1.32)

The total tensile reinforcement A_{st} has two components $A_{st1} + A_{st2}$ for M_{bal} and M', respectively. The equation of A_{st} is:

$Ast = Ast\ 1 + Ast\ 2$ (1.33)

where $A_{st1} = p_{t\,bal}\ ()$ (1.34)

and $A_{st\,2} = M'\ /\ \sigma_{st}\ (d - d')$ (1.35)

The compression reinforcement A_{sc} is expressed as a ratio of additional tensile reinforcement A_{st2}, as given below:

$(A_{sc}\ /\ A_{st2}) = \{p_c\ /(p_t - p_{t\,bal}\ )\}$

$(A_{sc} \quad / \quad A_{st2}) \quad = \quad \sigma_{st} \quad / \quad \{\sigma_{cbc} \quad (1.5m \quad - \quad 1) \quad (1 \quad - \quad)$
or, (1.36)

Table M of SP-16 presents the values of A_{st} / A_{st2} for different values of d' / d and σ_{cbc} for two values of σ_{st} =140 N/mm^2 and 230 N/mm^2. Selective values are furnished in Table 1.5 as a ready reference. Tables 72 to 79 of SP-16 provide values of p_t and p_c for four values of d' / d against M/bd^2 for four grades of concrete and two grades of steel.

Values of $A_{sc} / A_{st\,2}$

σst	***σcbc***	***d' / d***			
(N/mm^2)	(N/mm^2)	0.05	0.10	0.15	0.20
140	7.0	1.20	1.40	1.68	2.11
	8.5	1.22	1.42	1.70	2.13
	10.0	1.23	1.44	1.72	2.15
230	7.0	2.09	2.65	3.60	5.54
	8.5	2.12	2.68	3.64	5.63
	10.0	2.14	2.71	3.68	5.76

Philosophy of design of limit state

Introduction

In any method of design, the following are the common steps to be followed:

- To assess the dead loads and other external loads and forces likely to be applied on the structure,
- To determine the design loads from different combinations of loads,
- To estimate structural responses (bending moment, shear force, axial thrust etc.) due to the design loads,
- To determine the cross-sectional areas of concrete sections and amounts of reinforcement needed.
- Many of the above steps have lot of uncertainties. Estimation of loads and evaluation of material properties are to name a few. Hence, some suitable

factors of safety should be taken into consideration depending on the degrees of such uncertainties.

Limit state method is one of the three methods of design as per IS456:2000. While accommodating the working stress method in Annex B of the code (IS 456). Considering rapid development in concrete technology and simultaneous development in handling problems of uncertainties, the limit state method is a superior method where certain aspects of reality can be explained in a better manner.

Limit State Method

What are limit states?

- Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs.
- The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended.
- It is worth mentioning that structures will not just fail or collapse by violating (exceeding) the limit states. Failure, therefore, implies that clearly defined limit states of structural usefulness has been exceeded.
- Limit state of collapse was found / detailed in several countries in continent fifty years ago. In 1960 Soviet Code recognized three limit states: (i) deformation, (ii) cracking and (iii) collapse.

Assumptions

Design for the limit state of collapse in flexure shall be based on the assumptions given *below:*

a) Plane sections normal to the axis remain plane after bending.

b) The maximum strain in concrete at the outer most compression fibre is taken as 0.0035

in bending.

c) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, Trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress strain curve is given For design purposes. the compressive strength of concrete in the structure shall be assumed to be 0.67times the characteristic strength. The partial safety factor y. = 1.5 shall be applied in addition to this.

d) The tensile strength of the concrete is ignored.

e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. For design purposes the partial safety factor Ym• equal to 1.15 shall be applied.

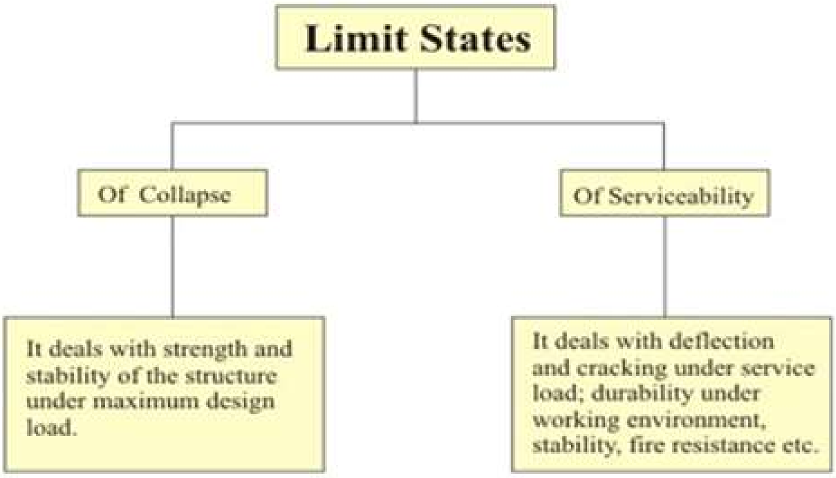
Characteristic Strength of Materials

The term 'characteristic strength' means that value of the strength of the material below which not more than 5 percent of the test results are expected to fall. Until the relevant Indian Standard Specifications for reinforcing steel are modified to include the concept of characteristic strength, the characteristic value shall be assumed as the minimum yield stress *10.2* percent proof stress Specified in the relevant Indian Standard Specifications.

Characteristic load

The term 'characteristic load' means that value of load which has a *95* percent probability of not being exceeded during the life of the structure. Since data are not available to express loads in statistical terms, for the purpose of this standard, dead loads given in IS 875 (Part 1).imposed loads given in IS 875 (Part 2), wind loads given in IS 875 (Part 3), snow load as given in IS 875 (Part 4) and seismic forces given in IS 1893 Shall be assumed as the characteristic loads.

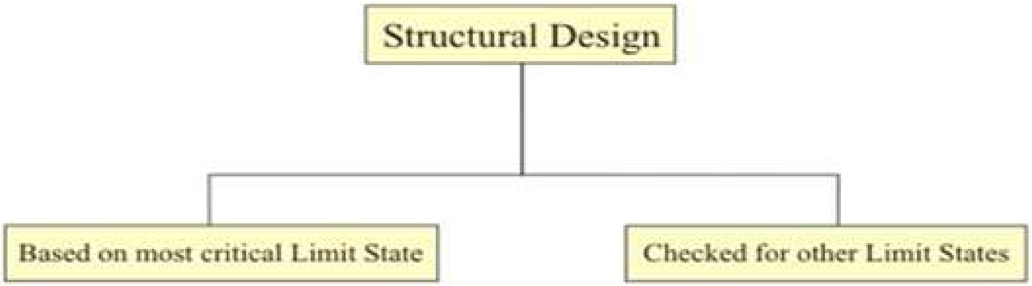
How many limit states are there?



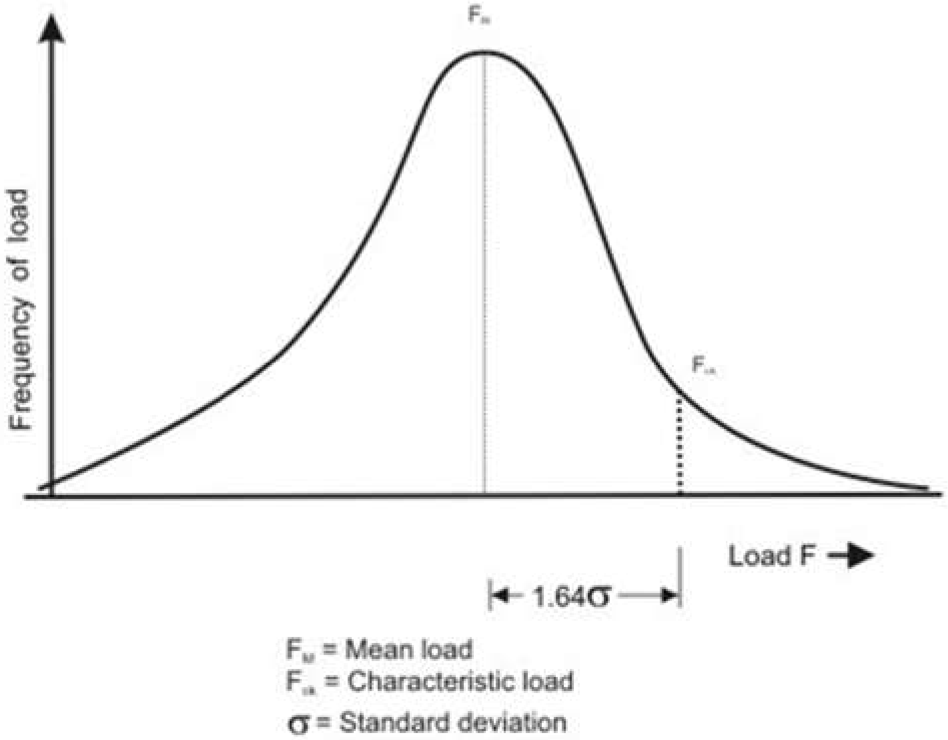
There are two main limit states: (i) limit state of collapse and (ii) limit state of serviceability in above figure

Limit state of collapse deals with the strength and stability of structures subjected to the maximum design loads out of the possible combinations of several types of loads. Therefore, this limit state ensures that neither any part nor the whole structure should collapse or become unstable under any combination of expected overloads.

Limit state of serviceability deals with deflection and cracking of structures under service loads, durability under working environment during their anticipated exposure conditions during service, stability of structures as a whole, fire resistance etc.



Partial safety factors



It is assumed that in ninety-five per cent cases the characteristic loads will not be exceeded during the life of the structures. However, structures are subjected to overloading also. Hence, structures should be designed with loads obtained by multiplying the characteristic loads with suitable factors of safety depending on the nature of loads or their combinations, and the limit state being considered. These factors of safety for loads are termed as partial safety factors *(γf)* for loads. Thus, the design loads are calculated as
(Design load *Fd*) =(Characteristic load *F*)/(Partial safety factor for load γ*f*)

Respective values of γ*f* for loads in the two limit states as given in Table 18 of IS 456 for different combinations of loads are furnished in below table

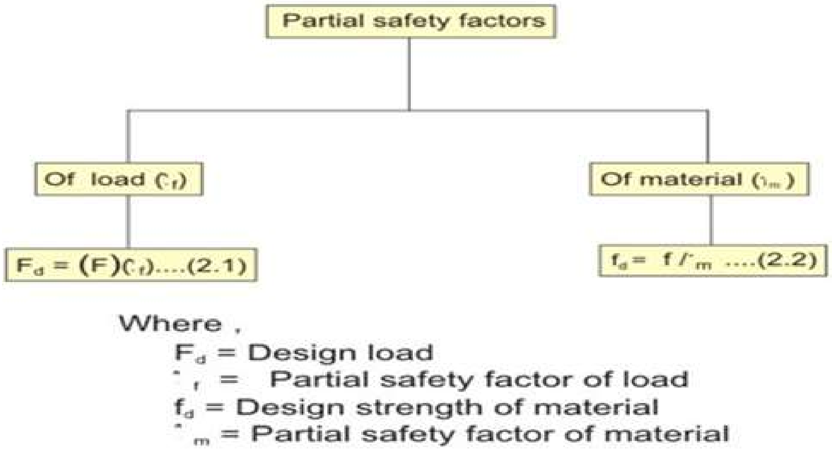
Values of partial safety factor γ*f* for loads

Load combinations	**Limit state of collapse**			**Limit state of serviceability** **(for short term effects only)**		
	DL	*IL*	*WL*	*DL*	*IL*	*WL*
DL +	1.5		1.0	1.0	1.0	-
DL + *WL*	1.5 or 0.91)	-	1.5	1.0	-	1.0
DL + *IL* + *WL*	1.2			1.0	0.8	0.8

NOTES:
- While considering earthquake effects, substitute *EL* for *WL.*

- For the limit states of serviceability, the values of γ*f* given in this table are applicable for short term effects. While assessing the long term effects due to creep the dead load and that part of the live load likely to be permanent may only be considered.

This value is to be considered when stability against overturning or stress reversal is critical.



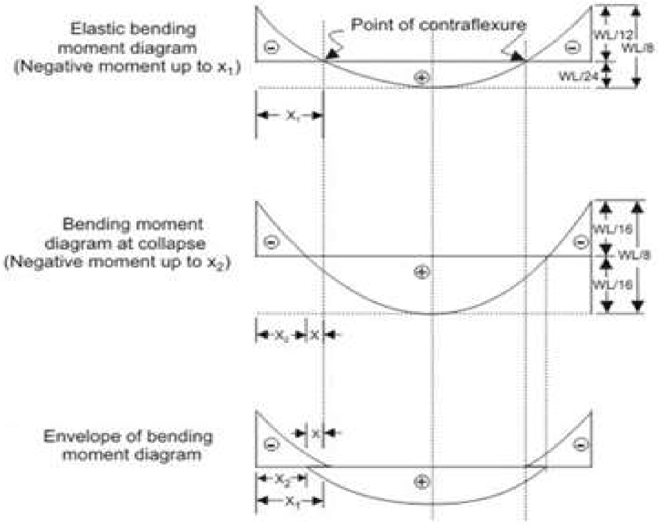
Where ,
F_d = Design load
γ_f = Partial safety factor of load
f_d = Design strength of material
γ_m = Partial safety factor of material

Similarly, the characteristic strength of a material as obtained from the statistical approach is the strength of that material below which not more than five per cent of the test results are expected to fall .However, such characteristic strengths may differ from sample to sample also. Accordingly, the design strength is calculated dividing the characteristic strength further by the partial safety factor for the material (γ*m*), where γ*m* depends on the material and the limit state being considered.Thus,Design Strength of the material f_d = *(characteristic strength of the material)/(partial safety factor of the material* γ_m*)* Both the partial safety factors are shown schematically Clause 36.4.2 of IS 456 states that γ*m* for concrete and steel should be taken as 1.5 and 1.15, respectively when assessing the strength of the structures or structural members employing limit state of collapse. However, when assessing the deflection, the material properties such as modulus of elasticity should be taken as those associated with the characteristic strength of the material. It is worth mentioning that partial safety factor for steel (1.15) is comparatively lower than that of concrete (1.5) because the steel for reinforcement is produced in steel plants and commercially available in specific diameters with expected better quality control than that of concrete. Further, in case of concrete the characteristic strength is calculated on the basis of test results on 150 mm standard cubes. But the concrete in the structure has different sizes.

Analysis

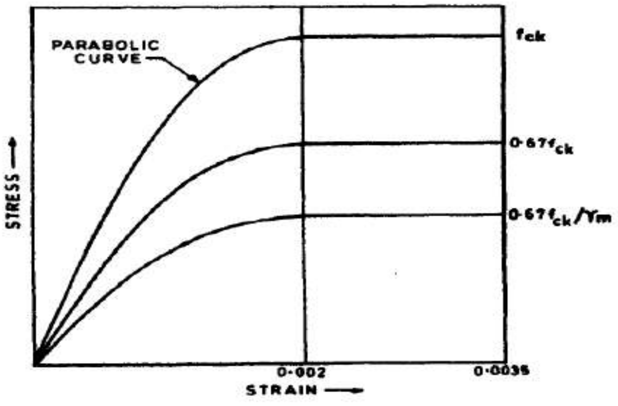
The analysis of structure, in the two limit states (of collapse and of serviceability), is taken up. In the limit state of collapse, the strength and stability of the structure or part of the structure are ensured. The resistances to bending moment, shear force, axial thrust, torsional moment at every section shall not be less than their appropriate values at that section due to the probable most unfavorable combination of the design loads on the structure. Further, the structure or part of the structure should be assessed for rupture of one or more critical sections and buckling due to elastic or plastic instability considering the effects of sway, if it occurs or overturning.

Linear elastic theory is recommended in cl. 22 of IS 456 to analyse the entire structural system subjected to design loads. The code further stipulates the adoption of simplified analyses for frames (cl. 22.4) and for continuous beams (cl.22.5). For both the limit states the material strengths should be taken as the characteristic values in determining the elastic properties of members.

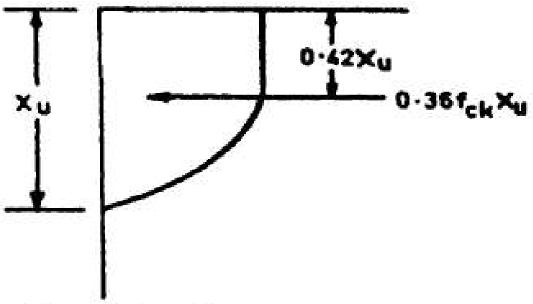


The design of structure, therefore, should also ensure that the less stressed sections can absorb further moments with a view to enabling the structure to rotate till their full capacities. This will give sufficient warming to the users before the structures collapse. Accordingly, there is a need to redistribute moments in continuous beams and frames. The analysis of slabs spanning in two directions at right angles should be performed by employing yield line theory or any other acceptable method. IS456:2000 has illustrated alternative provisions for the simply supported and restrained slabs spanning in two directions in Annex D giving bending moment coefficients of these slabs for different possible boundary conditions. These provisions enable to determine the reinforcement needed for bending moments in two directions and torsional reinforcement wherever needed.

Stress Strain curve for concrete



Stress-Strain Curve for Concrete



Stress Block Parameters

The diagram shows the distribution of compressive stress in concrete across the depth x_u of the section is termed as stress block.

Since the strain diagram is linear over the depth x_u but the shape of the stress block is same as the idealized stress strain curve of concrete. It has zero stress at the neutral axis it varies parabolic ally up to a height of 4/7 x_u and has constant value equal to the design stress $0.45f_{ck}$.

DESIGN &DRAWING OF R.C.STRUCTURES

UNIT - II

ANALYSIS OF BEAMS

Learning Material

Analysis of Structures

Structures when subjected to external loads (actions) have internal reactions in the form of bending moment, shear force, axial thrust and torsion in individual members. As a result, the structures develop internal stresses and undergo deformations. Essentially, we analyze a structure elastically replacing each member by a line (with EI values) and then design the section using concepts of limit state of collapse. The external loads to be applied on the structures are the design loads and the analysis of structures is based on linear elastic theory.

Analysis of singly reinforced rectangular beams:

In this type of problems the dimensions of the beam section (b, d), area of reinforcement (A_{st}) and grades or characteristic strengths of materials (f_{ck} , f_y) are given. The following types of problems are generally encountered in the analysis of concrete beams reinforced in tension only.

Type I: Determination of limiting or ultimate moment carrying capacity of a beam section.

The steps involved are:

1. Find the position of actual neutral axis x_u from the known values of b, d, A_{st}, f_{ck} and f_y.
   (Ref: Annex G page no 96, IS456:2000)
2. Find the position of critical neutral axis $x_{u,max}$.
3. Compare x_u with $x_{u,max}$ to determine the type of beam section:
   (a) If $x_u > x_{u,max}$, the section is over-reinforced section.
   (b) If $x_u < x_{u,max}$, the section is under-reinforced section.
4. Calculate the moment carrying capacity for the appropriate type of beam section.
   For under-reinforced section ie.,($x_u < x_{u,max}$) we use

   $M_u = T_u z = 0.87 f_y A_{st}(d - 0.42 x_u)$

   For balanced section ie.,($x_u = x_{u,max}$) we can use both the formulas.
   i.e.,
   $M_{u,lim} = 0.36 (1-0.42) bd^2 f_{ck}$
   $M_{u,lim} = T_u z = 0.87 f_y A_{st}(d - 0.42 x_{u,max})$

   For the over-reinforced section i.e.,($x_u > x_{u,max}$)(Ref: Annex G page no 96, IS456:2000) we use

$$M_{u,lim} = 0.36\ (1-0.42)\ bd^2 f_{ck}$$

Type II: Determination of load carrying capacity of a beam section. If in Type-I problems the effective span and support conditions of the beam are known, load-carrying capacity can be computed.

Analysis of doubly reinforced rectangular beams:

Determination of limiting moment of resistance or load carrying capacity:

In this case, cross sectional dimensions, area of reinforcements in tension and compression, and grades of materials used are known. The various steps involved are:

1. Determine the depth of the neutral axis of the section, $x_{u,max}$, by considering it to be a balanced section.
2. Determine the total compressive and tensile forces:

   $$C_u = 0.362\ f_{ck}\ b\ x_{u,max} + A_{sc}\ (f_{sc} - f_{cc})$$

   Where the stresses f_{sc} and f_{cc} correspond to the strain ε_{sc} at the level of compression steel which is given by

   $$\varepsilon_{sc} =$$

   With mild steel reinforcement, for d'/d ≤ 0.2,

   $$f_{cc} = 0.447\ f_{ck} \quad \text{and} \quad f_{sc} = 0.87\ f_y$$

   Tensile force, $T_u = 0.87\ f_y\ A_{st}$

   Whereas, in the case of Fe415 or Fe500 grade steel reinforcement, the stresses f_{sc} and f_{cc} are obtained from design stress-strain curves of the steel and the concrete, respectively.
3. Compare C_u with T_u to ascertain whether the section is balanced, under-reinforced or over-reinforced.
   i) If $C_u = T_u$ it is a balanced section. The limiting moment of resistance with respect to compressive force is given by:

   $$M_{u,lim} = 0.36\ f_{ck}\ b\ x_{u,max}\ (d - 0.42\ x_{u,max}) + (f_{sc} - f_{cc}) A_{sc} (d - d')$$

   ii) If $C_u > T_u$, it is an under-reinforced section. In this case, tension steel reaches its yield strength and the extreme compression fibre reaches its ultimate strain.

   iii) If $C_u < T_u$ it is an over-reinforced section.
4. Obtain the depth of the actual neutral axis from the internal force equilibrium relation:

$$C_u = T_u$$

i) For the under-reinforced section, the internal force equilibrium equation is:

$$0.36\, f_{ck}\, b\, x_u + (f_{sc} - f_{cc})A_{sc} = 0.87\, f_y\, A_{st}$$

Or

$$x_u =$$

Where f_{sc} *and* f_{cc} correspond to strain ε_{sc} which given by

$$\varepsilon_{sc} =$$

The value x_u satisfying the above two equations can be obtained by an interactive procedure, starting with $x_{u,max}$.

ii) For the over-reinforced section, the internal equilibrium equation is:

$$0.36\, f_{ck}\, b\, x_u + (f_{sc} - f_{cc})A_{sc} = f_y\, A_{st}$$

Or

$$x_u =$$

Where stresses f_{sc} *and* f_{cc} correspond to strain ε_{sc} at the level of compression steel which is given by

$$\varepsilon_{sc} =$$

And the stress f_s corresponds to the strain in tension steel ε_s given by

$$\varepsilon_{sc} =$$

The values of x_u, f_{sc} , f_{cc} and f_s satisfying the above three equations can be obtained by the iterative procedure, starting with $x_u = x_{u,max}$.

5. From the converged values of x_u, f_{sc} , f_{cc} and f_s , determine the limiting moment of resistance of the section:

   $$M_u = 0.36\, f_{ck}\, b\, x_u\, (d-0.42\, x_u) + (f_{sc} - f_{cc})A_{sc}(d-d')$$

6. If the effective span and the support conditions of the beam are known, compute the load – carrying capacity.

Analysis of the flanged beam section:

In many reinforced concrete structures, particularly in floor systems, a concrete slab is cast monolithically with and, connected to, rectangular beams. In such a construction a portion of the slab above the beam behaves structurally as a part of the beam in compression. The slab portion is called flange and the beam the web. If the flange projections are on either side of the rectangular *web* or *rib*, the resulting cross-section resembles the T- shape and hence is called a T-beam section. On the other hand, if the flange projects on one side, the resulting cross-section resembles an inverted L and hence is termed as L-beam. The flanged beams are shown in fig.1. In the absence of more accurate determination the effective width of the flange, b_f that acts along with the rectangular rib, may be taken as stipulated by IS:456.

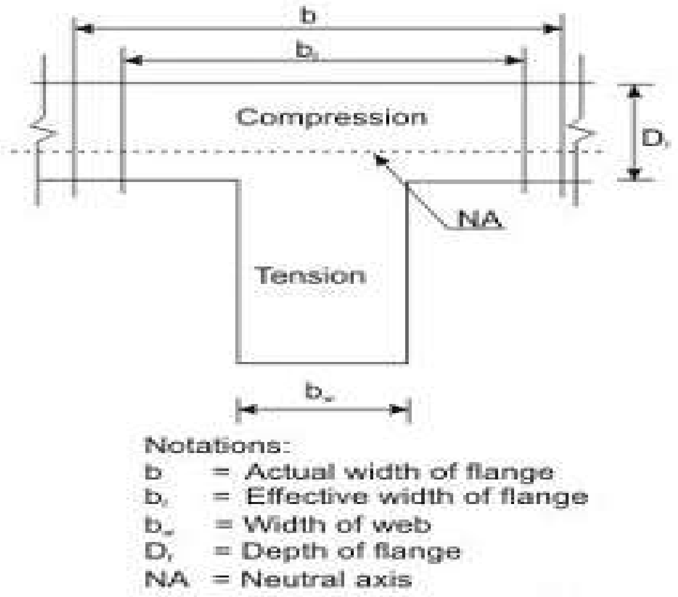


Fig.1: T-Beam

Clause 23.1.2 of IS 456 specifies the following effective widths of *T* and *L*-beams:

(a) For *T*-beams, the lesser of

(i) $b_f = lo/6 + bw + 6\,Df$

(ii) b_f = Actual width of the flange

(b) For *L*-beams, the lesser of

(i) $b_f = lo/12 + bw + 3\,Df$

(ii) b_f = Actual width of the flange

(c) For isolated *T*-beams, the lesser of

(i) $b_f = \; + b_w$

(ii) b_f = Actual width of the flange

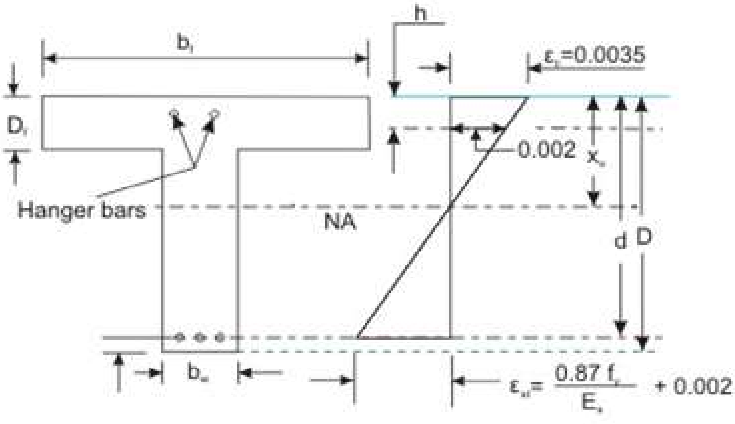
Where b_f = effective width of the flange,

lo = distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,

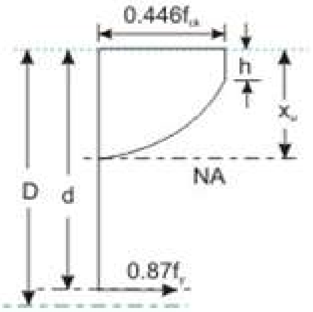
b_w = beadth of the web,

D_f = thickness of the flange

b = actual width of the flange.



(a) Cross section (b) Strain diagram



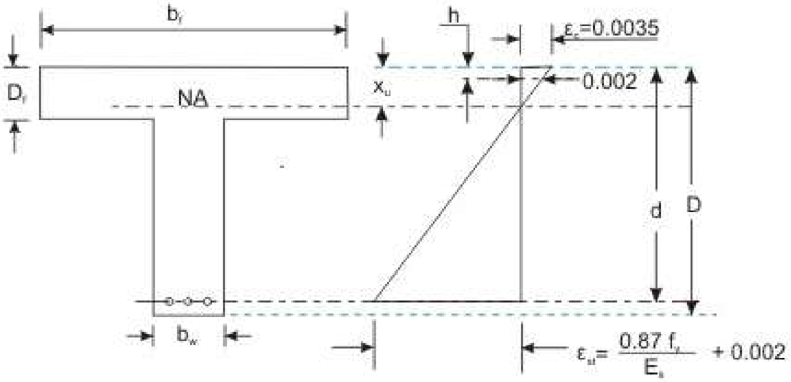
(c) Stress diagram

A typical T-beam section

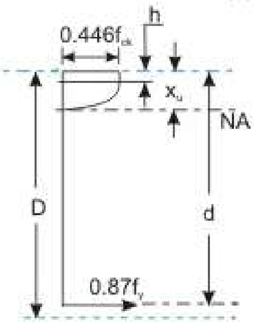
The neutral axis of a flanged beam may be either in the flange or in the web depending on the physical dimensions of the effective width of flange *bf*, effective width of web *bw*, thickness of flange *Df* and effective depth of flanged beam *d* (Fig). The flanged beam may be considered as a rectangular beam of width *bf* and effective depth *d* if the neutral axis is in the flange as the concrete in tension is ignored. However, if the neutral axis is in the web, the compression is taken by the flange and a part of the web. To ascertain the type of section consider it to be a balanced section. The limiting depth of the neutral axis for this case, $x_{u,max}$, can be obtained as:

=

Case I: Neutral axis is in the flange ($x_u < Df$) and the beam is analyzed as a rectangular beam of width b_f instead of a beam width b_w and depth d. Such an idealization is possible since the concrete in tension can be ignored. In this case the expression developed earlier for rectangular beam can be utilized. The depth of the actual neutral axis x_u must be evaluated by using the internal force equilibrium condition:



(a) Cross section (b) Strain diagram



(c) Stress diagram

T-beam, case (i), when $x_u < D_f$

$$C_u = T_u \qquad or \qquad 0.36\, f_{ck}\ x_u\, b_f\ = 0.87\, f_y\ A_{st}$$

Thus,

x_u =

The limiting or ultimate moment capacity of the beam can be obtained as:

$$M_u = T_u\, z = 0.87\, f_y\, A_{st}(\, d - 0.42\ x_u)$$

Case II: $x_u > D_f \leq 0.429x_u$. for this case $x_u > D_f$ the neutral axis lies in the web and the section will be analyzed as a flanged section. However, for the condition $D_f \leq 0.429x_u$, the stresses in the flange is uniform. The T-beam can be consider as a rectangular beam of width b_w and depth d, and remaining portion of flange as abeam of width ($b_f - b_w$) and depth D_f.

The depth of neutral axis x_u can be determined by using force equilibrium equation:

Compressive force in web + Compressive force in flange = total tension

$$C_{uw} + C_{uf} = T_u$$

Or

$$0.36\, f_{ck}\ x_u\, b_f\ + 0.447\, f_{ck}\, (b_f\ - b_w)\, D_f = 0.87\, f_y\ A_{st}$$

The limiting or ultimate moment capacity of the beam can be obtained as:

$$M_u = M_{u,web} + M_{u,flange}$$

$$M_u = 0.36\, f_{ck}\ x_u\, b_w(\, d - 0.42x_u) + 0.447\, f_{ck}\, (b_f - b_w)\, D_f\, (d - D_f / 2)$$

Case III: $x_u > D_f > 0.429x_u$. for this case $x_u > D_f$ the neutral axis lies in the web and the analysis shall be based on the flanged section. Moreover, since $D_f > 0.429x_u$,the stresses in the flange is nonlinear. For shallow flange with $D_f<0.2d$, IS:456 have given following formula for the calculation of the ultimate moment capacity:

$$M_u = M_{u,web} + M_{u,flange}$$

$$M_u = 0.36\, f_{ck}\ x_u\, b_w(\, d - 0.42x_u) + 0.447\, f_{ck}\, (b_f - b_w)\, Y_f\, (d - Y_f / 2)$$

Where $Y_f = (0.15x_u + 0.65D_f)$, but not greater than D_f.

To understand the basis of reduced flange depth in the above equation it may recalled that the depths of the neutral axis for the balanced section having Fe250,Fe415 and Fe500 grade steel as reinforcement are approximately 0.531d, 0.479d and 0.456d, respectively, and the parabolic portion of the parabolic-rectangular stress-strain curve exteds to a height of $0.002x_u$ / 0.0035= $0.571x_u$ from the neutral axis beyond which rectangular stress distribution extends to a height of $0.429x_u$. concidering the worst case, the rectangular stress distribution may extend to a depth 0.429X 0.456d = 0.196d. hence a depth of 0.2d has been chosen as the limiting depth for the shallow flanges.

The limiting value of Y_f is equal to D_f for the case when the bottom fibre of the flange is subjected to a compressive strain to or greater than 0.002 corrosponding to the design stress equal to $0.447f_{ck}$ .when the strain in the bottom fibre of the flange is less than 0.002. the design stress distribution in the flange is nonlinear. This makes it necessary to use the reduced flange depth D_f if the uniform design stress of $0.447f_{ck}$ is to be used over the entire flange depth.

The value of x_u in the preceding expression must be evaluated by using the internal force equilibrium equation, for which it is essential to assume for the first trail either $x_u < D_f$ or $x_u > D_f$. to reduce the calculations for the first trail, that the neutral axis coincides with the underneath of the flange slab, and calculate total compression concrete flange and total tension in steel, C_u and T_u respectively. Then

1. If $C_u > T_u$, the neutral axis lies in the flange.
2. If $C_u = T_u$, the neutral axis coincide with the bottom of the flange.
3. If $C_u < T_u$, the neutral axis lies in the web.

DESIGN AND DRAWING OF R.C STRUCTURE

UNIT-3

SHEAR AND DEFLECTION

SHEAR

FAILURE MODES DUE TO SHEAR

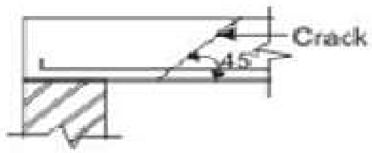


Fig. (a): Web shear progresses along dotted dotted lines

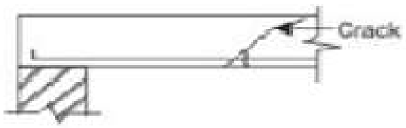


Fig. (b): Flexural tension (steel yields)

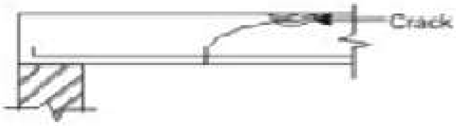


Fig. (c): Flexural compression
(concrete crushes in compression)

Fig : Failure modes

Bending in reinforced concrete beams is usually accompanied by shear, the exact analysis of which is very complex. However, experimental studies confirmed the following three different modes of failure due to possible combinations of shear force and bending moment at a given section (Figs a to c):

(i) Web shear (Fig a)

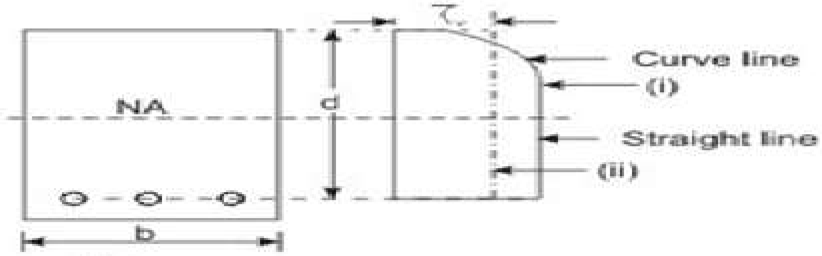
(ii) Flexural tension shear (Fig b)

(iii) Flexural compression shear (Fig c)

Web shear causes cracks which progress along the dotted line shown in Fig a. Steel yields in flexural tension shear as shown in Fig b, while concrete crushes in compression due to flexural compression shear as shown in Fig c. An in-depth presentation of the three types of failure modes is beyond the scope here. Only the salient points needed for the routine design of beams in shear are presented here.

SHEAR STRESS

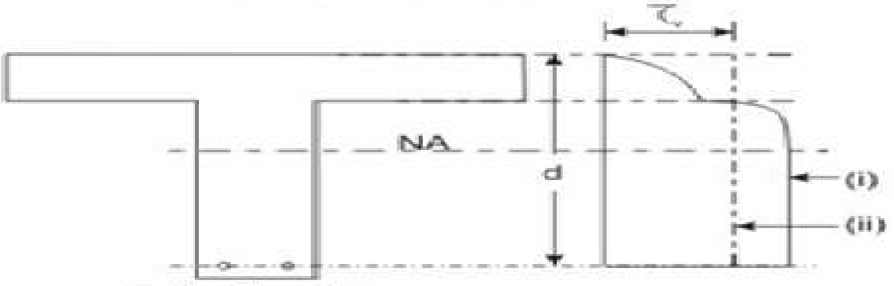
The distribution of shear stress in reinforced concrete rectangular, *T* and *L*-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τ_v is considered which is calculated as follows (IS 456, cls. 40.1 and 40.1.1):



Note:

(i) Actual distribution
(ii) Average distribution

(a) Rectangular beam



(b) T-beam

Note:

(i) Actual distribution
(ii) Average distribution

Fig : Distribution of shear stress and average shear stress

(i) In beams of uniform depth (Figs a and b):

Where Vu = shear force due to design loads,

b = breadth of rectangular beams and breadth of the web b_w for flanged beams, and

d = effective depth.

(ii) In beams of varying depth:

Where τ_v, Vu, b or b and d are the same as in (i),

Mu = bending moment at the section, and

β = angle between the top and the bottom edges.

The positive sign is applicable when the bending moment M_u decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment M_u increases numerically in the same direction as the effective depth increases

DESIGN SHEAR STRENGTH OF REINFORCED CONCRETE

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value τ_{cmax} depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

DESIGN SHEAR STRENGTH WITHOUT SHEAR REINFORCEMENT (IS 456, CL. 40.2.1)

Table: Design shear strength of concrete, τ_c in N/mm^2

(100 *As* /*b d*)	Grade of concrete				
	M 20	M 25	M 30	M 35	M40 and
≤ 0.15	0.28	0.29	0.29	0.29	0.30
0.25	0.36	0.36	0.37	0.37	0.38
0.50	0.48	0.49	0.50	0.50	0.51
0.75	0.56	0.57	0.59	0.59	0.60
1.00	0.62	0.64	0.66	0.67	0.68
1.25	0.67	0.70	0.71	0.73	0.74
1.50	0.72	0.74	0.76	0.78	0.79
1.75	0.75	0.78	0.80	0.82	0.84
2.00	0.79	0.82	0.84	0.86	0.88
2.25	0.81	0.85	0.88	0.90	0.92
2.50	0.82	0.88	0.91	0.93	0.95
2.75	0.82	0.90	0.94	0.96	0.98
≥ 3.00	0.82	0.92	0.96	0.99	1.01

In above table, *As* is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section considered except at support where the full area of tension reinforcement may be used provided the detailing is as per IS 456, cls. 26.2.2 and 26.2.3.

MAXIMUM SHEAR STRESS τ_c MAX WITH SHEAR REINFORCEMENT (cls. 40.2.3, 40.5.1 and 41.3.1)

Table 20 of IS 456 stipulates the maximum shear stress of reinforced concrete in beams τ_{cmax} as given below in Table 6.2. Under no circumstances, the nominal shear stress in beams τ_v shall exceed τc_{max} given in below table for different grades of concrete.

Grade of concrete	**M 20**	**M 25**	**M 30**	**M 35**	**M 40 and above**
τ_{cmax}, N/mm2	2.8	3.1	3.5	3.7	4.0

Table: Maximum shear stress, τ_{cmax} in N/mm2

CRITICAL SECTION FOR SHEAR

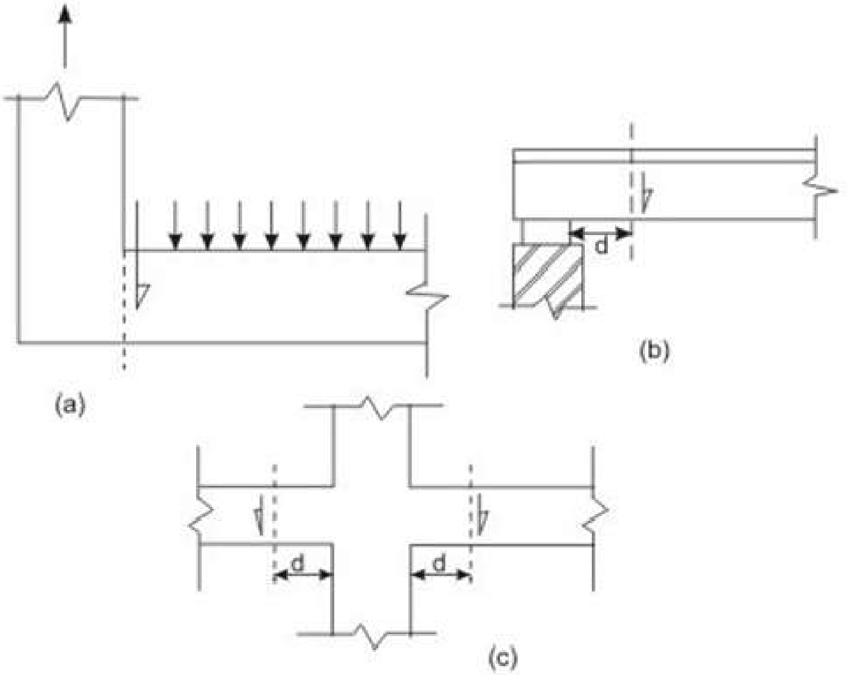


Fig : Support conditions for locating factored shear force

Clauses 22.6.2 and 22.6.2.1 stipulate the critical section for shear and are as follows:

For beams generally subjected to uniformly distributed loads or where the principal load is located further than 2d from the face of the support, where d is the effective depth of the beam, the critical sections depend on the conditions of supports as shown in Figs a, b and c and are mentioned below.

(i) When the reaction in the direction of the applied shear introduces tension (Fig a) into the end region of the member, the shear force is to be computed at the face of the support of the member at that section.

(ii) When the reaction in the direction of the applied shear introduces compression into the end region of the member (Figs b and c), the shear force computed at a distance *d* from the face of the support is to be used for the design of sections located at a distance less than *d* from the face of the support. The enhanced shear strength of sections close to supports, however, may be considered as discussed in the following section.

ENHANCED SHEAR STRENGTH OF SECTIONS CLOSE TO SUPPORTS (CL.40.5 OF IS 456)

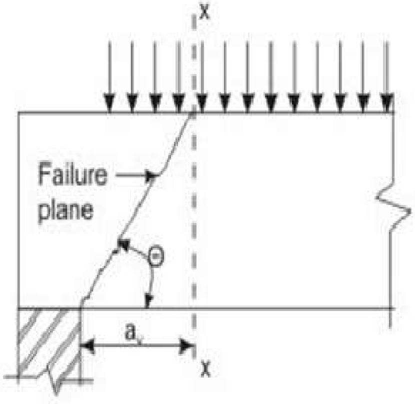


Fig : Shear failure near support

Above figure shows the shear failure of simply supported and cantilever beams without shear reinforcement. The failure plane is normally inclined at an angle of 30o to the horizontal. However, in some situations the angle of failure is steeper either due to the location of the failure section closed to a support or for some other reasons. Under these situations, the shear force required to produce failure is increased.

Such enhancement of shear strength near a support is taken into account by increasing the design shear strength of concrete to ($2d\tau_c/a_v$) provided that the design shear stress at the face of the support remains less than the value of τ_{cmax} given in above table (Table 20 of IS 456). In the above expression of the enhanced shear strength

d = effective depth of the beam,

τ_c = design shear strength of concrete before the enhancement as given in table (Table 19 of IS 456),

a_v = horizontal distance of the section from the face of the support

Similar enhancement of shear strength is also to be considered for sections closed to point loads. It is evident from the expression ($2d\tau_c / a_v$) that when a_v is equal to $2d$, the enhanced shear strength does not come into picture. Further, to increase the effectivity, the tension reinforcement is recommended to be extended on each side of the point where it is intersected by a possible failure plane for a distance at least equal to the effective depth, or to be provided with an equivalent anchorage.

MINIMUM SHEAR REINFORCEMENT (CLS. 40.3, 26.5.1.5 AND 26.5.1.6 OF IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c as recommended in cl. 40.3 of c. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that:

Where A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam bw, and

f_y = characteristic strength of the stirrup reinforcement in N/mm2 which shall not be taken greater than 415 N/mm2.

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value.

The minimum shear reinforcement is provided for the following:

(i) Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.

(ii) Brittle shear failure is arrested which would have occurred without shear reinforcement.

(iii) Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.

(iv) To hold the reinforcement in place when concrete is poured.

(v) Section becomes effective with the tie effect of the compression steel.

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than 0.75 d for vertical stirrups and d for inclined stirrups at 45o, where d is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

DESIGN OF SHEAR REINFORCEMENT (CL. 40.4 OF IS 456)

When τ_v is more than τ_c, shear reinforcement shall be provided in any of the three following forms:

(a) Vertical stirrups,
(b) Bent-up bars along with stirrups, and
(c) Inclined stirrups.

In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty per cent of that of the total shear reinforcement.

The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to

Where b is the breadth of rectangular beams or b_w *in* the case of flanged beams.

The strengths of shear reinforcement Vus for the three types of shear reinforcement are as follows:

(a) Vertical stirrups:

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

(c) For single bar or single group of parallel bars, all bent-up at the same cross- section:

Where Asv = total cross-sectional area of stirrup legs or bent-up bars within a distance sv,

sv = spacing of stirrups or bent-up bars along the length of the member,

τv = nominal shear stress,

τc = design shear strength of concrete,

b = breadth of the member which for the flanged beams shall be taken as the breadth of the web b_w.

fy = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm2,

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45o, and

d = effective depth.

The following two points are to be noted:

(i) The total shear resistance shall be computed as the sum of the resistance for the various types separately where more than one type of shear reinforcement is used.

(ii) The area of stirrups shall not be less than the minimum specified in cl. 26.5.1.6.

SHEAR REINFORCEMENT FOR SECTIONS CLOSE TO SUPPORTS

As stipulated in cl. 40.5.2 of IS 456, the total area of the required shear reinforcement A_s is obtained from:

And ≥

For flanged beams, b will be replaced by b_w, the breadth of the web of flanged beams.

This reinforcement should be provided within the middle three quarters of av, where av is less than d, horizontal shear reinforcement will be effective than vertical.

Alternatively, one simplified method has been recommended in cl. 40.5.3

of IS 456 and the same is given below.

The following method is for beams carrying generally uniform load or where the principal load is located further than $2d$ from the face of support. The shear stress is calculated at a section a distance d from the face of support. The value of τc is calculated in accordance with IS 456 and appropriate shear reinforcement is provided at sections closer to the support. No further check for shear at such sections is required.

BOND

INTRODUCTION

The bond between steel and concrete is very important and essential so that they can act together without any slip in a loaded structure. With the perfect bond between them, the plane section of a beam remains plane even after bending. The length of a member required to develop the full bond is called the anchorage length. The bond is measured by bond stress. The local bond stress varies along a member with the variation of bending moment. The average value throughout its anchorage length is designated as the average bond stress. In our calculation, the average bond stress will be used.

Thus, a tensile member has to be anchored properly by providing additional length on either side of the point of maximum tension, which is known as 'Development length in tension'. Similarly, for compression members also, we have 'Development length Ld in compression'.

It is worth mentioning that the deformed bars are known to be superior to the smooth mild steel bars due to the presence of ribs. In such a case, it is needed to check for the sufficient development length L_d only rather than checking both for the local bond stress and development length as required for the smooth mild steel bars. Accordingly, IS 456, cl. 26.2 stipulates the requirements of proper anchorage of reinforcement in terms of development length L_d only employing design bond stress τ_{bd}.

DESIGN BOND STRESS τ_{bd}

(a) Definition

The design bond stress τ_{bd} is defined as the shear force per unit nominal surface area of reinforcing bar. The stress is acting on the interface between bars and surrounding concrete and along the direction parallel to the bars.

This concept of design bond stress finally results in additional length of a bar of specified diameter to be provided beyond a given critical section. Though, the overall bond failure may be avoided by this provision of additional development length Ld, slippage of a bar may not always result in overall failure of a beam. It is, thus, desirable to provide end anchorages also to maintain the integrity of the structure and thereby, to enable it carrying the loads. Clause 26.2 of IS 456 stipulates, "The calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length or end anchorage or by a combination thereof."

(b) Design bond stress – values

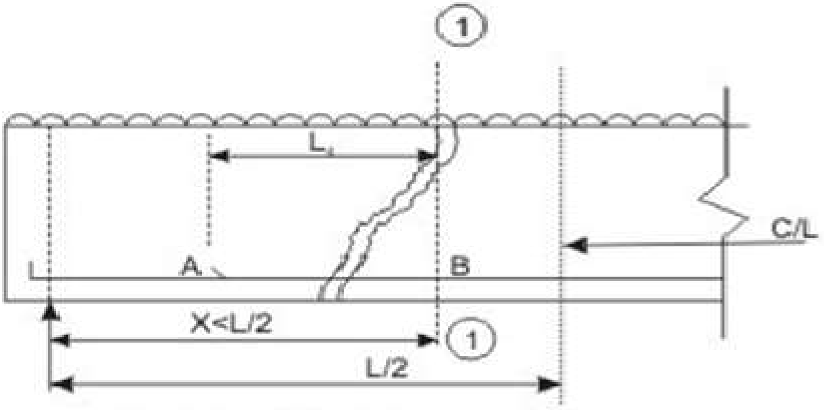
The local bond stress varies along the length of the reinforcement while the average bond stress gives the average value throughout its development length. This average bond stress is still used in the working stress method and IS 456 has mentioned about it in cl. B-2.1.2. However, in the limit state method of design, the average bond stress has been designated as design bond stress τ_{bd} and the values are given in cl. 26.2.1.1. The same is given below as a ready reference.

Table: τ_{bd} for plain bars in tension

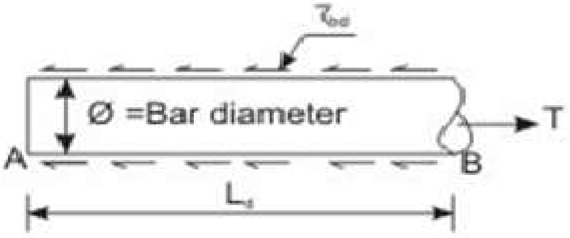
Grade of concrete	M 20	M 25	M 30	M 35	M 40 and
Design Bond Stress τ_{bd} in	1.2	1.4	1.5	1.7	1.9

For deformed bars conforming to IS 1786, these values shall be increased by 60 per cent. For bars in compression, the values of bond stress in tension shall be increased by 25 per cent.

DEVELOPMENT LENGTH



(a): Beam showing L_d of a bar



(b): Free body diagram of segment AB

Fig : Development length of bar

(a) A single bar

Figure above shows a simply supported beam subjected to uniformly distributed load. Because of the maximum moment, the A_{st} required is the maximum at $x = L/2$. For any section 1-1 at a distance $x < L/2$, some of the tensile bars can be curtailed. Let us then assume that section 1-1 is the theoretical cut-off point of one bar. However, it is necessary to extend the bar for a length L_d as explained earlier. Let us derive the expression to determine L_d of this bar.

Figure 6.15.1(b) shows the free body diagram of the segment AB of the bar. At B, the tensile force T trying to pull out the bar is of the value $T = (\pi \phi 2 \sigma s /4)$, where ϕ is the nominal diameter of the bar and σs is the tensile stress in bar at the section considered at design loads. It is necessary to have the resistance force to be developed by τ_{bd} for the length Ld to overcome the tensile force. The resistance force $= \pi \phi (Ld) (\tau bd)$. Equating the two, we get

Equation above, thus gives

The above equation is given in cl. 26.2.1 of IS 456 to determine the development length of bars.

The example taken above considers round bar in tension. Similarly, other sections of the bar should have the required L_d as determined for such sections. For bars in compression, the development length is reduced by 25 per cent as the design bond stress in compression τ_{bd} is 25 per cent more than that in tension (see the last lines below Table 6.4). Following the same logic, the development length of deformed bars is reduced by 60 per cent of that needed for the plain round bars. Tables 64 to 66 of SP-16 present the development lengths of fully stressed plain and deformed bars (when $\sigma s = 0.87 f_y$) both under tension and compression. It is to be noted that the consequence of stress concentration at the lugs of deformed bars has not been taken into consideration.

(b) Bars bundled in contact

The respective development lengths of each of the bars for two, three or four bars in contact are determined following the same principle. However, cl.26.2.1.2 of IS 456 stipulates a simpler approach to determine the development length directly under such cases and the same is given below:

"The development length of each bar of bundled bars shall be that for the individual bar, increased by 10 per cent for two bars in contact, 20 per cent for three bars in contact and 33 per cent for four bars in contact."

However, while using bundled bars the provision of cl. 26.1.1 of IS 456 must be satisfied. According to this clause:

- In addition to single bar, bars may be arranged in pairs in contact or in groups of three or four bars bundled in contact.
- Bundled bars shall be enclosed within stirrups or ties to ensure the bars remaining together.
- Bars larger than 32 mm diameter shall not be bundled, except in columns.

Curtailment of bundled bars should be done by terminating at different points spaced apart by not less than 40 times the bar diameter except for bundles stopping at support (cl. 26.2.3.5 of IS 456).

Checking of Development Lengths of Bars in Tension

The following are the stipulation of cl. 26.2.3.3 of IS 456.

(i) At least one-third of the positive moment reinforcement in simple members and one-fourth of the positive moment reinforcement in continuous members shall be extended along the same face of the member into the support, to a length equal to $L_d/3$.

(ii) Such reinforcements of (i) above shall also be anchored to develop its design stress in tension at the face of the support, when such member is part of the primary lateral load resisting system.

(iii) The diameter of the positive moment reinforcement shall be limited to a diameter such that the L_d computed for $\sigma_s = f_d$ in above equation does not exceed the following:

$$(L_d)_{\text{when } \sigma_s = f_d} \le \frac{M_1}{V} + L_o$$

Where M_1 = moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d,

V = shear force at the section due to design loads

Lo = sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support. At a point of inflection, Lo is limited to the effective depth of the member or $12Ø$, whichever is greater, and

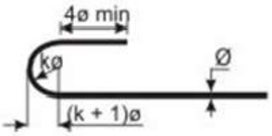
$Ø$ = diameter of bar.

It has been further stipulated that $M1/V$ in the above expression may be increased by 30 per cent when the ends of the reinforcement are confined by a compressive reaction.

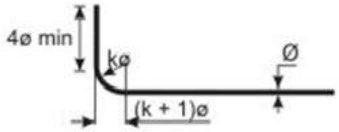
ANCHORING REINFORCING BARS

The bars may be anchored in combination of providing development length to maintain the integrity of the structure. Such anchoring is discussed below under three sub-sections for bars in tension, compression and shear respectively, as stipulated in cl. 26.2.2 of IS 456

(a) Bars in tension (cl. 26.2.2.1 of IS 456)



(a) Standard hook



Minimum k for (i) mild steel = 2, and
(ii) cold worked steel = 4

(b) Standard 90° bend

Fig : Standard hook and bend

The salient points are:

- Deformed bars may not need end anchorages if the development length requirement is satisfied.
- Hooks should normally be provided for plain bars in tension.
- Standard hooks and bends should be as per IS 2502 or as given in Table 67 of SP-16, which are shown in Figs a and b.
- The anchorage value of standard bend shall be considered as 4 times the diameter of the bar for each 45o bend subject to a maximum value of 16 times the diameter of the bar.
- The anchorage value of standard U-type hook shall be 16 times the diameter of the bar.

(b) Bars in compression (cl. 26.2.2.2 of IS 456)

Here, the salient points are:

- The anchorage length of straight compression bars shall be equal to its development length.
- The development length shall include the projected length of hooks, bends and straight lengths beyond bends, if provided.

(c) Bars in shear (cl. 26.2.2.4 of IS 456)

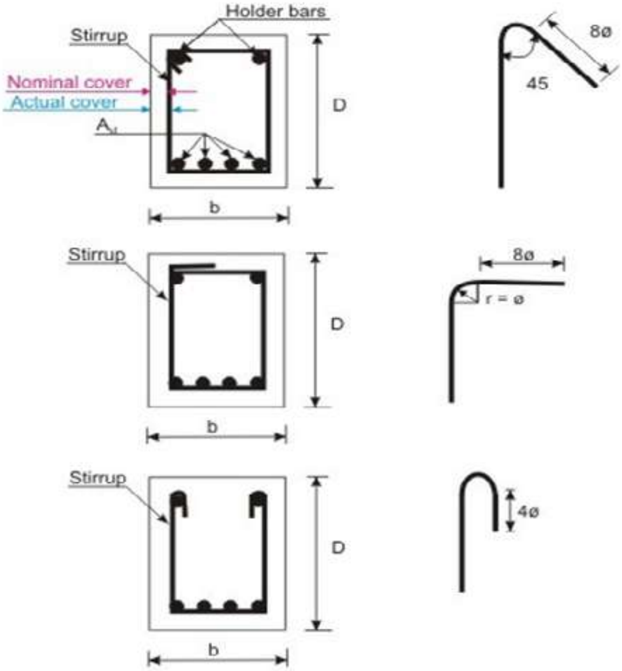


Fig : Anchorage of stirrups

The salient points are:

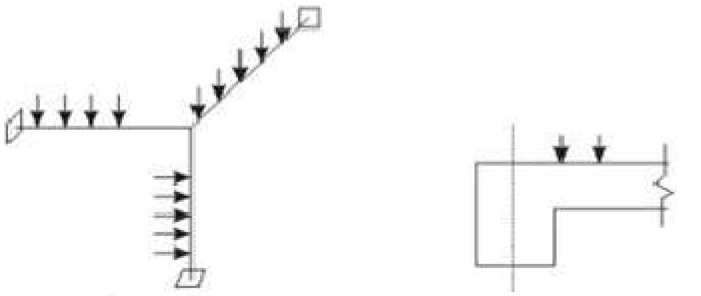
- Inclined bars in tension zone will have the development length equal to that of bars in tension and this length shall be measured from the end of sloping or inclined portion of the bar.
- Inclined bars in compression zone will have the development length equal to that of bars in tension and this length shall be measured from the mid- depth of the beam.
- For stirrups, transverse ties and other secondary reinforcement, complete development length and anchorage are considered to be satisfied if prepared as shown in figure above.

TORSION

INTRODUCTION

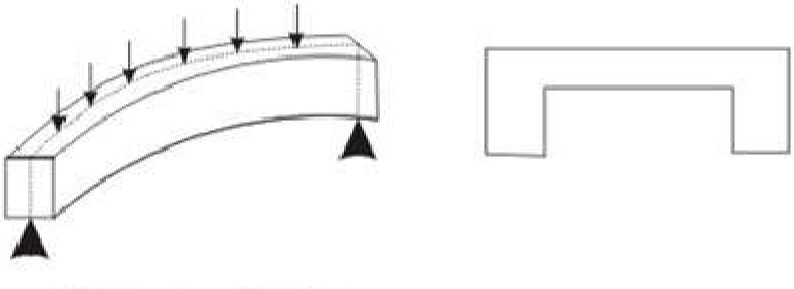
This lesson explains the presence of torsional moment along with bending moment and shear in reinforced concrete members with specific examples. The approach of design of such beams has been explained mentioning the critical section to be designed. Expressing the equivalent shear and bending moment, this lesson illustrates the step by step design procedure of beam under combined bending, shear and torsion. The requirements of IS 456 regarding the design are also explained. Numerical problems have been solved to explain the design of beams under combined bending, shear and torsion.

TORSION IN REINFORCED CONCRETE MEMBERS



(a) Space frame

(b) Inverted L-beam



(c) Beam curved in plan

(d) Edge beam in slab

Fig : Beams under combined bending, shear & torsion

On several situations beams and slabs are subjected to torsion in addition to bending moment and shear force. Loads acting normal to the plane of bending will cause bending moment and shear force. However, loads away from the plane of bending will induce torsional moment along with bending moment and shear. Space frames (Fig.a), inverted *L*-beams as in supporting sunshades and canopies (Fig.b), beams curved in plan (Fig.c), edge beams of slabs (Fig.6.16.1d) are some of the examples where torsional moments are also present.

Skew bending theory, space-truss analogy are some of the theories developed to understand the behaviour of reinforced concrete under torsion combined with bending moment and shear. These torsional moments are of two types:

(i) Primary or equilibrium torsion, and
(ii) Secondary or compatibility torsion.

The primary torsion is required for the basic static equilibrium of most of the statically determinate structures. Accordingly, this torsional moment must be considered in the design as it is a major component.

The secondary torsion is required to satisfy the compatibility condition between members. However, statically indeterminate structures may have any of the two types of torsions. Minor torsional effects may be ignored in statically indeterminate structures due to the advantage of having more than one load path for the distribution of loads to maintain the equilibrium. This may produce minor cracks without causing failure. However, torsional moments should be taken into account in the statically indeterminate structures if they are of equilibrium type and where the torsional stiffness of the members has been considered in the structural analysis. It is worth mentioning that torsion must be considered in structures subjected to unsymmetrical loadings about axes.

Clause 41 of IS 456 stipulates the above stating that, "In structures, where torsion is required to maintain equilibrium, members shall be designed for torsion in accordance with 41.2, 41.3 and 41.4. However, for such indeterminate structures where torsion can be eliminated by releasing redundant restraints, no specific design for torsion is necessary, provided torsional stiffness is neglected in the calculation of internal forces. Adequate control of any torsional cracking is provided by the shear reinforcement as per cl. 40".

ANALYSIS FOR TORSIONAL MOMENT IN A MEMBER

The behaviour of members under the effects of combined bending, shear and torsion is still a subject of extensive research.

We know that the bending moments are distributed among the sharing members with the corresponding distribution factors proportional to their bending stiffness EI/L where E is the elastic constant, I is the moment of inertia and L is the effective span of the respective members. In a similar manner, the torsional moments are also distributed among the sharing members with the corresponding distribution factors proportional to their torsional stiffness GJ/L, where G is the elastic shear modulus, J is polar moment of inertia and L is the effective span (or length) of the respective members.

The exact analysis of reinforced concrete members subjected to torsional moments combined with bending moments and shear forces is beyond the scope here. However, the codal provisions of designing such members are discussed below.

APPROACH OF DESIGN FOR COMBINED BENDING, SHEAR AND TORSION AS PER IS 456

As per the stipulations of IS 456, the longitudinal and transverse reinforcements are determined taking into account the combined effects of bending moment, shear force and torsional moment. Two empirical relations of equivalent shear and equivalent bending moment are given. These fictitious shear forces and bending moment, designated as equivalent shear and equivalent bending moment, are separate functions of actual shear and torsion, and actual bending moment and torsion, respectively. The total vertical reinforcement is designed to resist the equivalent shear *Ve* and the longitudinal reinforcement is designed to resist the equivalent bending moment Me1 *and* Me2, as explained in secs. 6.16.6 and 6.16.7, respectively. These design rules are applicable to beams of solid rectangular cross-section. However, they may be applied to flanged beams by substituting *bw* for *b*. IS 456 further suggests to refer to specialist literature for the flanged beams as the design adopting the code procedure is generally conservative.

Critical Section (cl. 41.2 of IS 456)

As per cl. 41.2 of IS 456, sections located less than a distance *d* from the face of the support is to be designed for the same torsion as computed at a distance *d*, where *d* is the effective depth of the beam.

SHEAR AND TORSION

(a) The equivalent shear, a function of the actual shear and torsional moment is determined from the following empirical relation:

Where *Ve* = equivalent shear,

Vu = actual shear,

Tu = actual torsional moment,

b = breadth of beam.

Reinforcement in Members subjected to Torsion

(a) Reinforcement for torsion shall consist of longitudinal and transverse reinforcement.

(b) The longitudinal flexural tension reinforcement shall be determined to resist an equivalent bending moment Me_1 as given below:

Where *Mu* = bending moment at the cross-section, and

Where *Tu* = Torsional moment,

D = overall depth of the beam, and

b = breadth of the beam.

(c) The longitudinal flexural compression reinforcement shall be provided if the numerical value of *Mt* as defined above in Eq. exceeds the numerical value of *Mu*. Such compression reinforcement should be able to resist an equivalent bending moment *Me2* as given below:

The *Me2* will be considered as acting in the opposite sense to the moment *Mu*.

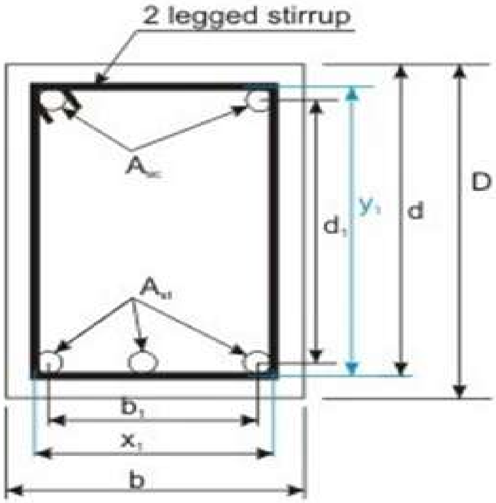


Fig : Stirrups in beams

(a) The transverse reinforcement consisting of two legged closed loops (Fig.6.16.2) enclosing the corner longitudinal bars shall be provided having an area of cross-section *Asv* given below:

$$A_{sv} = \frac{T_u\, s_v}{b_1\, d_1 (0.87\, f_y)} + \frac{V_u\, s_v}{2.5\, d_1 (0.87\, f_y)}$$

However, the total transverse reinforcement shall not be less than the following:

$$A_{sv} \geq (\tau_{ve} - \tau_c)\, b\, s_v / (0.87\, f_y)$$

Where *Tu* = torsional moment,

Vu = shear force,

sv = spacing of the stirrup reinforcement,

b1 = centre to centre distance between corner bars in the direction of the width,

d1 = centre to centre distance between corner bars,

b = breadth of the member,

fy = characteristic strength of the stirrup reinforcement,

τ_{ve}= equivalent shear stress and
τ_c = shear strength of concrete as per Table 19 of IS 456

REQUIREMENTS OF REINFORCEMENT

Beams subjected to bending moment, shear and torsional moment should satisfy the following requirements:

(a) Tension reinforcement (cl. 26.5.1.1 of IS 456)

The minimum area of tension reinforcement should be governed by

Where *As* = minimum area of tension reinforcement,

b = breadth of rectangular beam or breadth of web of *T*-beam,
d = effective depth of beam,

f_y = characteristic strength of reinforcement in N/mm2.

The maximum area of tension reinforcement shall not exceed 0.04 *bD*, where *D* is the overall depth of the beam.

(b) Compression reinforcement (cl. 26.5.1.2 of IS 456)

The maximum area of compression reinforcement shall not exceed 0.04 *bD*. They shall be enclosed by stirrups for effective lateral restraint.

(c) Side face reinforcement (cls. 26.5.1.3 and 26.5.1.7b)

Beams exceeding the depth of 750 mm and subjected to bending moment and shear shall have side face reinforcement. However, if the beams are having torsional moment also, the side face reinforcement shall be provided for the overall depth exceeding 450 mm. The total area of side face reinforcement shall be at least 0.1 per cent of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness, whichever is less.

(d) Transverse reinforcement (cl. 26.5.1.4 of IS 456)

The transverse reinforcement shall be placed around the outer-most tension and compression bars. They should pass around longitudinal bars located close to the outer face of the flange in *T*- and *I*-beams.

(e) Maximum spacing of shear reinforcement (cl. 26.5.1.5 of IS 456)

The centre to centre spacing of shear reinforcement shall not be more than 0.75 *d* for vertical stirrups and *d* for inclined stirrups at 45o, but not exceeding 300 mm, where *d* is the effective depth of the section.

(f) Minimum shear reinforcement (cl. 26.5.1.6 of IS 456)

This has been discussed in above sec.

(g) Distribution of torsion reinforcement (cl. 26.5.1.7 of IS 456)

The transverse reinforcement shall consist of rectangular close stirrups placed perpendicular to the axis of the member. The spacing of stirrups shall not be more than the least of *x1*, (*x1* + *y1*)/4 and 300 mm, where *x1* and *y1* are the short and long dimensions of the stirrups (Fig.6.16.2).

Longitudinal reinforcements should be placed as close as possible to the corners of the cross-section.

(h) Reinforcement in flanges of T- and L-beams (cl. 26.5.1.8 of IS 456)

For flanges in tension, a part of the main tensile reinforcement shall be distributed over the effective flange width or a width equal to one-tenth of the span, whichever is smaller. For effective flange width greater than one-tenth of the span, nominal longitudinal reinforcement shall be provided to the outer portion of the flange.

DEFLECTION

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SHORT AND LONG TERM DEFLECTIONS

As evident from the names, short-term deflection refers to the immediate deflection after casting and application of partial or full service loads, while the long-term deflection occurs over a long period of time largely due to shrinkage and creep of the materials. The following factors influence the short-term deflection of structures:

- magnitude and distribution of live loads,
- span and type of end supports,
- cross-sectional area of the members,
- amount of steel reinforcement and the stress developed in the reinforcement,
- characteristic strengths of concrete and steel, and
- amount and extent of cracking.

The long-term deflection is almost two to three times of the short-term deflection. The following are the major factors influencing the long-term deflection of the structures.

- humidity and temperature ranges during curing,
- age of concrete at the time of loading, and
- type and size of aggregates, water-cement ratio, amount of compression reinforcement, size of members etc., which influence the creep and shrinkage of concrete.

CONTROL OF DEFLECTION

Clause 23.2 of IS 456 stipulates the limiting deflections under two heads as given below:

The maximum final deflection should not normally exceed span/250 due to all loads including the effects of temperatures, creep and shrinkage and measured from the as-cast level of the supports of floors, roof and all other horizontal members.

The maximum deflection should not normally exceed the lesser of span/350 or 20 mm including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes. It is essential that both the requirements are to be fulfilled for every structure.

SELECTION OF PRELIMINARY DIMENSIONS

The two requirements of the deflection are checked after designing the members. However, the structural design has to be revised if it fails to satisfy any one of the two or both the requirements. In order to avoid this, IS 456 recommends the guidelines to assume the initial dimensions of the members which will generally satisfy the deflection limits. Clause 23.2.1 stipulates different span to effective depth ratios and cl. 23.3 recommends limiting slenderness of beams, a relation of b and d of the members, to ensure lateral stability. They are given below:

(A) For the deflection requirements

Different basic values of span to effective depth ratios for three different support conditions are prescribed for spans up to 10 m, which should be modified under any or all of the four different situations: (i) for spans above 10 m, (ii) depending on the amount and the stress of tension steel reinforcement, (iii) depending on the amount of compression reinforcement, and (iv) for flanged beams. These are furnished in below table

(B) For lateral stability

The lateral stability of beams depends upon the slenderness ratio and the support conditions. Accordingly cl. 23.3 of IS code stipulates the following:

4.8.6 For simply supported and continuous beams, the clear distance between the lateral restraints shall not exceed the lesser of $60b$ or $250b2/d$, where d is the effective depth and b is the breadth of the compression face midway between the lateral restraints.

4.8.7 For cantilever beams, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed the lesser of $25b$ or $100b2/d$.

Table: Span/depth ratios and modification factors

Sl. No	**Items**	**Cantilever**	**Simply supported**	**Continuous**
1	effective depth ratio for spans up to 10m	7	20	26
2	Modification factors for spans > 10 m	Not applicable as deflection calculations are to be done	Multiply values of row 1 by 10/span in meters	
3	Modification factors depending on	Multiply values of row 1 with the modification factor from Fig.4 of IS 456.		

	area and stress of steel	
4	Modification factors depending as area of compression steel	Further multiply the earlier respective value with that obtained from Fig.5 of IS 456.
5	Modification factors for flanged beams	(i)Modify values of row 1 or 2 as per Fig.6 of IS 456. (ii)Further modify as per row 3 and/or 4 where reinforcement percentage to be used on area of section equal to *bf d*.

CALCULATION OF SHORT-TERM DEFLECTION

Clause C-2 of Annex C of IS 456 prescribes the steps of calculating the short-term deflection. The code recommends the usual methods for elastic deflections using the short-term modulus of elasticity of concrete *Ec* and effective moment of inertia *Ieff* given by the following equation:

$$I_{eff} = \frac{I_r}{1.2 - (M_r/M)(z/d)(1 - x/d)(b_w/b)} \text{;but} I_r \leq I_{eff} \leq I_{gr}$$

Where Ir =moment of inertia of the cracked section

Mr = cracking moment equal to (*fcr Igr*)/ *yt* , where *fcr* is the modulus of rupture of concrete, *Igr* is the moment of inertia of the gross section about the centroidal axis neglecting the reinforcement, and *yt* is the distance from centroidal axis of gross section, neglecting the reinforcement, to extreme fibre in tension,

M = maximum moment under service loads,

z = lever arm,

x = depth of neutral axis,

d = effective depth,

b_w = breadth of web, and

b = breadth of compression face.

For continuous beams, however, the values of I_r, I_{gr} and M_r are to be modified by the following equation:

Where X_e = modified value of X,

X_1, X_2 = values of X at the supports

X_o = value of X at mid span,

k_1 = coefficient given in Table 25 of IS 456 and in Table 7.2 here, and

X = value of Ir,IgrorMras appropriate

Values of coefficient k_1

$k1$	0.5 or less	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
$k2$	0	0.03	0.08	0.16	0.30	0.50	0.73	0.91	0.97	1.0

Note: k_2 is given by $(M_1 + M_2)/(M_{F1} + M_{F2})$, where M_1 and M_2 = support moments, and M_{F1} and M_{F2} = fixed end moments

DEFLECTION DUE TO SHRINKAGE

Clause C-3 of Annex C of IS 456 prescribes the method of calculating the deflection due to shrinkage *a cs* from the following equation:

where k_3 is a constant which is 0.5 for cantilevers, 0.125 for simply supported members, 0.086 for members continuous at one end, and 0.063 for fully continuous members ;ψ_{cs} is shrinkage curvature equal to k4 ε_{cs} /D where ε_{cs} is the ultimate shrinkage strain of concrete. For ε_{cs}, cl. 6.2.4.1 of IS 456 recommends an approximate value of 0.0003 in the absence of test data

$$
\begin{aligned}
k_4 &= 0.72(p_t - p_c)/\sqrt{p_t} \leq 1.0, \text{ for } 0.25 \leq p_t - p_c < 1.0 \\
&= 0.65(p_t - p_c)/\sqrt{p_t} \leq 1.0, \text{ for } p_t - p_c \geq 1.0
\end{aligned}
$$

Where p_t =100Ast/bd and pc=100A_{sc}/bd, D is the total depth of the section, and l is the length of span.

DEFLECTION DUE TO CREEP

Clause C-4 of Annex C of IS 456 stipulates the following method of calculating deflection due to creep. The creep deflection due to permanent loads α cc(perm) is obtained from the following equation:

Where α1 cc(perm) =initial plus creep deflection due to permanent loads obtained using an elastic analysis with an effective modulus of elasticity,

$E_{ce} = E_c / (1 + \theta)$, θ being the creep coefficient, and

α1 (perm) = short-term deflection due to permanent loads using Ec.

Design & Drawing of R.C Structures

UNIT – IV: Design of Beams (using Limit State Method)

Objective:

- To design and detailing of singly reinforced and doubly reinforced rectangular and flanged beams

Syllabus:

Design of singly reinforced, doubly reinforced rectangular and flanged beams; with different end condition (simply supported, cantilever and continuous beams) and also shear and deflection checks- Examples with reinforcement detailing.

Learning Outcomes:

At the end of this lesson, the student should be able to

- design singly reinforced rectangular and flanged beams
- design doubly reinforced rectangular and flanged beams

Learning Material

Design Type of Problems

The designer has to make preliminary plan lay out including location of the beam, its span and spacing, estimate the imposed and other loads from the given functional requirement of the structure. The dead loads of the beam are estimated assuming the dimensions b and d initially. The bending moment, shear force and axial thrust are determined after estimating the different loads. In this illustrative problem, let us assume that the imposed and other loads are given. Therefore, the problem is such that the designer has to start with some initial dimensions and subsequently revise them, if needed. The following guidelines are helpful to assume the design parameters initially.

(i) Selection of breadth of the beam *b*

Normally, the breadth of the beam b is governed by: (i) proper housing of reinforcing bars and (ii) architectural considerations. It is desirable that the width of the beam should be less than or equal to the width of its supporting structure like column width, or width of the wall etc. Practical aspects should also be kept in mind. It has been found that most of the requirements are satisfied with b as 150, 200, 230, 250 and 300 mm. Again, width to overall depth ratio is normally kept between 0.5 and 0.67.

(ii) Selection of depths of the beam *d* and *D*

The effective depth has the major role to play in satisfying (i) the strength requirements of bending moment and shear force, and (ii) deflection of the beam. The initial effective depth of the beam, however, is assumed to satisfy the deflection requirement depending on the span and type of the reinforcement. IS 456 stipulates the basic ratios of span to effective depth of beams for span up to 10 m as (Clause 23.2.1)

Cantilever 7
Simply supported 20
Continuous 26

For spans above 10 m, the above values may be multiplied with 10/span in meters, except for cantilevers where the deflection calculations should be made. Further, these ratios are to be multiplied with the modification factor depending on reinforcement percentage and type. Figures 4 and 5 of IS 456 give the different values of modification factors. The total depth D can be determined by adding 40 to 80 mm to the effective depth.

(iii) Selection of the amount of steel reinforcement A_{st}

The amount of steel reinforcement should provide the required tensile force T to resist the factored moment M_u of the beam. Further, it should satisfy the minimum and maximum percentages of reinforcement requirements also. The minimum reinforcement A_{st} is provided for creep, shrinkage, thermal and other environmental requirements irrespective of the strength requirement. The minimum reinforcement A_{st} to be provided in a beam depends on the f_y of steel and it follows the relation: (cl. 26.5.1.1a of IS 456)

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

The maximum tension reinforcement should not exceed 0.04 bD (cl. 26.5.1.1b of IS 456), where D is the total depth.

Besides satisfying the minimum and maximum reinforcement, the amount of reinforcement of the singly reinforced beam should normally be 75 to 80% of $p_{t,min}$. This will ensure that

$0.87 f$

strain in steel will be more than ($\frac{y}{E_s} + 0.002$) as the design stress in steel will be $0.87 f_y$.

Moreover, in many cases, the depth required for deflection becomes more than the limiting depth required to resist $M_{u.lim}$. Thus, it is almost obligatory to provide more depth. Providing more depth also helps in the amount of the steel which is less than that required for $M_{u,lim}$. This helps to ensure ductile failure. Such beams are designated as under-reinforced beams.

(iv) Selection of diameters of bar of tension reinforcement

Reinforcement bars are available in different diameters such as 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 36 and 40 mm. Some of these bars are less available. The selection of the diameter of bars depends on its availability, minimum stiffness to resist while persons walk over them during construction, bond requirement etc. Normally, the diameters of main tensile bars are chosen from 12, 16, 20, 22, 25 and 32 mm.

(v) Selection of grade of concrete

Besides strength and deflection, durability is a major factor to decide on the grade of concrete. Table 5 of IS 456 recommends M 20 as the minimum grade under mild environmental exposure and other grades of concrete under different environmental exposures also.

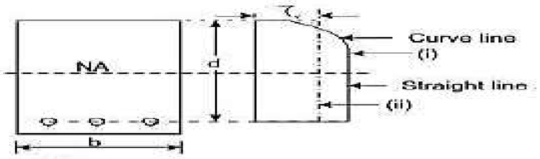
(vi) Selection of grade of steel

Normally, Fe 250, 415 and 500 are in used in reinforced concrete work. Mild steel (Fe 250) is more ductile and is preferred for structures in earthquake zones or where there are possibilities of vibration, impact, blast etc.

Shear Stress

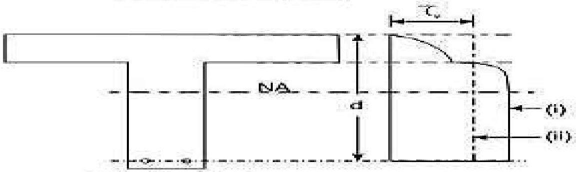
The distribution of shear stress in reinforced concrete rectangular, *T* and *L*-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τ_v is considered which is calculated as follows (IS 456, cls.40.1 and 40.1.1):

(i) In beams of uniform depth (Figs.):



Note:
(i) Actual distribution
(ii) Average distribution

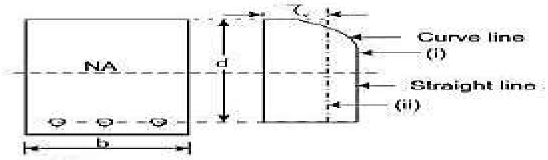
(a) Rectangular beam



(b) T-beam

Note:
(i) Actual distribution
(ii) Average distribution

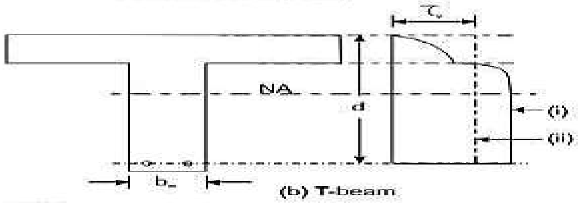
$$\tau_v = \frac{V_u}{bd}$$



Note:

(i) Actual distribution
(ii) Average distribution

(a) Rectangular beam



(b) T-beam

Note:

(i) Actual distribution
(ii) Average distribution

Figure 1: Distribution of shear stress and average shear stress

(ii) In beams of varying depth:

$$\tau_v = \frac{V_u \pm \frac{M_u}{d} \tan \beta}{bd}$$

where η_v, Vu, b or b_w and d are the same as in (i),

M_u = bending moment at the section, and

β = angle between the top and the bottom edges.

The positive sign is applicable when the bending moment M_u decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment M_u increases numerically in the same direction as the effective depth increases.

Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value τ_c depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c given in Table 3.

Design shear strength of concrete, $\tau_{c,max}$

100A_s / *bd*	**Grade of**				
	M 20	**M 25**	**M 30**	**M 35**	**M40 and above**
≤ 0.15	0.28	0.29	0.29	0.29	0.30
0.25	0.36	0.36	0.37	0.37	0.38
0.50	0.48	0.49	0.50	0.50	0.51
0.75	0.56	0.57	0.59	0.59	0.60
1.00	0.62	0.64	0.66	0.67	0.68
1.25	0.67	0.70	0.71	0.73	0.74
1.50	0.72	0.74	0.76	0.78	0.79
1.75	0.75	0.78	0.80	0.82	0.84
2.00	0.79	0.82	0.84	0.86	0.88
2.25	0.81	0.85	0.88	0.90	0.92
2.50	0.82	0.88	0.91	0.93	0.95
2.75	0.82	0.90	0.94	0.96	0.98
≥ 3.00	0.82	0.92	0.96	0.99	1.01

In Table, A_{sv} is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section considered except at support where the full area of tension reinforcement may be used provided the detailing is as per IS 456, cls. 26.2.2 and 26.2.3.

Maximum shear stress τ_{cmax} with shear reinforcement (cls. 40.2.3, 40.5.1 and 41.3.1)

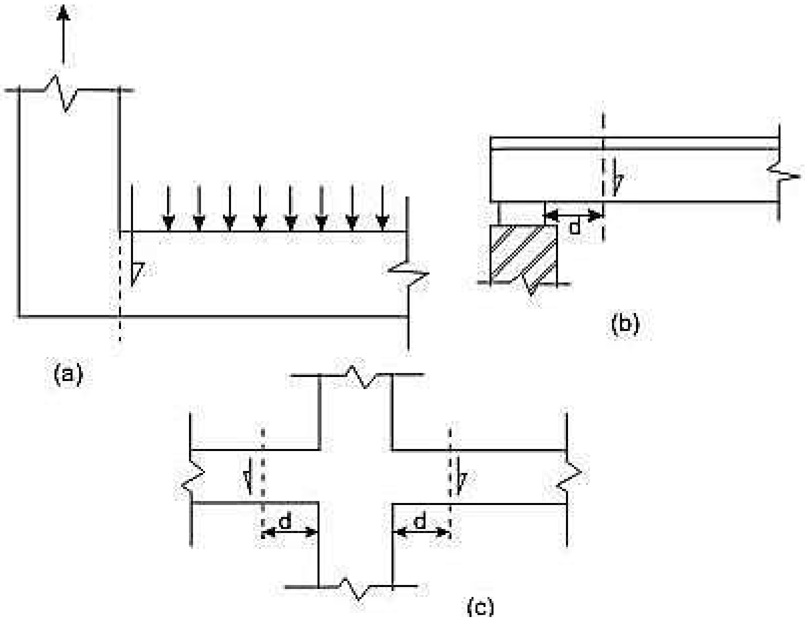
Table 20 of IS 456 stipulates the maximum shear stress of reinforced concrete in beams τ_{cmax} as given below in Table 6.2. Under no circumstances, the nominal shear stress in beams τ_v shall exceed τ_{cmax} given in Table 6.2 for different grades of concrete

Grade of	**M 20**	**M 25**	**M 30**	**M 35**	**M 40 and above**
$\tau_{c,\max}$ N/mm^2	2.8	3.1	3.5	3.7	4.0

Maximum shear stress, τ_{cmax} in N/mm^2

Critical Section for Shear

Figure 2.Support condition for locating factored shear force



Clauses 22.6.2 and 22.6.2.1 stipulate the critical section for shear and are as follows:

For beams generally subjected to uniformly distributed loads or where the principal load is located further than 2d from the face of the support, where d is the effective depth of the beam, the critical sections depend on the conditions of supports as shown in Figs. 2 are mentioned below.

(i) When the reaction in the direction of the applied shear introduces tension (Fig. 2a) into the end region of the member, the shear force is to be computed at the face of the support of the member at that section.

(ii) When the reaction in the direction of the applied shear introduces compression into the end region of the member (Figs. 2b and c), the shear force computed at a distance d from the face of the support is to be used for the design of sections located at a distance less than d from the face of the support. The enhanced shear strength of sections close to supports, however, may be considered as discussed in the following section.

Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

he minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{f_y}$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam b

and

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 taken greater than 415 N/mm^2 .

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value.

The minimum shear reinforcement is provided for the following:

Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.

Brittle shear failure is arrested which would have occurred without shear reinforcement.

Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams. To hold the reinforcement in place when concrete is poured. Section becomes effective with the tie effect of the compression steel.

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than 0.75 *d* for vertical stirrups and *d* for inclined stirrups at 45^0 , where *d* is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

Design of Shear Reinforcement (cl. 40.4 of IS 456)

When τ_v is more than τ_c given in Table, shear reinforcement shall be provided in any of the three following forms:

(a) Vertical stirrups,

(b) Bent-up bars along with stirrups, and

(c) Inclined stirrups.

In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty per cent of that of the total shear reinforcement. The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to

$$V_{us} = V_u - \tau_c b d$$

where b is the breadth of rectangular beams.

The strengths of shear reinforcement V_{us} for the three types of shear reinforcement are as follows:

(a) Vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$0.87 f A d$$

$$V_{us} = {}^{y\,sv} S_v (\sin\alpha + \cos\alpha)$$

(c) For single bar or single group of parallel bars, all bent-up at the same cross-section:

$V_{us} = 0.87 f_y A_{sv} d$

$\sin\alpha$

Doubly Reinforced Beam

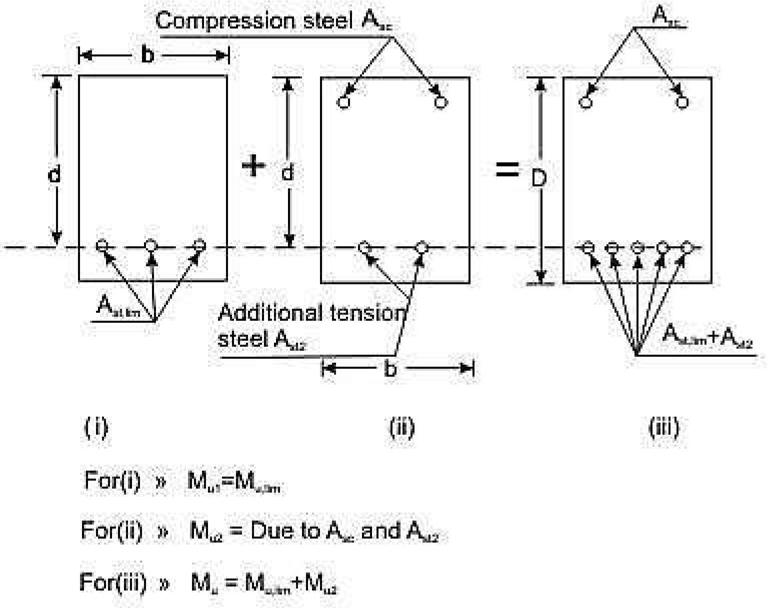


Figure 3 Doubly reinforced beam

Concrete has very good compressive strength and almost negligible tensile strength. Hence, steel reinforcement is used on the tensile side of concrete. Thus, singly reinforced beams reinforced on the tensile face are good both in compression and tension. However, these beams have their respective limiting moments of resistance with specified width, depth and grades of concrete and steel. The amount of steel reinforcement needed is known as $A_{st,lim}$.

Problem will arise, therefore, if such a section is subjected to bending moment greater than its limiting moment of resistance as a singly reinforced section.

There are two ways to solve the problem. First, we may increase the depth of the beam, which may not be feasible in many situations. In those cases, it is possible to increase both the compressive and tensile forces of the beam by providing steel reinforcement in compression face and additional reinforcement in tension face of the beam without increasing the depth (Fig. 3). The total compressive force of such beams comprises (i) force due to concrete in compression and (ii) force due to steel in compression. The tensile force also has two components: (i) the first provided by $A_{st,lim}$ which is equal to the compressive force of concrete in compression. The second part is due to the additional steel in tension - its force will be equal to the compressive force of steel in compression. Such reinforced concrete beams having steel reinforcement both on tensile and compressive faces are known as doubly reinforced beams.

Doubly reinforced beams, therefore, have moment of resistance more than the singly reinforced beams of the same depth for particular grades of steel and concrete. In many practical situations, architectural or functional requirements may restrict the overall depth of the beams. However, other than in doubly reinforced beams compression steel reinforcement is provided when:

(i) Some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone or vice versa.

(ii) The ductility requirement has to be followed.

(iii) The reduction of long term deflection is needed.

Basic Principle

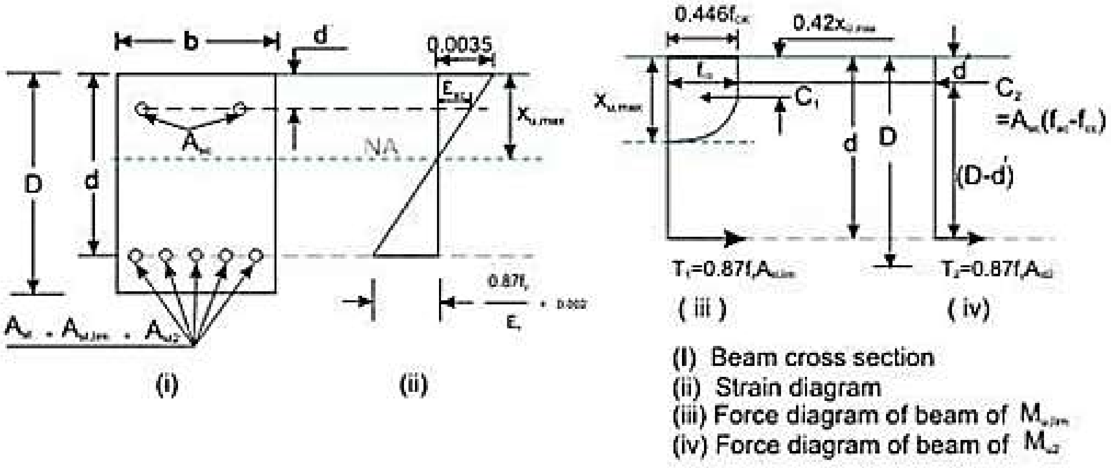


Figure 4 Stress, strain and force diagrams of doubly reinforced beam

The moment of resistance M_u of the doubly reinforced beam consists of (i) $M_{u,lim}$ of singly reinforced beam and (ii) M_{u2} because of equal and opposite compression and tension forces (C_2 and T_2) due to additional steel reinforcement on compression and tension faces of the beam (Figs. 2.6 and 7). Thus, the moment of resistance M_u of a doubly reinforced beam is

$M_u = M_{u,lim} + M_{u2}$

$x_{u,\max} \Big(\qquad x_{u,\max} \Big) \qquad 2$

$$M_{u,\mathrm{lim}} = 0.36 \quad d \quad \Big| 1 - 0.42 \quad d \quad \Big| f_{ck} b d$$

$$\mathrm{a}$$

$$\left(\qquad \qquad \right)$$

Also, $M_{u\,lim}$, can be written

$$M_{u,\lim} = 0.87 A_{st,\lim} f_y \left(d - 0.416 x_{u,\max}\right)$$

The additional moment M_{u2} can be expressed in two ways (Fig. 2.7): considering (i) the compressive force C_2 due to compression steel and (ii) the tensile force T_2 due to additional steel on tension face. In both the equations, the lever arm is $(d - d')$. Thus, we have

$$M_u = A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$M_u = A_{st}(0.87 f_y)(d - d')$$

where A_{sc} = area of compression steel reinforcement

f_{sc} = stress in compression steel reinforcement

f_{cc} = compressive stress in concrete at the level of centroid of compression steel reinforcement

A_{st2} = area of additional steel reinforcement

Since the additional compressive force C_2 is equal to the additional tensile force T_2, we have

$$A_{sc}(f_{sc} - f_{cc}) = A_{st2}(0.87 f_y)$$

Any two of the three equations (Eqs. 6 - 8) can be employed to determine A_{sc} and A_{st2}.

The total tensile reinforcement A_{st} is then obtained from:

$$Ast = Ast1 + Ast\,2$$

$$A_{st_1} = p_{t,\lim} \; bd \; = \qquad M_{u,\lim}$$

$100 \quad 0.87 f_y (d - 0.42 x_{u,\max})$

Determination of f_{sc} and f_{cc}

It is seen that the values of f_{sc} and f_{cc} should be known before calculating A_{sc}. The following procedure may be followed to determine the value of f_{sc} and f_{cc} for the design type of problems (and not for analyzing a given section). For the design problem the depth of the neutral axis may be taken as $x_{u,max}$ as shown in Fig. 2.7. From Fig. 2.7, the strain at the level of compression steel reinforcement ε_{sc} may be written as

$$\left(\quad d' \right)$$

$\varepsilon_{sc} =$
$0.0035 \left|\right| 1 -$

$$\Big(\qquad x_{u,\max} \Big)$$

f_{sc} for Cold worked bars Fe 415 and Fe 500

Table Values of f_{sc} and ε_{sc}

Stress level	Fe 415		Fe 500	
	Strain ε_{sc}	Stress f_{sc} (N/mm^2)	Strain ε_{sc}	Stress f_{sc} (N/mm^2)
$0.80 f_{Yd}$	0.00144	288.7	0.00174	347.8
$0.85 f_{Yd}$	0.00163	306.7	0.00195	369.6
$0.90 f_{Yd}$	0.00192	324.8	0.00226	391.3
$0.95 f_{yd}$	0.00241	342.8	0.00277	413.0
$0.975 f_{yd}$	0.00276	351.8	0.00312	423.9
$1.0 f_{yd}$	0.00380	360.9	0.00417	434.8

Design type of problems

In the design type of problems, the given data are b, d, D, grades of concrete and steel. The designer has to determine A_{sc} and A_{st} of the beam from the given factored moment.

Step 1: To determine $M_{u, lim}$ and $A_{st, lim}$

Step 2: To determine M_{u2}, A_{sc}, A_{st2} and A_{st}.

Step 3: To select the number and diameter of bars from known values of A_{sc} and A_{st}.

Analysis type of problems

In the analysis type of problems, the data given are b, d, d', D, f , f, A and A . It is required *ck y sc st*

to determine the moment of resistance M_u of such beams.

Step 1: To check if the beam is under-reinforced or over-reinforced.

First, $x_{u,max}$ is determined assuming it has reached limiting stage using $\frac{x_{u,\max}}{d}$ coefficients as given in cl. 38.1, Note of IS 456. The strain of tensile steel ε_{st} is computed from $\varepsilon_{st} = \frac{\varepsilon_c\,(d - x_{u.\max})}{x_{u,\max}}$ and is checked if ε_{st} has reached the yield strain of steel:

$$\varepsilon_{st\ at\ yield} = \frac{f_y}{E} + 0.002$$

The beam is under-reinforced or over-reinforced if ε_{st} is less than or more than the yield strain.

Step 2: To determine M_{ulim} and $A_{st\ lim}$ from the $p_{t,lim}$

Step 3: To determine A_{st} *and* A_{sc}

Step 4: To determine M_{u2} and M_u .

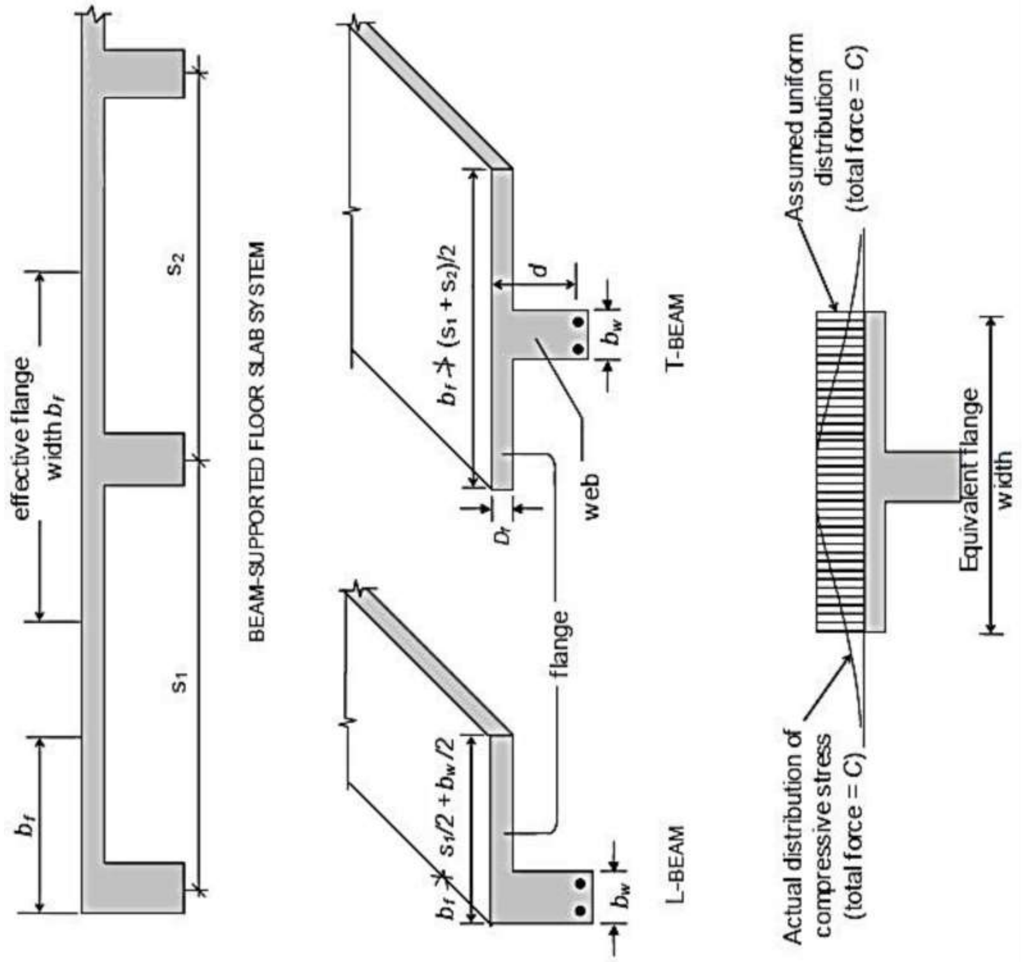
T−beams and L−beams

Beams having effectively T-sections and L-sections (called T-beams and L-beams) are commonly encountered in beam-supported slab floor systems.In such situations, a portion of the slab acts integrally with the beam and bends in the longitudinal direction of the beam. This slab portion is called the flange of the T- or L-beam. The beam portion below the flange is often termed the web, although, technically, the web is the full rectangular portion of the beam other than the overhanging parts of the flange. Indeed, in shear calculations, the web is interpreted in this manner.

When the flange is relatively wide, the flexural compressive stress is not uniform over its width. The stress varies from a maximum in the web region to progressively lower values at points farther away from the web. In order to operate within the framework of the theory of flexure, which assumes a uniform stress distribution across the width of the section, it is necessary to define a reduced effective flange.

The effective width of flange‘ may be defined as the width of a hypothetical flange that resists in-plane compressive stresses of uniform magnitude equal to the peak stress in the original wide flange, such that the value of the resultant longitudinal compressive force is the same .

Figure 4 T-beams and L-beams in beam-supported floor slab systems



The effective flange width is found to increase with increased span, increased web width and increased flange thickness. It also depends on the type of loading (concentrated, distributed, etc.) and the support conditions (simply supported, continuous, etc.). Approximate formulae for estimating the effective width of flange‘ b_f (Cl. 23.1.2 of Code) are given as follows:

$$b_f = \begin{cases} l_0 / 6 + b_w + 6D_f & \text{for } T-\text{Beam} \\ l_0 / 12 + b_w + 3D_f & \text{for } L-\text{Beam} \end{cases}$$

where b_w is the breadth of the web, D_f is the thickness of the flange,and l_0 is the —distance between points of zero moments in the beam‖ (which may be assumed as 0.7 times the effective span in continuous beams and frames). Obviously, b_f cannot extend beyond the slab portion tributary to a beam, i.e., the actual width of slab available. Hence, the calculated b_f should be restricted to a value that does not exceed $(s_1+s_2)/2$ in the case of T−beams, and

$s/2 + b\ /2$ in the case of L−beams, where the spans s and s of the slab are as marked in Fig.

$l \qquad w \qquad\qquad\qquad\qquad\qquad\qquad 1 \qquad 2$

In some situations, *isolated* T−beams and L−beams are encountered, i.e., the slab is discontinuous at the sides, as in a footbridge or a _stringer beam‘ of a staircase. In such cases, the Code [Cl. 23.1.2(c)] recommends the use of the following formula to estimate the _effective width of flange‘ b_f:

$$\lceil \underline{\quad l_0 \quad} + b \quad \textit{for isolated } T - Beams$$

$$= \; l \;/\, b + 4$$

$$bf \quad \Big\{ 0 \qquad w$$

$| \quad 0.5l_0 \quad + b_w$ *for isolated* $L - Beam$

$| \lfloor l_0 / b + 4$

where b denotes the *actual* width of flange; evidently, the calculated value of b_f should not exceed b.

Analysis of Singly Reinforced Flanged Sections

The procedure for analysing flanged beams at ultimate loads depends on whether the neutral axis is located in the flange region or in the web region.

If the neutral axis lies within the flange (i.e., $x_u \leq D_f$), then as in the analysis at service loads all the concrete on the tension side of the neutral axis is assumed ineffective, and the T-section may be analysed as a rectangular section of width b_f and effective depth d

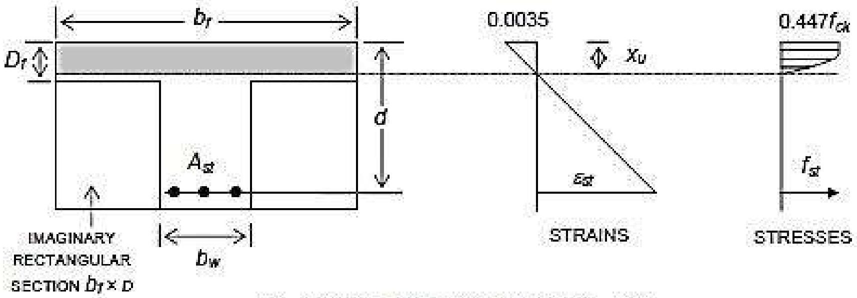
If the neutral axis lies in the web region (i.e., $x_u > D_f$), then the compressive stress is carried by the concrete in the flange and a portion of the web, as shown in. It is convenient to consider the contributions to the resultant compressive force C_u, from the *web* portion (b $\times x$) and the *flange* portion (width $b - b$) separately, and to sum up these effects.

w u f w

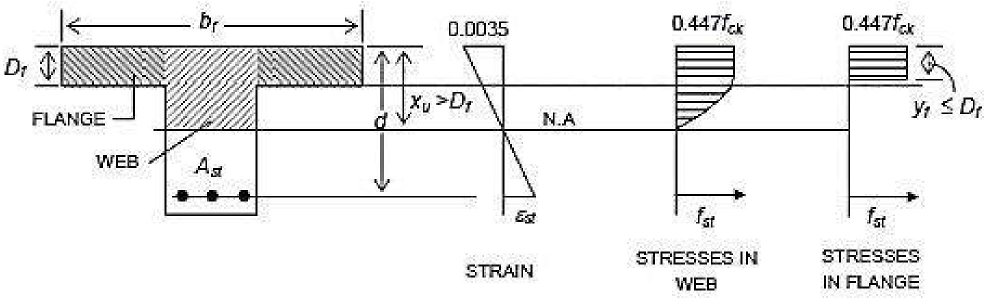
Estimating the compressive force C_{uw} in the ‗web‘ and its moment contribution M_{uw} is easy, as the full stress block is operative:

$$C_{uw} = 0.361\, f_{ck} b_w x_u$$

$$M_{uw} = C_{uw} (d - 0.416 x_u)$$



(a) neutral axis within flange $x_u \leq D_f$



(b) neutral axis outside flange $x_u > D_f$

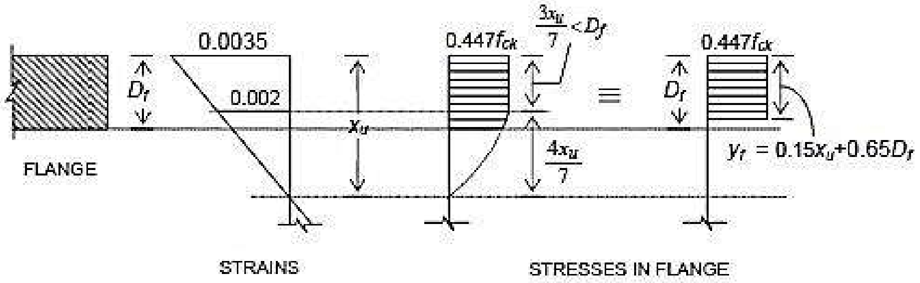


Figure 6 Behaviour of flanged beam section at ultimate limit state

However, estimating the compressive force C_{uf} in the flange is rendered difficult by the fact that the stress block for the flange portions may comprise a rectangular area plus a truncated parabolic area [Fig5].

A general expression for the total area of the stress block operative in the flange, as well as an expression for the centroidal location of the stress block, is evidently not convenient to derive for such a case. However, when the stress block over the flange depth contains only a rectangular area (having a uniform stress 0.447 f_{ck}), which occurs when $\frac{3}{7}x_u \geq D_f$, an expression for C_{uf} and its moment contribution M_{uf} can easily be formulated. For the case, $1 < x_u / D_f < 7/3$, an *equivalent* rectangular stress block (of area $0.447 f_{ck} y_f$) can be conceived, for convenience, with an equivalent depth $y_f \leq D_f$, as shown in Fig. The expression for y_f given in the Code (Cl. G – 2.2.1) is necessarily an approximation, because it cannot satisfy the *two* conditions of _equivalence', in terms of area of stress block as well as centroidal location. A general expression for y_f may be specified for any $x_u > D_f$:

$$y_f = \begin{cases} 0.15x_u + 0.65D_f \; for 1 < x_u / D_f < 7/3 \\ \_{f} \qquad \ldots \_{f} \geq 7/3 \end{cases}$$

The expressions for C_{uf} and M_{uf} are accordingly obtained as:

$$C_{uf} = 0.447 f_{ck}(b_f - b_w)y_f \;\; for\; x_u > D_f$$

$$M_{uf} = C_{uf}(d - y_f / 2)$$

The location of the neutral axis is fixed by the force equilibrium condition (with y_f expressed in terms of x_u)

$$C_{uf} + C_{uf} = f_{st} A_{st}$$

where $f_{st} = 0.87 f_y$ for $x_u \leq x_{u,max}$. Where $x_u > x_{u,max}$, the strain compatibility method has to be employed to determine x_u.

The final expression for the ultimate moment of resistance M_{uR} is obtained as:

$$M_{uR} = M_{uw} + M_{uf}$$

$$\Rightarrow M_{uR} = 0.361 f_{ck} b_w x_u (d - 0.416x_u) + 0.447 f_{ck}(b_f - b_w)y_f(d - y_f / 2)$$

Limiting Moment of Resistance

The limiting moment of resistance $M_{u,lim}$ is obtained for the condition $x_u = x_{u,max}$, where $x_{u,max}$ takes the values of 0.531*d*, 0.479*d* and 0.456*d* for Fe 250, Fe 415 and Fe 500 grades of tensile steel reinforcement. The condition $x_u/D_f \geq 7/3$ in Eq. , for the typical case of Fe 415, works out, for $x_u = x_{u,max}$, as $0.479d/D_f \geq 7/3$, i.e., *Ddf*≤0205.. The Code (Cl. G–2.2) suggests a simplified condition of $d/D_f \leq 0.2$ for *all* grades of steel — to represent the condition $x_u/D_f \geq 7/3$.

$$M_{u,lim} = 0.361 f_{ck} b_w x_{u,max} (d - 0.416 x_{u,max})$$
$$+0.447 f_{ck} (b_f - b_w) y_f (d - y_f/2) \text{ for } x_{u,max} > D_f$$

The advantage of using Eq in lieu of the more exact Eq (with $x = x_{u,max}$) is that the estimation of y_f is made somewhat simpler. Of course, for $x_{u,max} \leq D_f$ (i.e., neutral axis within the flange),

$$M_{u,\mathrm{lim}} = 0.361 f_{ck} b_f x_{u,\mathrm{max}} (d - 0.416 x_{u,\mathrm{max}}) \text{ for } x_{u,\mathrm{max}} \leq D_f$$

As mentioned earlier, when it is found by *analysis* of a given T-section that $x_u > x_{u,max}$, then the strain compatibility method has to be applied. As an approximate and conservative estimate, $M_{u,lim}$ may be taken as $M_{u,lim}$. From the point of view of *design*, $M_{u,lim}$ provides a measure of the ultimate moment capacity that can be expected from a T-section of given proportions. If the section has to be designed for a *factored moment* $M_u > M_{u,lim}$, then this calls for the provision of compression reinforcement in addition to extra tension reinforcement.

Design Procedure

In the case of a continuous flanged beam, the negative moment at the face of the support generally exceeds the maximum positive moment (at or near the midspan), and hence governs the proportioning of the beam cross-section. In such cases of negative moment, if the slab is located on top of the beam (as is usually the case), the flange is under flexural tension and hence the concrete in the flange is rendered ineffective. The beam section at the support is therefore to be designed as a rectangular section for the factored negative moment. Towards the midspan of the beam, however, the beam behaves as a proper flanged beam (with the flange under flexural compression).

The determination of the actual reinforcement in a flanged beam depends on the location of the neutral axis x_u, which, of course, should be limited to $x_{u,max}$. If M_u exceeds $M_{u,lim}$ for a singly reinforced flange section, the depth of the section should be suitably increased; otherwise, a doubly reinforced section is to be designed.

Neutral Axis within Flange ($x_u \leq D_f$):

This is, by far, the most common situation encountered in building design. Because of the very large compressive concrete area contributed by the flange in T-beam and L-beams of usual proportions, the neutral axis lies within the flange ($x_u \leq D_f$), whereby the section behaves like a rectangular section having width b_f and effective depth d.

A simple way of first checking $x_u \leq D_f$ is by verifying $M_u \leq (M_{uR})_{x_u = D_f}$ where $(M_{uR})_{x_u = D_f}$

is the limiting ultimate moment of resistance for the condition $x_u = D_f$ and is given by

$$(M_{uR})_{x_u = D_f} = 0.361\, f_{ck} b_f D_f (d - 0.416 D_f)$$

It may be noted that the above equation is meaning only if $x_{u,\max} > D_f$. In rare situations

involving very thick flanges and relatively shallow beams, $x_{u,\max}$ may be less than D_f. in such cases, $M_{u,lim}$ is obtained by substituting $x_{u,max}$ in place of D_f

Neutral Axis within Web **($x_u > D_f$):**

When $M_u > (M_{uR})_{x_u=D_f}$, it follows that $x_u > D_f$. The accurate determination of x_u can be laborious. The contributions of the compressive forces C_{uw} and C_{uf} in the _web' and _flange' may be accounted for separately as follows:

$$M_{uR} = C_{uw}(d - 0.416x_u) + C_{uf}(d - y_f\ /2)$$

$C_{uw} = 0.361 f_{ck} b_w x_u$

$C_{uf} = 0.447 f_{ck} (b_f - b_w) y_f$

and the equivalent flange thickness y_f is equal to or less than D_f depending on whether x_u exceeds $7D_f/3$ or not.

For $x_{u,max} \geq 7D_f/3$, the value of the ultimate moment of resistance $(M_{uR})_{x_u = 7D_f/3}$ corresponding to $x_u = 7D_f/3$ and $y_f = D_f$ may be first computed. If the factored moment $M_u \geq (M_{uR})_{x_u = 7D_f/3}$, it follows that $x_u > 7D_f/3$ and $y_f = D_f$. Otherwise, $D_f < x_u > 7D_f/3$ for $(M_{uR})_{x_u = D_f} < M_u < (M_{uR})_{x_u = 7D_f/3}$ and

$y_f = 0.15x_u + 0.65D_f$

Inserting the appropriate value — Df or the expression for y_f, the resulting quadratic equation (in terms of the unknown x_u) can be solved to yield the correct value of x_u. Corresponding to this value of x_u the values of C_{uw} and C_{uf} can be computed. and the required A_{st} obtained by solving the force equilibrium equation.

$T_u = 0.87 f_f A_{st} = C_{uw} + C_{uf}$

Design of Singly Reinforced Beam – Rectangular section.

Given data: Live load, span, grade of Concrete, Grade of steel

- Assume width of beam i.e support width or 300mm
- Assume depth of beam from serviceability point of view as per IS code 456-2000.Page No.37

i.e L/d = 20 x modification factor for simply supported

= 7 x modification factor for Cantilever

= 26 x modification factor for Continuous beam

- Calculate the effective span as per IS code 456-2000.Page No.34 and 35
- Calculate the design Constants X_u max/d as per IS code 456-2000.Page No.70

= 0.48 (for Fe 415 steel)

= 0.46 (for Fe 500 steel)

= 0.53 (for Fe 250 steel)

- and $R_U = 0.36\, f_{ck}\, X_u \max/d(1-0.42\, X_u \max/d)$
- Calculate Self weight of beam =25*b*D
- Calculate Design load W =1.5(LL+DL)

- Calculate Bending Moment $M=WL^2/8$ for simply supported beam

  Bending Moment $M=WL^2/2$ for Cantilever beam
- Check the effective depth required as per bending point of view d = , providing 25 mm clear cover and selecting dia. of bar and dia. of stirrup bar and fix Overall depth and Effective depth.
- Calculate the area of steel $=A_{st}=0.5\ (1-)bd$
- Check the Area of Steel with Min and Max Area of steel
- $\frac{A_{stmin}}{b\ d} = \frac{0.85}{f_y}$ and 0.04 bD for

$\therefore A_{stmin} < Ast < A_{stmax}$.

Calculate no. of bars required by assuming dia. of bar.

$$\text{No of bars} = \frac{Ast}{\frac{\pi}{4} \times \Phi^2}$$

- **Curtailment of Reinforcement**: At least one-third of positive Reinforcement for simple members and 1/4th of +ve reinforcement in continuous members shall be extended in to the support to a length = $\underline{Ld}$ (pg.44, 26.2.3.3 clause)

And calculate Theoretical curtailment point X1 from the support by equating B.M at X_1 to Two third of Max BM for remaining bars. And actual cut of point is X_1 - d or 12ϴ which ever more .

- **SHEAR REINFORCEMENT**: Critical section will be at a distance d from face of support (pg:36;Clause 22.6.2) $v_u = \frac{w_u l}{2} - w_u\left(d + \frac{d}{2}\right)$

From (pg.72: Clause 40.1) $\tau_v = \frac{v_u}{bd}$

Calculate $\frac{100Ast}{bd}$ & Calculate τ_c from table 19. Calculate τ_{cmax} (pg.73, Table 20)

Case(i) : If $\tau_v < \tau_c < \tau_{cmax}$

No shear reinforcement is required but nominal shear reinforcement should be provided according to (26.5.1.6)

$$\frac{Asv}{bs_v} \geq \frac{0.4}{0.8fy}$$

Preferably provide 2-ledge stirrups and calculate spacing S_v.

But max spacing ≤0.75 d or 300mm (pg.47; 26.5.1.5)

Case (ii): If $\tau_v > \tau_c$

We have to provide shear reinforcement according to 40.4 (pg.72).

- **CHECK FOR DEVOLOPEMENT LENGTH:**

$\underline{1.3M_1} + L_0 \geq$ Ld (pg.42; 26.2.1)

- Detailing of reinforcement

Design of Doubly Reinforced Beam – Rectangular section.

Same as in singly reinforced section
But here $M_{ud} > M_{u\ limit}$

Steel reinforcement details:

Calculation of Ast :

$0.87fy\ Ast_l\ (0 - 0.42\ xu\ max) = Mu\ limit$

$$Ast_l = \frac{0.5fck}{fy}\ \ 1 - \frac{\ }{fck\ \ bd^2}\ \ bd$$

Calculation of Ast_2 :

$(M_{uD} - M_u\ limit) = 0.8 > fy\ A_{st2}\ \ (d - d^1)$

$\therefore\ A_{st} = A_{stl} + A_{st2}$

Calculation of compression reinforcement : $A_{sc}\ \ (f_{sc} - 0.444\ fck)\ \ A_{sc}\ (d - d^1) = (M_{uD} - M_u limit)$

Trail & error methods.

$$Esc = 0.0035\ 1 - \frac{dc}{xu\ max}$$

& calculate f_{sc} from stress strain curve (pg70) & then A_{sc} value.

Remaining checks for shear reinforcement & development length is same as in singly reinforced section.

Curtailment of tensile & Compression reinforcement:

(i) Calculte L_{dT} in tension = $\frac{0.87\ fy\Phi}{4\tau\ bd}$ pg:42 26.2.1

(ii) Calculate L_{dT} compression = $\frac{0.87fy\Phi}{4(1.25\tau bd)}$

Hence the tension, compression steel cannot be curtailed less than L_{dT} & L_{dc} respectively from the centre curtail the bars (should satisfy the code conditions given in clause 26.2.3.3) pg.44

Actual cut off from the centre of span can be extended by d or 12 Φ

Singly reinforced – T beam or doubly.

Step 1: Assume suitable value of b_w

(ex: Generally b_w should be sufficient to accommodate tengile reinforce $b_w = [(5x25) + (4x25) + (2x8) + (2x25)]$

Step 2: Computation of bf: pg.36; 23.1.2

Step 3: Effective length of span (pg:34) (a) or (b)

Assume total depth (D) of beam; equal to

$\frac{L}{13}$ to $\frac{L}{15}$ simply supported

$\frac{L}{15}$ to $\frac{L}{20}$ Continuous (light loads)

$\frac{L}{12}$ to $\frac{L}{15}$ Continuous (medium loads)

$\frac{L}{10}$ to $\frac{L}{12}$ Continuous (heavy loads)

Compute load on beam & them w_u

Step 4: Compute M_{UD} & Vu

$$M_{UD} = \frac{WL^3}{8}$$

$$Vu = \frac{Wu^L}{2}$$

Step 5: Fixation of effective depth

d = 2/3
Rubw

Check if for deflection criteria $\frac{L}{d}$ = (Value) x F_L x E_T x F_b (pg.38,39)

Step 6: At this stage, bw, bf, d & Df (thickness of slab) are known (pg.96)

Case(i) : Assume : $x_u = D_f$

$$M_{u1} = 0.36 f_{ck} bf D_f (d - 0.42D_f)$$

Step 7: if $M_{u1} > M_{uD}$, $(xu \le Df)$ in this case $A_{st} = \frac{0.5fck}{fy} \quad 1 - \frac{1.4.6\ M_{uD}}{fck\ bfd^2} \quad b_f d$

Step 8: if $M_{U1} < M_{uD}$ $(xu \ge Df)$ assume; $x_u = 7/3Df$

$$M_{u2} = 0.36\ fck\ b_w\ x_u\ (\ d - 0.42x_u) + 0.446\ fck\ (b_f - b_w)Df\ (d - df/2) \text{ here } xu = 7/3\ Df$$

Step 9: If $M_{u2} < M_{uD;}$ $x_u > 7/3$ Df then compute $Asw = \frac{0.36\ f_{ck}\ b_w\ x_u}{0.87fy}$

$$A_{sf} = \frac{0.446\ f_{ck}\ (b_f - b_w)\ D_f}{0.87fy}$$

$A_{st} = A_{sw} + A_{sf}$

If $M_{u2} > M_{uD}$; $x_u < 7/3$ Df. Then design procedure will be of trail & error.

With the following steps: (pg.97)

(i) Assume $x_u < 7/3$ Df
(ii) Compute $y_f = 0.15\ x_u + 0.65\ D_f$ (sub max of D_f)
(iii) Compute $M_u = 0.36\ fck\ bw\ x_u\ (\ d - 0.42\ x_u) + 0.45\ f_{ck}\ (b_f - b_w)\ (\ d- 2_f/2)\ y_f$
(iv) If $M_u = M_{uD}$ assumed x_u is correct
If $M_{uD} > M_u$ increase x_u for next trail
If $M_{uD} < M_u$ decrease x_u for next trail
Repeat till $M_u = M_{uD}$
(v) Knowing x_u, compute $Cu = 0.36f_{ck}\ x_u b_w + 0.446\ f_{ck}\ (b_f - b_w)y_f$

$$A_{st} = \frac{C_u}{0.87fy}$$

Step 10: Check for shear & design shear reinforcement exactly in same way as done of rectangular beam.

Step 11: Check for ancharge & L_d at the supports.

DDRCS

Unit-5

DESIGN OF SLABS

Introduction

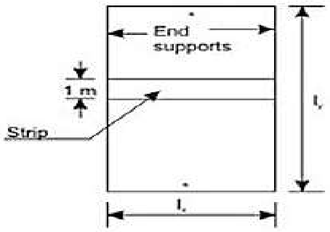


Fig. One span

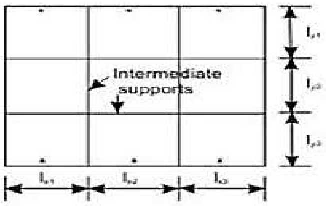


Fig. Continuous in both directions

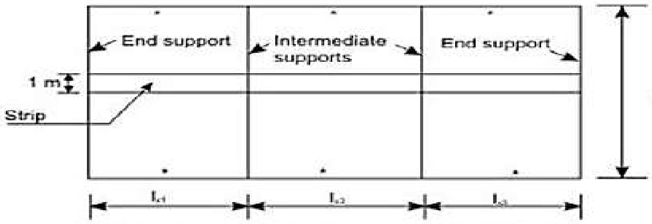


Fig. Continuous in one direction

Slabs, used in floors and roofs of buildings mostly integrated with the supporting beams, carry the distributed loads primarily by bending. The integrated slab is considered as flange of *T*- or *L*-beams because of monolithic construction. However, the remaining part of the slab needs design considerations. These slabs are either single span or continuous having different support conditions like fixed, hinged or free along the edges (Figs.a,b and c).

One-way Slabs

Figures a and b explain the share of loads on beams supporting solid slabs along four edges when vertical loads are uniformly distributed. It is evident from the figures that the share of loads on beams in two perpendicular directions depends upon the aspect ratio *ly /lx* of the slab, *lx* being the shorter span. For large values of *ly*, the triangular area is much less than the trapezoidal area (Fig.a). Hence, the share of loads on beams along shorter span will gradually reduce with increasing ratio of *ly /lx*. In such cases, it may be said that the loads are primarily taken by beams along longer span. The deflection profiles of the slab along both directions are also shown in the figure. The deflection profile is found to be constant along the longer span except near the edges for the slab panel of Fig.These slabs are designated as one-way slabs as they span in one direction (shorter one) only for a large part of the slab when *ly/lx* >2.

On the other hand, for square slabs of *ly /lx* = 1 and rectangular slabs of *ly /lx*up to 2, the deflection profiles in the two directions are parabolic (Fig.b). Thus, they are spanning in two directions and these slabs with *ly /lx* upto 2 are designated as two-wayslabs,when supported on all edges.

It would be noted that an entirely one-way slab would need lack of support on short edges.

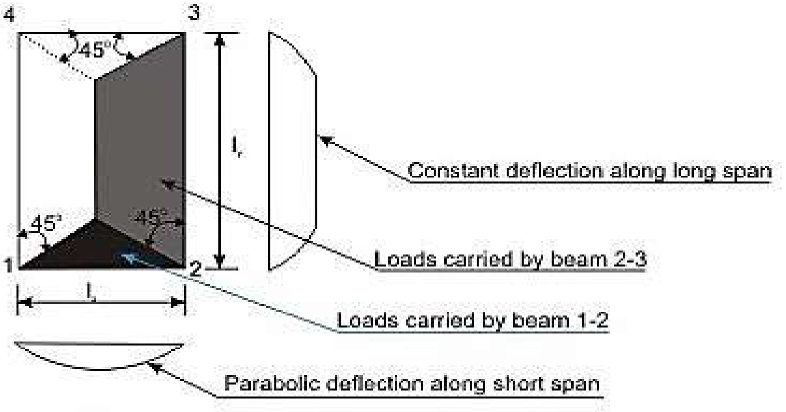


Fig. 8.18.4(a): One-way slab ($l_y/l_x > 2$) Gtg

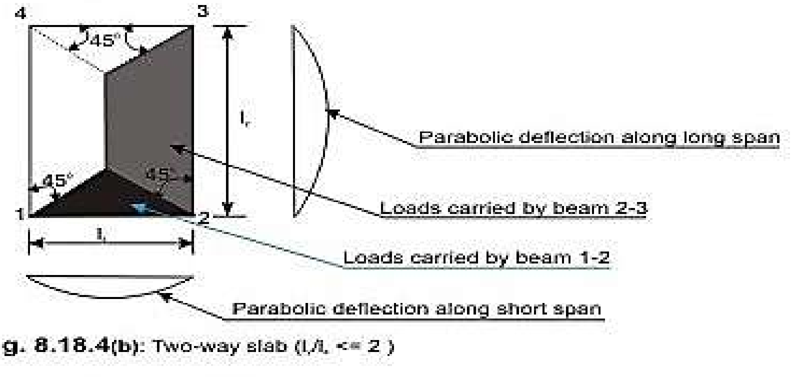


Fig. 8.18.4(b): Two-way slab ($l_y/l_x <= 2$)

Fig. 8.18.4: Sharing of loads

Also, even for *ly /lx* < 2, absence of supports in two parallel edges will render the slab one-way. In Fig. b, the separating line at 45 degree is tentative serving purpose of design.

Design Shear Strength of Concrete in Slabs

Experimental tests confirmed that the shear strength of solid slabs up to a depth of 300 mm is comparatively more than those of depth greater than 300 mm. Accordingly, cl.40.2.1.1 of IS 456 stipulates the values of a factor *k* to be multiplied with given in Table 19 of IS 456 for different overall depths of slab.

Table: presents the values of *k* as a ready reference below

Overall depth of slab (mm)	300 or more	275	250	225	200	175	150 or less
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

Thin slabs, therefore, have more shear strength than that of thicker slabs. It is the normal practice to choose the depth of the slabs so that the concrete can resist the shear without any stirrups for slab subjected to uniformly distributed loads. However, for deck slabs, culverts, bridges and fly over, shear reinforcement should be provided as the loads are heavily concentrated in those slabs. Though, the selection of depth should be made for normal floor and roof slabs to avoid stirrups, it is essential that the depth is checked for the shear for these slabs taking due consideration of enhanced shear strength as discussed above depending on the overall depth of the slabs.

Design Considerations

The primary design considerations of both one and two-way slabs are strength and deflection. The depth of the slab and areas of steel reinforcement are to be determined from these two aspects. The detailed procedure of design of one-way slab is taken up in the next section. However, the following aspects are to be decided first.

Effective span (cl.22.2 of IS456)

The effective span of a slab depends on the boundary condition. Table gives the guidelines stipulated in cl.22.2 of IS 456 to determine the effective span of a slab.

Effective span of slab (cl.22.2 of IS 456)

Sl.No.	Support condition	Effective span
1	Simply supported not built integrally with its supports	Lesser of (i) clear span + effective depth of slab, and (ii) centre to centre
2	Continuous when the width of the support is < 1/12thof clear span	Do
3	Continuous when the width of the support is > lesser of 1/12thof clear span or 600 mm (i) for end span with one end fixed and the other end continuous or for intermediatespans, (ii) for end span with one end freeand the	(i) Clear span between the supports (ii) Lesser of (a) clear span + half the effective depth of slab, and (b) clear span + half the width of the discontinuous support

	other endcontinuous, (iii) spans with roller or rocker bearings.	(iii) The distance between the centres ofbearings
4	Cantilever slab at the end of a continuous slab	Length up to the centre of support
5	Cantilever span	Length up to the face of the support + half the effective depth
6	Frames	Centre to centre distance

Effective span to effective depth ratio (cls.23.2.1a-e of IS456)

The deflection of the slab can be kept under control if the ratios of effective span to effective depth of one-way slabs are taken up from the provisions in cl.23.2.1a-e of IS 456. These stipulations are for the beams and are also applicable for one-way slabs as they are designed considering them as beam of unit width.

Nominal cover (cl.26.4 of IS456)

The nominal cover to be provided depends upon durability and fire resistance requirements. Table 16 and 16A of IS 456 provide the respective values. Appropriate value of the nominal cover is to be provided from these tables for the particular requirement of thestructure.

Minimum reinforcement (cl.26.5.2.1 of IS456)

Both for one and two-way slabs, the amount of minimum reinforcement in either direction shall not be less than 0.15 and 0.12 per cents of the total cross- sectional area for mild steel (Fe 250) and high strength deformed bars (Fe 415 and Fe 500)/weldedwire fabric, respectively.

Maximum diameter of reinforcing bars(cl.26.5.2.2)

The maximum diameter of reinforcing bars of one and two-way slabs shall not exceed one-eighth of the total depth of the slab.

Maximum distance between bars (cl.26.3.3 of IS456)

The maximum horizontal distance between parallel main reinforcing bars shall be the lesser of (i) three times the effective depth, or (ii) 300 mm. However, the same for secondary/distribution bars for temperature, shrinkage etc. shall be the lesser of (i) five times the effective depth, or (ii) 450 mm.

Design of One-waySlabs

The procedure of the design of one-way slab is the same as that of beams. However, the amounts of reinforcing bars are for one meter width of the slab as to be determined from either the governing design moments (positive or negative) or from the requirement of minimum reinforcement. The different steps of the design are explainedbelow.

Step 1: Selection of preliminary depth of slab

The depth of the slab shall be assumed from the span to effective depth ratios as given

Step 2: Design loads, bending moments and shear forces

The total factored (design) loads are to be determined adding the estimated dead load of the slab, load of the floor finish, given or assumed live loads etc. after multiplying each of them with the respective partial safety factors. Thereafter, the design positive and negative bending moments and shear forces are to be determined using the respective coefficients given in Tables 12 and 13 of IS 456.

Step 3: Determination/checking of the effective and total depths of slabs

The effective depth of the slab shall be determined employing Eq. is given below as a ready reference here,

$$Mu,lim=R,limbd2$$

where the values of R,lim for three different grades of concrete and three different grades of steel are given in Table The value of b shall be taken as one metre.

The total depth of the slab shall then be determined adding appropriate nominal cover (Table 16 and 16A of cl.26.4 of IS 456) and half of the diameter of the larger bar if the bars are of different sizes. Normally, the computed depth of the slab comes out to be much less than the assumed depth in Step 1. However, final selection of the depth shall be done after checking the depth for shear force.

Step 4: Depth of the slab for shear force

Theoretically, the depth of the slab can be checked for shear force if the design shear strength of concrete is known. Since this depends upon the percentage of tensile reinforcement, the design shear strength shall be assumed.

Step 5: Determination of areas of steel

Area of steel reinforcement along the direction of one-way slab should be determined employing Eq.

$$Mu = 0.87 fy\ Astd\ \{1-(Ast)(fy)/(fck)(bd)$$

The above equation is applicable as the slab in most of the cases is under- reinforced due to the selection of depth larger than the computed value in Step 3. The area of steel so determined should be checked whether it is at least the minimum area of steel as mentioned in cl.26.5.2.1 of IS 456

Alternatively, tables and charts of SP-16 may be used to determine the depth of the slab and the corresponding area of steel. Tables 5 to 44 of SP-16 covering a wide range of grades of concrete and Chart 90 shall be used for determining the depth and reinforcement of slabs. Tables of SP-16 take into consideration of maximum diameter of bars not exceeding one-eighth the depth of the slab. Zeros at the top right hand corner of these tables indicate the region where the percentage of reinforcement would exceed pt,lim. Similarly, zeros at the lower left and corner indicate the region where the reinforcement is less than the minimum stipulated in the code. Therefore, no separate checking is needed for the allowable maximum diameter of the bars or the computed area of steel exceedingtheminimumareaofsteelwhileusingtablesandchartsofSP-16.

The amount of steel reinforcement along the large span shall be the minimum amount of steel as per cl.26.5.2.1 of IS 456

Detailing of Reinforcement

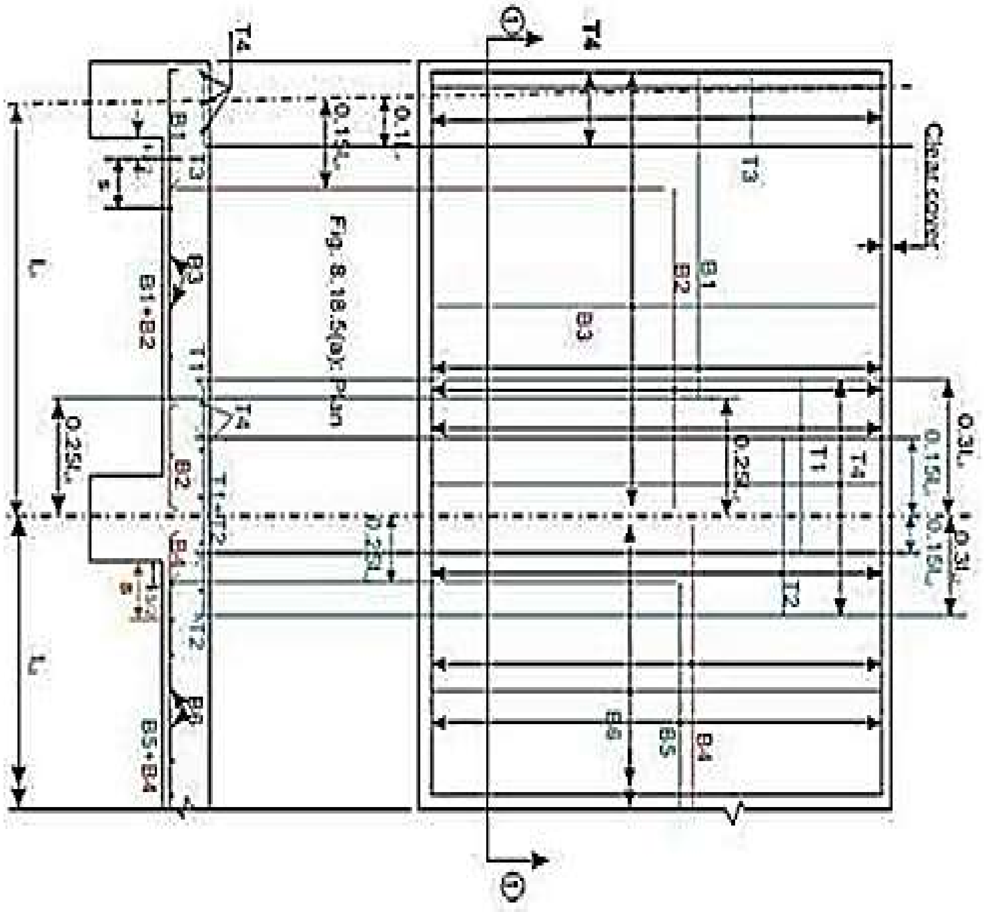


Fig. Reinforcement of one-way slab

Figures a and b present the plan and section 1-1 of one-way continuous slab showing the different reinforcing bars in the discontinuous and continuous ends (DEP and CEP, respectively) of end panel and continuous end of adjacent panel (CAP). The end panel has three bottom bars B1, B2 and B3 and four top bars T1, T2, T3 and T4. Only three bottom bars B4, B5 and B6 are shown in the adjacent panel. Table 8.3 presents these bars mentioning the respective zone of their placement (DEP/CEP/CAP), direction of the bars (along *x* or *y*) and the resisting moment for which they shall be designed or if to be provided on the basis of minimum reinforcement. These bars are explained below for the three types of ends of the twopanels.

Table - Steel bars of one-way slab (Figs a and b)

Sl.No.	Bars	Panel	Along	Resisting moment
1	B1, B2	DEP	*x*	+ 0.5 *Mx* for each,
2	B3	DEP	*y*	Minimum steel
3	B4, B5	CAP	*x*	+ 0.5 *Mx* for each,
4	B6	CAP	*y*	Minimum steel
5	T1, T2	CEP	*x*	- 0.5 *Mx* for each,
6	T3	DEP	*x*	+ 0.5 *Mx*
7	T4	DEP	*y*	Minimum steel

Notes: (i) DEP = Discontinuous End Panel
(ii) CEP = Continuous EndPanel
(iii) CAP = Continuous AdjacentPanel

Design of One Way Slab

Type of Problems : a) One way slab simply supported
b) One way slab Cantilever
c) One way slab Continuous

One way slab ($L_y/L_x > 2$) : L_y – Long span L_x – Short span

Given data: L_x, L_y, L_s = Thickness of support W_d = Dead load

W_l = Live load w_{ff} = Floor finishing f_{ck} & f_y

Step 1: Calculation of design constant:

(i) Neutral Axis depth factor $\frac{x_{u\,max}}{d}$ (pg: 70, IS 456)

= 0.48 (for Fe 415 steel)
= 0.46 (for Fe 500 steel)
= 0.53 (for Fe 250 steel)

(ii) Moment factor $R_U = 0.36 f_{ck} \frac{x_{umax}}{d} (1-0.42 \frac{x_{umax}}{d})$

$= 0.138 f_{ck}$ (for Fe 4.15 steel)

Step 2: Calculation of effective depth:

(i) Assume depth of slab from serviceability point of view as per IS code 456-2000.Page No.37

i.e L/d = 20 x modification factor for simply supported
= 7 x modification factor for Cantilever
= 26 x modification factor for Continuous beam

Modification factor is calculated from graphs (pg 38)

Note: In limit state design, effective depth required from the point of view of bending will be very Much less than the one required from deflection point of view. This will result in an Under –reinforced section. Hence while taking the modification factor (F2) from fig.4 (Pg.38). The value of P_t lim for Ms bars & 30% of P_t lim for HYSD bars.are assumed

(ii) Calculate the effective span as per IS code 456-2000.Page No.34 and 35

- Calculate Self weight of beam =25*1*D
- Calculate Design load W =1.5(LL+DL)
- Calculate Bending Moment $M=WL^2/8$ for simply supported beam

  Bending Moment $M=WL^2/2$ for Cantilever beam
- Check the effective depth required as per bending point of view d = , assume b= 1000mm providing 15 mm clear cover and selecting dia. of bar and fix Overall depth and Effective depth.

Step 3: Calculate of steel reinforcement:

Calculate $A_{st} = \dfrac{0.5\, f_{ck}}{f_y} \quad 1 -$

$\sqrt{}\ f_{ck}\, bd^2 \qquad bd$

Check for $A_{st\,min}$ = 0.12% of gross c.s.area (for HYSD bars)

= 0.15% of G.C.S.area (for MS bars)

- Chose the dia (Φ) of the bar
- Calculate the spacing $S = \dfrac{\frac{\pi \times \Phi^2}{4}}{A_{st}} \times 1000$ But S < 3d or 300mm.

Provide the above reinforcement along the short span direction (i.e. L_x) as main reinforcement.

- Bend alternate bars at 0.15l from centre of simple support & 0.25l at continues support
- Provide minimum reinforcement along long span direction as distribution steel and

  Calculate the spacing

  $\pi \times \Phi^2$

  S = 4 X1000 But S < 5d or 450mm. (Which over is less)

  A_{stmin}

Step 4: Check for shear:

Critical section will be at a distance ‘d’ from the face of support.

Calculate $V_{ud} = \dfrac{w_\mu L_x}{2} - w_u \left(d + \dfrac{d}{2} \right)$

Calculate $\tau_v = \frac{V_u}{bd}$

& τ_c from table 19 to corresponding $100 \frac{A_{st}}{bd}$ (at support) i.e 50% of p_t

check: $\tau_v < k\, t_c$; where k is calculated from 40.2.1.1

Step 5: Check for development length:

Calculate $Ld = \frac{0.87\, fy\, \Phi}{4\, \tau_{bd}}$

$\frac{1.3M_{lu}}{v_{lu}} + Lo > Ld$

Where $M_{lu} = 0.87\, f_y\, A_{st1}\, (d - 0.42x_u)$ & $Ast_1 = \frac{Ast}{2}$

$V_{lu} = \frac{w_u\, L_{ex}}{2}$

$Lo = \frac{Ls}{2} - (\text{Clear cover})$

Otherwise provide anchorages.

Step 6: Reinforcement details

Draw longitudinal section about XX and YY

Design Of One Way Continuous Slab

Step 1: Same as in one way slab. i.e

Calculation of design constant:

(i) NA depth factor $\frac{xu\,max}{D}$ = (pg: 70; IS 456)

= 0.48 (for Fe 415 steel)

= 0.46 (for Fe 500 steel)

= 0.53 (for Fe 250 steel)

(ii) Moment factor $R_U = 0.36\, fck \frac{xumax}{d} (1 - 0.42 \frac{xumax}{d})$

= 0.138 fck (for Fe 4.15 steel)

Step 2: Arrangement of spans. (Pg.35, 22.2 (b))

Step 3: Calculation of 'effective depth'

(i) Deflection criteria $\frac{L}{d} = 26$ for continuous.

$= 20$ for s.supported spans.

Chose modification factor from pg:38 & Calculate d,D

Step 4: Computation of design BM & effective depth

wd = 25 x D = _________

wff = _________

Total w_d = _________

Live load = wl

Calculate the BM values for end span

(i) Near the centre M_1

(ii) At support next it the end support M_2.

(iii) At middle M_3

(iv) At interior support M_4 from Table 12 pg.36.

Out of 4 values of moments, the effective depth will be determined for max value M_{ud}

$$d = \frac{}{Ru.b}$$

(generally this calculated d will be less than the previous one calculated i.e from deflection criteria)

Step 5: Determination of reinforcement

(a) Better fallow the following type of reinforcement.

Calculate Ast for end & intermediate spans and supports using

$$Ast = \frac{0.5\, fck}{fy} \quad 1- \quad \frac{4\text{-}6\, Mu}{fck\, bd^2}$$

$$\text{Spacing } S = \frac{\frac{\pi}{4} \times \Phi^2}{Ast} \times 1000$$

Distribution reinforcement i.e min reinfo of G.C.S area

Ast min = 0.12% for HYSD

= 0.15% of GCS area for MS.

Step 5: check for development length at end support.

(i) $L_d = 47\Phi$

$$\frac{L_d}{3} < \left(\frac{L_s}{2} - \text{clear cover}\right)$$

(ii) $\frac{1.3\,M_{lu}}{V_u} + L_o \geq$

V_u is calculated using Table 13; pg.36

$V_u = 1.5\,[0.4\,w_d + 0.45\,w_s]^L$

$M_{u1} = 0.87\,f_y\,A_{st}\,(d - 0.42\,x_u)$

Here $A_{st} = \frac{A_{st1}}{2}$

$L_o = \frac{L_s}{2} - \text{clear cover}$

Step 6: Check for shear:

Check for shear at support next to end support where SF is max (Table.13) pg.36

$v_u = 1.5\,[0.6\,w_3 + 0.6\,w_d]\,L$

$\tau_v = \frac{v_u}{bd}$; calculate $\frac{100A_s}{bd}$ & τ_c from table 19.

Here $\tau_v < k\,\tau_c$ (k is calculated from 40.2.1.1)

Step 7: Details of reinforcement (shown in step 3)

Two-way Slabs

Two-way slabs subjected mostly to uniformly distributed loads resist them primarily by bending about both the axis. However, as in the one-way slab, the depth of the two-way slabs should also be checked for the shear stresses to avoid any reinforcement for shear. Moreover, these slabs should have sufficient depth for the control deflection. Thus, strength and deflection are the requirements of design of two-way slabs.

Design of Two-way Slabs

The procedure of the design of two-way slabs will have all the six steps mentioned for the design of one-way slabs except that the bending moments and shear forces are determined by different methods for the two types of slab.

While the bending moments and shear forces are computed from the coefficients given in Tables 12 and 13 (cl. 22.5) of IS 456 for the one-way slabs, the same are obtained from Tables r the bending moment in the two types of two-way slabs and the shear forces are computed from Eq. for the two-way slabs.

Further, the restrained two-way slabs need adequate torsional reinforcing bars at the corners to prevent them from lifting. There are three types of corners having three different requirements. Accordingly, the determination of torsional reinforcement is discussed in Step 7, as all the other six steps are common for the one and two-way slabs.

Step 7: Determination of torsional reinforcement

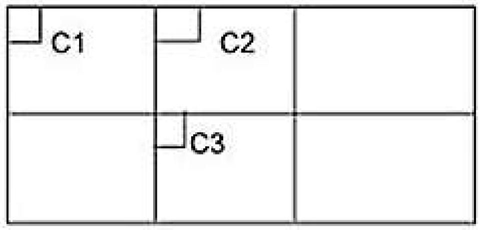


Fig. Three types of corners

Three types of corners, C1, C2 and C3, shown in Fig., have three different requirements of torsion steel as mentioned below.

(a) At corner C1 where the slab is discontinuous on both sides, the torsion reinforcement shall consist of top and bottom bars each with layers of bar placed parallel to the sides of the slab and extending a minimum distance of one- fifth of the shorter span from the edges. The amount of reinforcement in each of the four layers shall be 75 per cent of the area required for the maximum mid-span moment in the slab. This provision is given incl.D-1.8ofIS456.

(b) At corner C2 contained by edges over one of which is continuous, the torsional reinforcement shall be half of the amount of (a) above. This provision is given in cl. D-1.9 of IS456.

(c) At corner C3 contained by edges over both of which the slab is continuous, torsional reinforcing bars need not be provided, as stipulated in cl.D-1.10 of IS 456.

Detailing of Reinforcement

Step 5 explains the two methods of determining the required areas of steel required for the maximum positive and negative moments. The two methods are (i) employing Eqas given in Step 5 or (ii) using tables and charts of SP-16. Thereafter, Step 7 explains the method of determining the areas steel for corners of restrained slab depending on the type of corner. The detailing of torsional reinforcing bars is explained in Step In the following, the detailings of reinforcing bars for (i) restrained slabs and (ii) simply supported slabs are discussed separately for the bars either for the maximum positive or negative bending moments or to satisfy the requirement of minimum amount of steel.

(i) Restrained slabs

The maximum positive and negative moments per unit width of the slab calculatedare applicable only to the respective middle strips (Fig.). There shall be no redistribution of these moments. The reinforcing bars so calculated from the maximum moments are to be placed satisfying the following stipulations of IS 456.

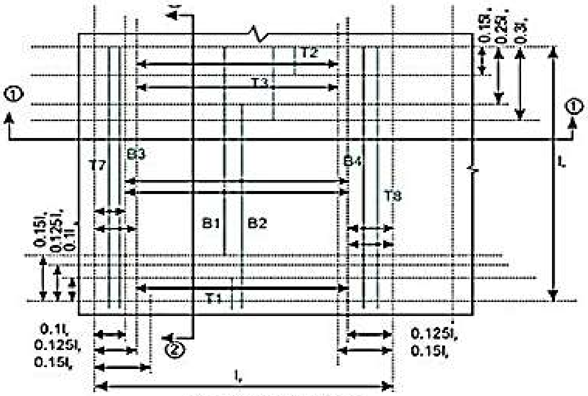


Fig.: Bars along l_x only

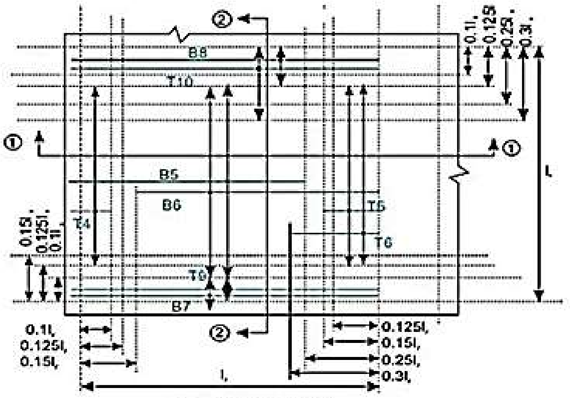
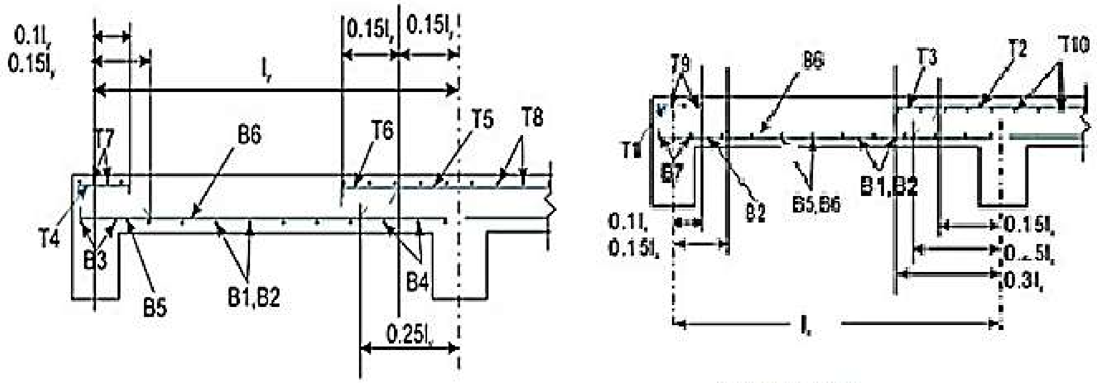


Fig. Bars along l_y only



Section 1-1

Section 2-2

Fig Reinforcement of Two-way slab, $l_x < l_y$ (except torsional reinforcement)

- Bottom tension reinforcement bars of mid-span in the middle strips hall extent in the lower part of the slab to within 0.25*l* of a continuous edge, or 0.15*l* of a discontinuous edge (cl. D-1.4 of IS 456). Bars marked as B1, B2, B5 and B6 in Figs. a and b are these bars.

- Top tension reinforcement bars over the continuous edges of middle strip shall extend in the upper part of the slab for a distance of 0.15*l* from the support, and at least fifty per cent of these bars shall extend a distance of 0.3*l* (cl. D-1.5 of IS 456). Bars marked as T2, T3, T5 and T6 in Figs. a and b are these bars.

- To resist the negative moment at a discontinuous edge depending on the degree of fixity at the edge of the slab, top tension reinforcement bars equal to fifty per cent of that provided at mid-span shall extend 0.1*l* into the span (cl. D-1.6 of IS 456). Bars marked as T1 and T4 in Figs .a and b are these bars.

- The edge strip of each panel shall have reinforcing bars parallel to that edge satisfying the requirement of minimum amount (cls. D-1.7 to D-1.10 of IS 456). The bottom and top bars of the edge strips are explained below.

- Bottom bars B3 and B4 (Fig. a) are parallel to the edge along *lx* for the edge strip for span *ly*, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS456)

- Bottom bars B7 and B8 (Fig. b) are parallel to the edge along *ly* for the edge stripfor span *lx*, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS456)

- Top bars T7 and T8 (Fig. a) are parallel to the edge along *lx* for the edge strip for span *ly*, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS456).

- Top bars T9 and T10 (Figb) are parallel to the edge along *ly* for the edge strip for span *lx*, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS456).

The above explanation reveals that there are eighteen bars altogether comprising eight bottom bars (B1 to B8) and ten top bars (T1 to T10). Tables present them separately for the bottom and top bars, respectively, mentioning the respective zone of their placement (MS/LDES/ACES/BDES to designate Middle Strip/Left Discontinuous Edge Strip/Adjacent Continuous Edge Strip/Bottom Discontinuous Edge Strip), direction of the bars (along *x* or *y*), the resisting moment for which they shall be determined or if to be provided on the basis of minimum reinforcement clause number of IS 456 and Fig. No. For easy understanding, plan views in (a) and (b) of Fig.8.19.5 show all the bars separately along *x* and *y* directions, respectively. Two sections (1-1 and 2-2), however, present the bars shown in the two plans. Torsional reinforcements are not included in Tables and Figs.a and b.

Table Details of eight bottom bars

S.No.	Bars	Into	Along	Resisting Moment	Cl.No. of IS 456	Fig.No.
1	B1, B2	MS	*x*	Max. +*Mx*	D-1.3,1.4	8.19.5a, c,d
2	B3	LDES	*x*	Min.Steel	D-1.7	8.19.5a, c
3	B4	ACES	*x*	Min.Steel	D-1.7	8.19.5a, c
4	B5, B6	MS	*y*	Max. +*My*	D-1.3,1.4	8.19.5b, c,d
5	B7	BDES	*y*	Min. Steel	D-1.7	8.19.5b, d
6	B8	ACES	*y*	Min. Steel	D-1.7	8.19.5b, d

Notes: (i) MS = Middle Strip
(ii) LDES = Left Discontinuous EdgeStrip
(iii) ACES = Adjacent Continuous EdgeStrip
(iv) BDES = Bottom Discontinuous Edge Strip

Table Details of eight top bars

S.No.	Bars	Into	Along	Resisting Moment	Cl.No. of IS	Fig.No.
1	T1	BDES	*x*	+ 0.5 *Mx*	D-1.6	8.19.5a,d
2	T2, T3	ACES	*x*	- 0.5 *Mx* for each	D-1.5	8.19.5a,d
3	T4	LDES	*y*	+ 0.5 *My*	D-1.6	8.19.5b,c
4	T5, T6	ACES	*y*	-0.5 *My* for each	D-1.5	8.19.5b,c
5	T7	LDES	*x*	Min. Steel	D-1.7	8.19.5a,c
6	T8	ACES	*x*	Min. Steel	D-1.7	8.19.5a,c
7	T9	LDES	*y*	Min. Steel	D-1.7	8.19.5b,d
8	T10	ACES	*y*	Min. Steel	D-1.7	8.19.5b,d

Notes: (i) MS = Middle Strip
(ii) LDES = Left Discontinuous EdgeStrip
(iii) ACES = Adjacent Continuous EdgeStrip
(iv) BDES = Bottom Discontinuous EdgeStrip

(ii) Simply supported slabs

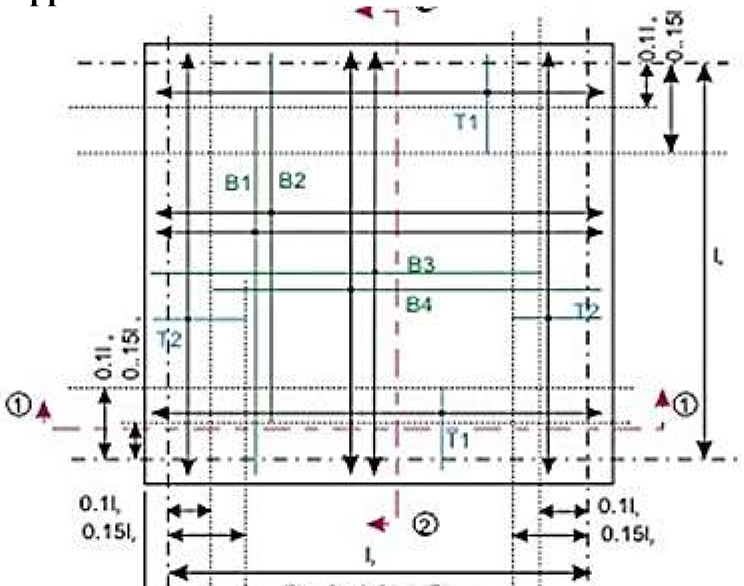


Fig. Plan

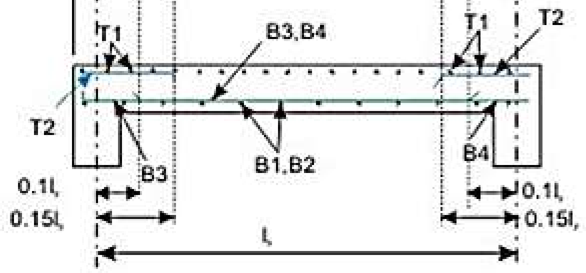


Fig. Section 1-1

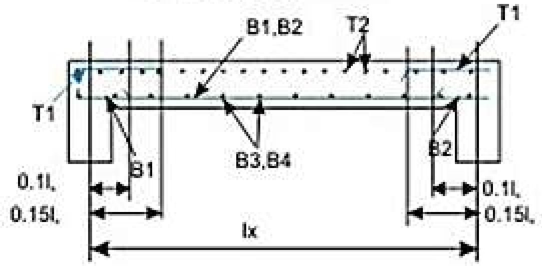


Fig. : Section 2-2

Fig. Simply supported two-way slab, corners not held down

Figures a, b and c present the detailing of reinforcing bars of simply supported slabs not having adequate provision to resist torsion at corners and to prevent corners from lifting. Clause D-2.1 stipulates that fifty per cent of the tension reinforcement provided at mid-span should extend to the supports. The remaining fifty per cent should extend to within 0.1*lx* or 0.1*ly* of the support, as appropriate.

Design Of Two Way Slab (Ly/Lx < 2)

(i) With corners held down (Restrained slabs) Pg:90

Step 1: Design constants Same as in one way slab i.e

Calculation of design constant:

(i) NA depth factor $\frac{xu\ max}{d}$ = (pg: 70; IS 456)

= 0.48 (for Fe 415 steel)
= 0.46 (for Fe 500 steel)
= 0.53 (for Fe 250 steel)

(ii) Moment factor $R_U = 0.36\ fck\ \frac{xumax}{d}\left(1-0.42\ \frac{xumax}{d}\right)$

= 0.138 fck (for Fe 4.15 steel)

Step 2: Calculation of effective depth.

(i) Deflection criteria: same as in one way slab.

(i) Assume depth of slab from serviceability point of view as per IS code 456-2000.Page No.37

i.e L/d = 20 x modification factor for simply supported
= 7 x modification factor for Cantilever
= 26 x modification factor for Continuous beam
Modification factor is calculated from graphs (pg 38)

Note: In limit state design, effective depth required from the point of view of bending will be very Much less than the one required from deflection point of view. This will result in an under –reinforced section. Hence while taking the modification factor (F2) from fig.4 (Pg.38). The value of P_t lim for Ms bars 35% & 30% of P_t lim for HYSD bars.are assumed

Calculate total design load w_u

Calculate $\frac{L_{ey}}{L_{ex}}$ & From Table 26, chose α x & αy

Calculate $Mx = \alpha_x w_u lx^2$ $My = \alpha_y w_u lx^2$

(ii) BM criteria d =

$R_u b$

Fix d, D

Step 3: Calculation of reinforcement

(i) in short span

(ii) in long span using $Ast = \frac{0.5fck}{fy} \quad 1- \frac{1-4.6\, Mu_x \text{ c or y}}{fck\, bd^2} \quad bd$

for long span $d^1 = d - \Phi$

Provide ¾ of calculated reinforcement in middle strips and min reinfo in edge strips. Calculate the spacing & bent the alternate bars as specified by code (pg.40)

Step 4: Check for shear: $r = \frac{Ly}{Lx}$

(i) In short span:

$Vu = SF = Wu\, lx \ \frac{r}{2+r}$ (long edged)

$\tau v = \frac{Vu}{bd}$

% of steel = $\frac{100\, Ast}{bd}$ & hence calculate τc

$\tau c > \tau v$

(ii) In long span:

$Vu = 1/3\ wulx$ (short edges)

remaining is same : check $\tau_c > \tau_v$

Step 5: Check for development length:

$Ld = 47\Phi$

Check $\frac{1.3\, M_{lu}}{vu} + Lo > L_d$

Step 6: Torsion reinforcement at corners: size of torsional mash = $\frac{lx}{5}$ from centre of support.

Torsional reinforcement = 3/4 Ast x.

(ii) When corners are free to lift:

Step 1: Design constant

(i) NA depth factor $\frac{xu\ max}{D}$ = (pg: 70; IS 456)

= 0.48 (for Fe 415 steel)
= 0.46 (for Fe 500 steel)
= 0.53 (for Fe 250 steel)

(ii) Moment factor $R_U = 0.36\, fck \frac{xumax}{d} (1-0.42 \frac{xumax}{d})$

= 0.138 fck (for Fe 4.15 steel)

Step 2: Computation of loading & BM
w Total load / unit run.

In long direction $w_L = \frac{w}{1+r^4}$ $r = \frac{L}{B}$
ly

$w_B = \frac{W}{1+r^4} \quad x\, r^4$
$M_L = \frac{W_L\ L^2}{8};\ M_B = \frac{W_B B^2}{8}$

Step3: Effective depth by considering max BM

d =
Ru b b = 1m

Step 4: Steel reinforce

$Ast_B = 0.5 \frac{fck}{fy} \quad 1- \frac{\_}{fck\ bd^2} \quad bd$

$Ast_L = 0.5 \frac{fck}{fy} \quad 1- \frac{\_}{fck\ bd^{12}} \quad bd^1$

$d^1 = d - \Phi$

Calculate spacing = $\frac{1000\ A\Phi}{Ast}$ and bent up alternate bars at L/7 from centre of each support.

Step 5: Check for shear
$Vub = 1/3 wu\ B$ $Vul = w_u\ B \frac{r}{2+r}$

$\tau\ vb = \frac{V_{UB}}{bd_L}$; $\tau_{UL} = \frac{V_{UL}}{bd_{B..}}$

check: $\tau v < \tau c$ (Table 4) $d_L - d_B - \Phi$

Step 6: Check for development length.

$$\frac{1.3\ M_{lu}}{Vu} + Lo > Ld$$

For the ends of short span:

$Vu = V_{UL}$

$LO = \frac{Ls}{2} - x^1$

$Ast\ 1\ B = \frac{1000 \times A\Phi}{2 \times 5_B}$

$M_{lu} = 0.87\ fy\ Ast_{1B}\ (d_B - 0.42\ xu)$

$Xu = \frac{0.87\ fy\ Ast1B}{0.36\ fck\ b}$

Step 7: Torsional reinforce at corners.

Area of Torsional reinforcement = $\frac{3}{4} \times Ast_B$

Size of mesh = $\frac{Lx}{5}$ from centre of support.

For the ends of short span:

$Vu = V_{UB}$

$Ast_{1L} = \frac{1000\ A\Phi}{2 \times 5_L}$

$M_{lu} = 0.87\ fy\ Ast_{1L}\ (d_L - 0.42\ xo)$

STAIR CASE

Types of Staircases

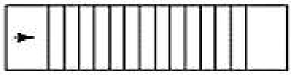


Fig. (a): Single flight staircase

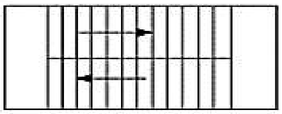


Fig. (b): Two flight staircase

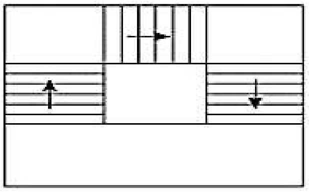


Fig. (c): Open-well staircase

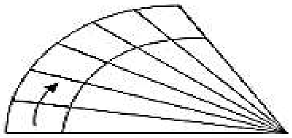


Fig. (e): Helicoidal staircase

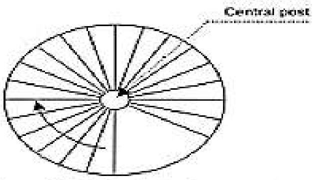


Fig. (d): Spiral staircase

Fig. Types of staircases

Figures a to e present some of the common types of staircases based on geometrical configurations:

(a) Single flight staircase (Fig..a)
(b) Two flight staircase (Fig.b)
(c) Open-well staircase (Fig.c)
(d) Spiral staircase (Fig.d)
(e) Helicoidal staircase (Fig.e)

Architectural considerations involving aesthetics, structural feasibility and functional requirements are the major aspects to select a particular type of the staircase. Other influencing parameters of the selection are lighting, ventilation, comfort, accessibility, space etc.

A Typical Flight

Figures a to d present plans and sections of a typical flight of different possibilities. The different terminologies used in the staircase are given below:

(a) Tread: The horizontal top portion of a step where foot rests (Fig.b) is known as tread. The dimension ranges from 270 mm for residential buildings and factories to 300 mm for public buildings where large number of persons use thestaircase.

(b) Nosing: In some cases the tread is projected outward to increase the space. This projection is designated as nosing

(Fig.b).

(c) Riser: The vertical distance between two successive steps is termed as riser (Fig.b). The dimension of the riser ranges from 150 mm for public buildings to 190 mm for residential buildings andfactories.

(d) Waist: The thickness of the waist-slab on which steps are made is known as waist (Fig.b). The depth (thickness) of the waist is the minimum thickness perpendicular to the soffit of the staircase (cl. 33.3 of IS 456). The stepsofthestaircaserestingonwaist-slabcanbe madeofbricksorconcrete.

(e) Going: Going is the horizontal projection between the first and the last riser of an inclined flight(Fig.a).

The flight shown in Fig.a has two landings and one going. Figures b to d present the three ways of arranging the flight as mentioned below:

(i) waist-slab type(Fig.b),
(ii) tread-riser type (Fig.c), or free-standing staircase,and
(iii) Isolated tread type(Fig.d).

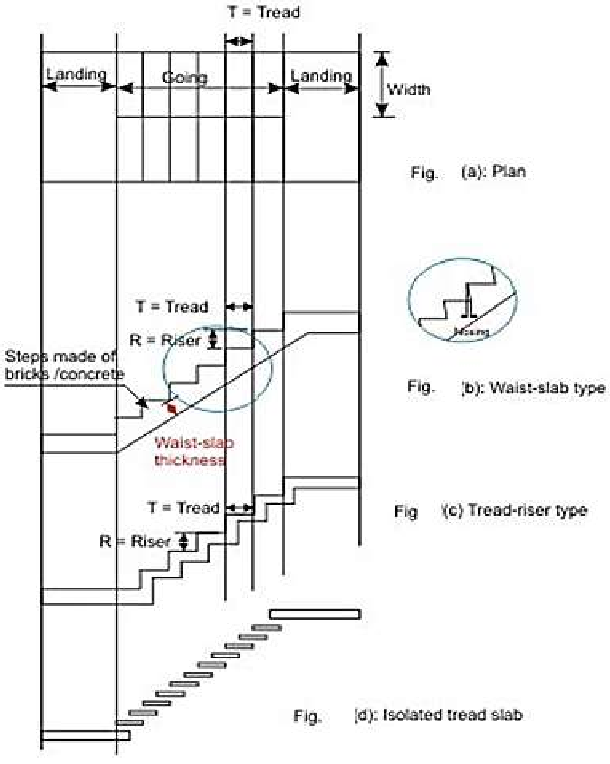


Fig. A typical flight

General Guidelines

The following are some of the general guidelines to be considered while planning a staircase:

- The respective dimensions of tread and riser for all the parallel steps should be the same in consecutive floor of a building.
- The minimum vertical headroom above any step should be 2m.
- Generally, the number of risers in a flight should be restricted to twelve.
- The minimum width of stair (Fig.) should be 850 mm, though it is desirable to

have the width between 1.1 to 1.6 m. In public building, cinema halls etc., large widths of the stair should be provided.

- Effective Span of Stairs

The stipulations of clause 33 of IS 456 are given below as a ready reference regarding the determination of effective span of stair. Three different cases are given to determine the effective span of stairs without stringer beams.

(i) The horizontal centre-to-centre distance of beams should be considered as the effective span when the slab is supported at top and bottom risers by beams spanning parallel with the risers.

(ii) The horizontal distance equal to the going of the stairs plus at each end either half the width of the landing or one meter, whichever is smaller when the stair slab is spanning on to the edge of a landing slab which spans parallel with the risers. See Table 9.1 for the effective span for this type of staircases shown inFig.

Table-Effective span of stairs shown in Fig.

Sl. No.	x	Y	Effective span in metres
1	< 1 m	< 1 m	$G + x + y$
2	< 1 m	>1 m	$G + x + 1$
3	>1 m	< 1 m	$G + y + 1$
4	>1 m	>1 m	$G + 1 + 1$

Note: G = Going, as shown in Fig.

Distribution of Loadings on Stairs

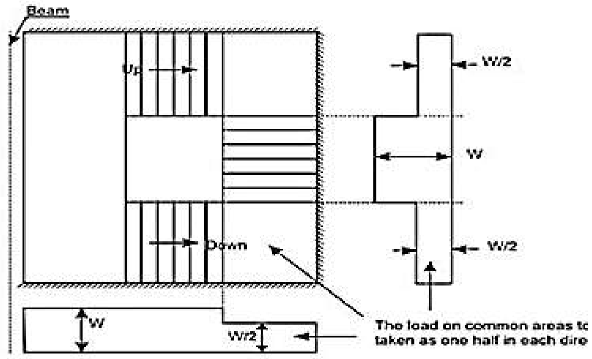


Fig. Loadings on open-well staircases

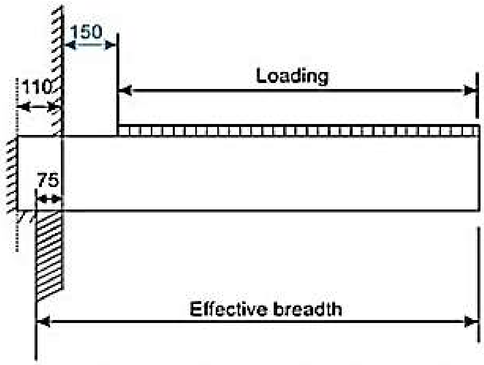


Fig. Loading on staircases built into walls

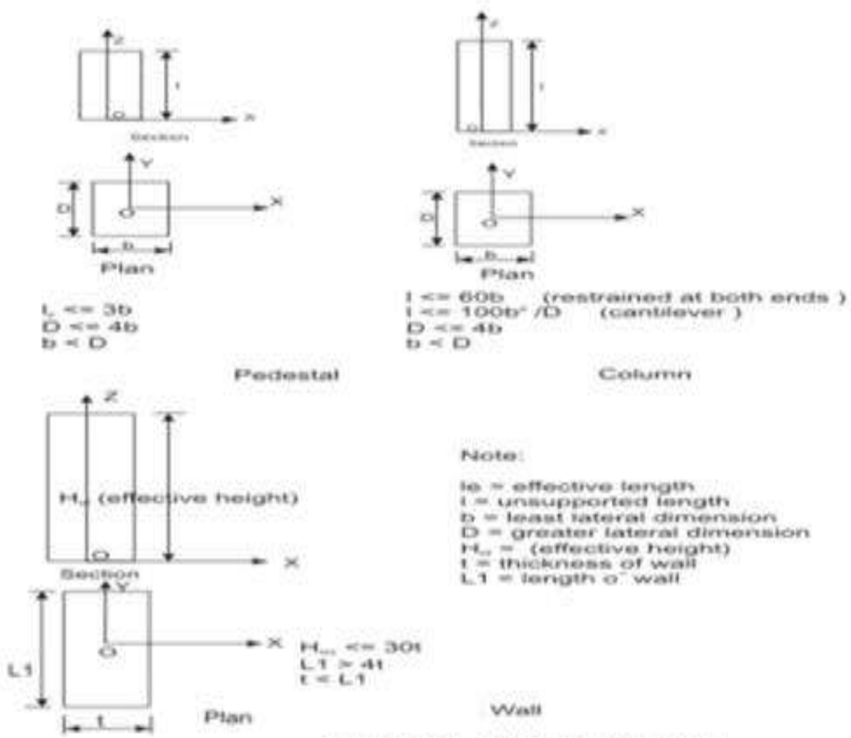
Figure shows one open-well stair where spans partly cross at right angle. The load in such stairs on areas common to any two such spans should be taken as fifty per cent in each direction as shown in Fig. Moreover, one 150 mm strip may be deducted from the loaded area and the effective breadth of the section is increased by 75 mm for the design where flights or landings are embedded into walls for a length of at least 110 mm and are designed to span in the direction of the flight (Fig).

DDRCS

UNIT-6 Compression Members

Introduction

Compression members are structural elements primarily subjected to axial compressive forces and hence, their design is guided by considerations of strength and buckling. show their examples: pedestal, column, wall and strut. While pedestal, column and wall carry the loads along its length *l* in vertical direction, the strut in truss carries loads in any direction. The letters *l*, *b* and *D* represent the unsupported vertical length, horizontal lest lateral dimension, width and the horizontal longer lateral dimension, depth. These compression members may be made of bricks or reinforced concrete. Herein, reinforced concrete compression members are only discussed.



Pedestal, column and wall

Salient Points

(a) Effective length: The vertical distance between the points of inflection of the compression member in the buckled configuration in a plane is termed as effective length l_e of that compression member in that plane. The effective length is different from the unsupported length *l* of the member, though it depends on the unsupported length and the type of end restraints. The relation between the effective and unsupported lengths of any compression member is

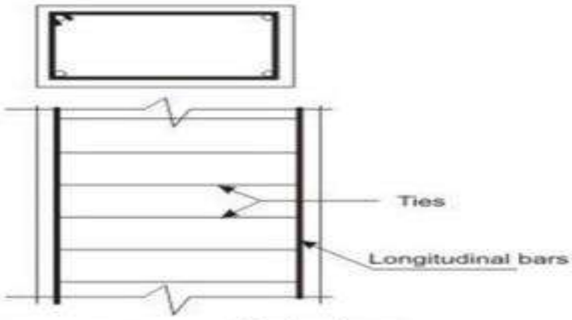
$$\mathbf{l_e = k\,l}$$

where *k* is the ratio of effective to the unsupported lengths. Clause 25.2 of IS 456 stipulates the effective lengths of compression members (vide Annex E of IS 456). This parameter is needed in classifying and designing the compression members

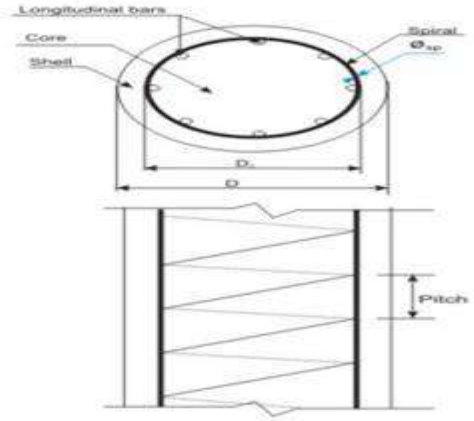
(b) **Pedestal:** Pedestal is a vertical compression member whose effective length l_e does not exceed three times of its least horizontal dimension *b*. The other horizontal dimension *D* shall not exceed four times of *b*.

(c) **Column:** Column is a vertical compression member whose unsupported length *l* shall not exceed sixty times of *b* (least lateral dimension), if restrained at the two ends. Further, its unsupported length of a cantilever column shall not exceed $100b^2/D$, where *D* is the larger lateral dimension which is also restricted up to four times of *b*

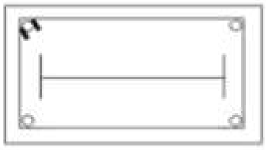
Classification of Columns Based on Types of Reinforcement



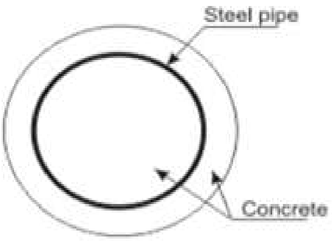
Tied column



Column with helical reinforcement



Composite column (steel section)



Composite column (steel pipe)

Tied, helically bound & composite columns

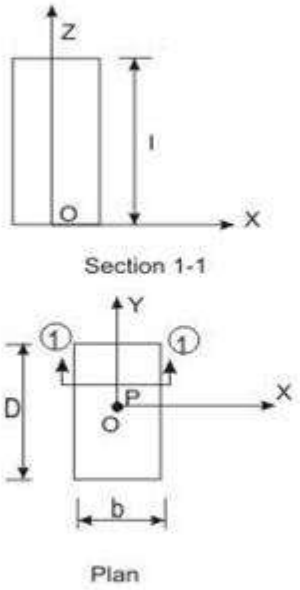
Based on the types of reinforcement, the reinforced concrete columns are classified into three groups:

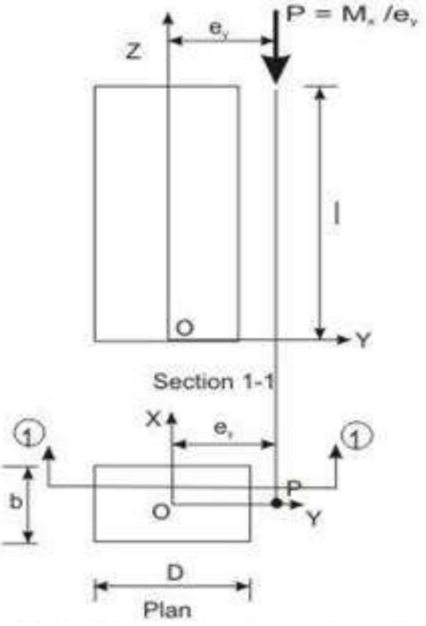
(i) Tied columns: The main longitudinal reinforcement bars are enclosed within closely spaced lateral ties

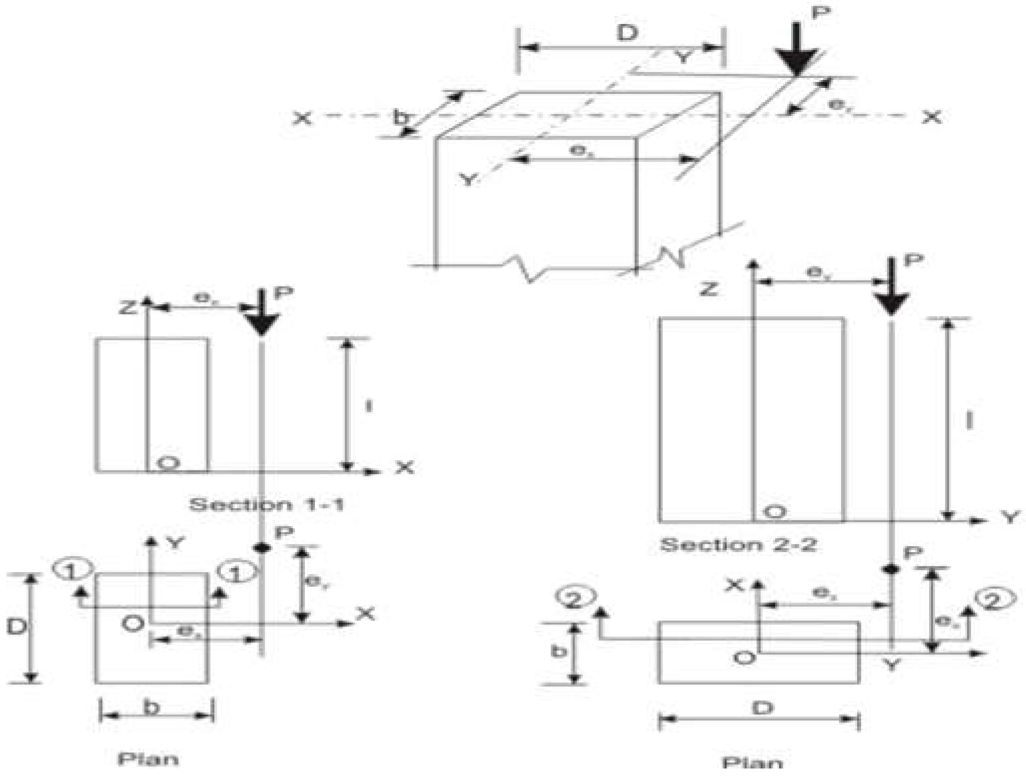
(ii) Columns with helical reinforcement: The main longitudinal reinforcement bars are enclosed within closely spaced and continuously wound spiral reinforcement. Circular and octagonal columns are mostly of this type

(iii) Composite columns: The main longitudinal reinforcement of the composite columns consists of structural steel sections or pipes with or without longitudinal bars

Classification of Columns Based on Loadings







Columns are classified into the three following types based on the loadings:

(i) Columns subjected to axial loads only (concentric),

(ii) Columns subjected to combined axial load and uniaxial bending,

(iii) Columns subjected to combined axial load and bi-axial bending.

Classification of Columns Based on Slenderness Ratios

Columns are classified into the following two types based on the slenderness ratios:

(i) Short columns

(ii) Slender or long columns

Longitudinal Reinforcement

The longitudinal reinforcing bars carry the compressive loads along with the concrete. Clause 26.5.3.1 stipulates the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc. The following are the salient points:

(a) The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.

(b) The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.

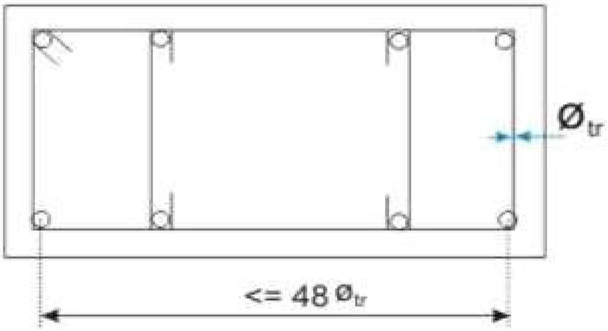
(c) Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.

(d) The diameter of the longitudinal bars should be at least 12 mm.

(e) Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.

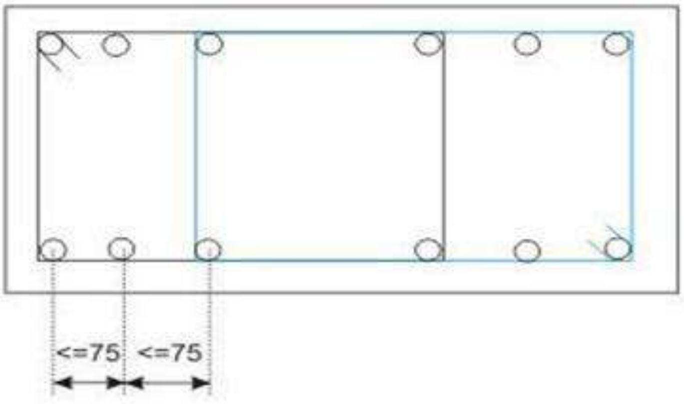
(f) The bars shall be spaced not exceeding 300 mm along the periphery of the column.

(g) The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross-sectional area provided.

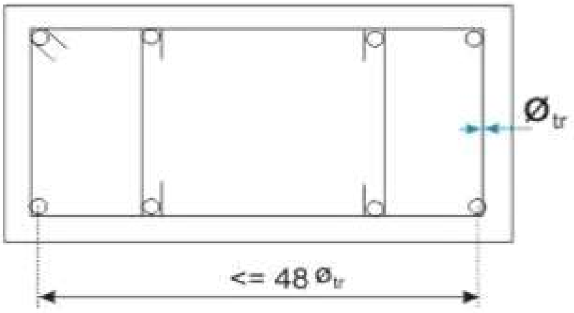


Transverse Reinforcement

Transverse reinforcing bars are provided in forms of circular rings, polygonal links (lateral ties) with internal angles not exceeding 135° or helical reinforcement. The transverse reinforcing bars are provided to ensure that every longitudinal bar nearest to the compression face has effective lateral support against buckling. Clause 26.5.3.2 stipulates the guidelines of the arrangement of transverse reinforcement. The salient points are:



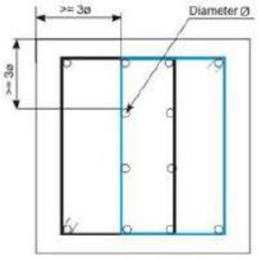
Lateral tie (Scheme 1)



Lateral tie (Scheme 2)

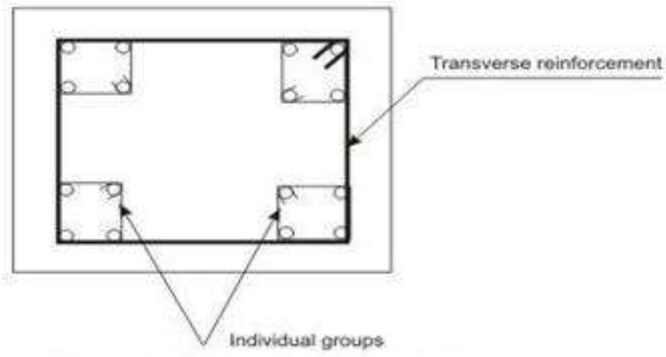
(a) Transverse reinforcement shall only go round corner and alternate bars if the longitudinal bars are not spaced more than 75 mm on either side

(b) Longitudinal bars spaced at a maximum distance of 48 times the diameter of the tie shall be tied by single tie and additional open ties for in between longitudinal bars



Lateral tie (Scheme 3)

(c) For longitudinal bars placed in more than one row (i) transverse reinforcement is provided for the outer-most row in accordance with (a) above, and (ii) no bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row.



Lateral tie (Scheme 4)

Pitch and Diameter of Lateral Ties

(a) Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:

(i) the least lateral dimension of the compression members;

(ii) sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and

(iii) 300 mm.

(b) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

Minimum Eccentricity

In practical construction, columns are rarely truly concentric. Even a theoretical column loaded axially will have accidental eccentricity due to inaccuracy in construction or variation of materials etc. Accordingly, all axially loaded columns should be designed considering the minimum eccentricity as stipulated in cl. 25.4 of IS 456

$$e_{x\,min} \geq \text{greater of })l/500 + D/30) \text{ or } 20 \text{ mm}$$

$$e_{y\,min} \geq \text{greater of })l/500 + b/30) \text{ or } 20 \text{ mm}$$

where l, D and b are the unsupported length, larger lateral dimension and least lateral dimension, respectively.

Governing Equation for Short Axially Loaded Tied Columns

Factored concentric load applied on short tied columns is resisted by concrete of area A_c and longitudinal steel of areas A_{sc} effectively held by lateral ties at intervals. Assuming the design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, we can write

$$P_u = 0.4f_{ck}A_c + 0.67f_y A_{sc}$$

where P_u = factored axial load on the member,

f_{ck} = characteristic compressive strength of the concrete,

A_c = area of concrete,

f_y = characteristic strength of the compression reinforcement, and

A_{sc} = area of longitudinal reinforcement for columns.

The above equation, given in cl. 39.3 of IS 456, has two unknowns Ac and A_{sc} to be determined from one equation. The equation is recast in terms of A_g, the gross area of concrete and p, the percentage of compression reinforcement employing.

Governing Equation of Short Axially Loaded Columns with Helical Ties

Columns with helical reinforcement take more load than that of tied columns due to additional strength of spirals in contributing to the strength of columns. Accordingly, cl. 39.4 recommends a multiplying factor of 1.05 regarding the strength of such columns. The code further recommends that the ratio of volume of helical reinforcement to the volume of core shall not be less than $0.36\,(A_g/A_c - 1)\,(f_{ck}/f_y)$, in order to apply the additional strength factor of 1.05 (cl. 39.4.1). Accordingly, the governing equation of the spiral columns may be written as

$$P_u = 1.05(0.4\,f_{ck}A_c + 0.67\,f_y A_{sc})$$

Behaviour of Short Columns under Axial Load and Uniaxial Moment

Normally, the side columns of a grid of beams and columns are subjected to axial load P and uniaxial moment M_x causing bending about the major axis xx, hereafter will be written as M. The moment M can be made equivalent to the axial load P acting at an eccentricity of e (= M/P). Let us consider a symmetrically reinforced short rectangular column subjected to axial load P_u at an eccentricity of e to have M_u causing failure of the column.

Two strain profiles IN and EF. For the strain profile IN, the depth of the neutral axis kD is less than D, i.e., neutral axis is within the section resulting the maximum compressive strain of 0.0035 on the right edge and tensile strains on the left of the neutral axis forming cracks.

This column is in a state of collapse for the axial force P_u and moment M_u for which IN is the strain profile. Reducing the eccentricity of the load P_u to zero, we get the other strain profile EF resulting in the constant compressive strain of 0.002, which also is another collapse load. This axial load P_u is different from the other one, i.e., a pair of P_u and M_u, for which IN is the profile. For the strain profile

EF, the neutral axis is at infinity ($k = \alpha$).

The strain profile EF with two more strain profiles IH and JK intersecting at the fulcrum point V. The strain profile IH has the neutral axis depth $kD = D$, while other strain profile JK has $kD > D$. The load and its eccentricity for the strain profile IH are such that the maximum compressive strain reaches 0.0035 at the right edge causing collapse of the column, though the strains throughout the depth is compressive and zero at the left edge. The strain profile JK has the maximum compressive strain at the right edge between 0.002 and 0.0035 and the minimum compressive strain at the left edge. This strain profile JK also causes collapse of the column since the maximum compressive strain at the right edge is a limiting strain satisfying assumption

Interaction Diagram

It is now understood that a reinforced concrete column with specified amount of longitudinal steel has different carrying capacities of a pair of P_u and M_u before its collapse depending on the eccentricity of the load. One such interaction diagram giving the carrying capacities ranging from P_o with zero eccentricity on the vertical axis to M_o (pure bending) on the horizontal axis. The vertical axis corresponds to load with zero eccentricity while the horizontal axis represents infinite value of eccentricity. A radial line joining the origin O of to point 2 represents the load having the minimum eccentricity. In fact, any radial line represents a particular eccentricity of the load. Any point on the interaction diagram gives a unique pair of P_u and M_u that causes the state of incipient failure. The interaction diagram has three distinct zones of failure: (i) from point 1 to just before point 5 is the zone of compression failure, (ii) point 5 is the balanced failure and (iii) from point 5 to point 6 is the zone of tension failure. In the compression failure zone, small eccentricities produce failure of concrete in compression, while large eccentricities cause failure triggered by yielding of tension steel. In between, point 5 is the critical point at which both the failures of concrete in compression and steel in yielding occur simultaneously.

The interaction diagram further reveals that as the axial force P_u becomes larger the section can carry smaller M_u before failing in the compression zone. The reverse is the case in the tension zone, where the moment carrying capacity M_u increases with the increase of axial load P_u. In the compression failure zone, the failure occurs due to over straining of concrete. The large axial force produces high compressive strain of concrete keeping smaller margin available for additional compressive strain line to bending. On the other hand, in the tension failure zone, yielding of steel initiates failure. This tensile yield stress reduces with the additional compressive stress due to additional axial load. As a result, further moment can be applied till the combined stress of steel due to axial force and increased moment reaches the yield strength.

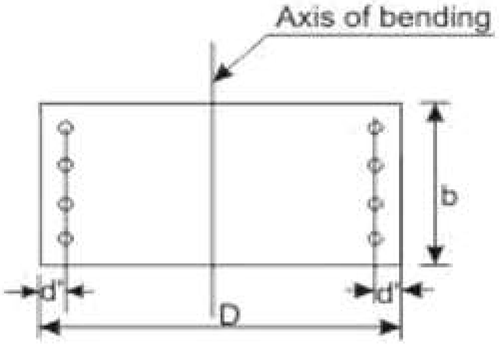
Therefore, the design of a column with given P_u and M_u should be done following the three steps, as given below:

1. Selection of a trial section with assumed longitudinal steel,
2. Construction of the interaction diagram of the selected trial column section by successive choices of the neutral axis depth from infinity (pure axial load) to a very small value (to be found by trial to get $P = 0$ for pure bending),
3. Checking of the given P_u and M_u, if they are within the diagram.

We will discuss later whether the above procedure should be followed or not. Let us first understand the corresponding compressive stress blocks of concrete for the two distinct cases of the depth of the neutral axis: (i) outside the cross-section and (ii) within the cross-section in the following sections.

Design of Short Columns under Axial Load with Uniaxial Bending

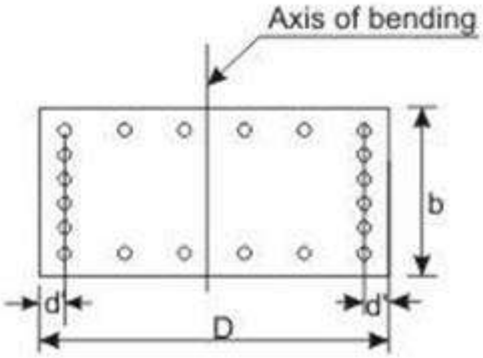
It is known that the design of columns by direct computations involves several trials and hence, time taking. On the other hand, design charts are very useful in getting several alternative solutions quickly. Further, design charts are also used for the analysis of columns for safety etc. However, there are limitations of using the design charts, which are mentioned in this lesson. Several numerical problems are solved in this lesson for the purpose of illustration covering both analysis and design types of problems using the design charts of SP-16.



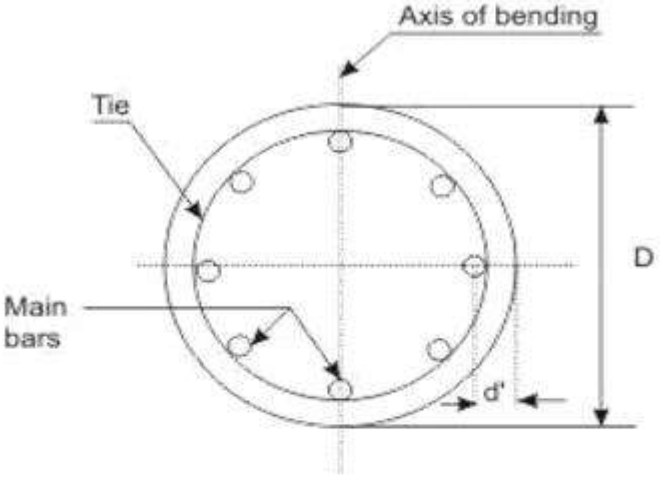
Design Charts of SP-16

SP-16 has three sets of design charts prepared by following the procedure explained in Lesson 24 for rectangular and circular types of cross-sections of columns. The three sets are as follows:

(i) Charts 27 to 38 are the first set of twelve charts for rectangular columns having symmetrical longitudinal steel bars in two rows for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).



(ii) Charts 39 to 50 are the second set of twelve charts for rectangular columns having symmetrical longitudinal steel bars (twenty numbers) distributed equally on four sides (in six rows,) for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).



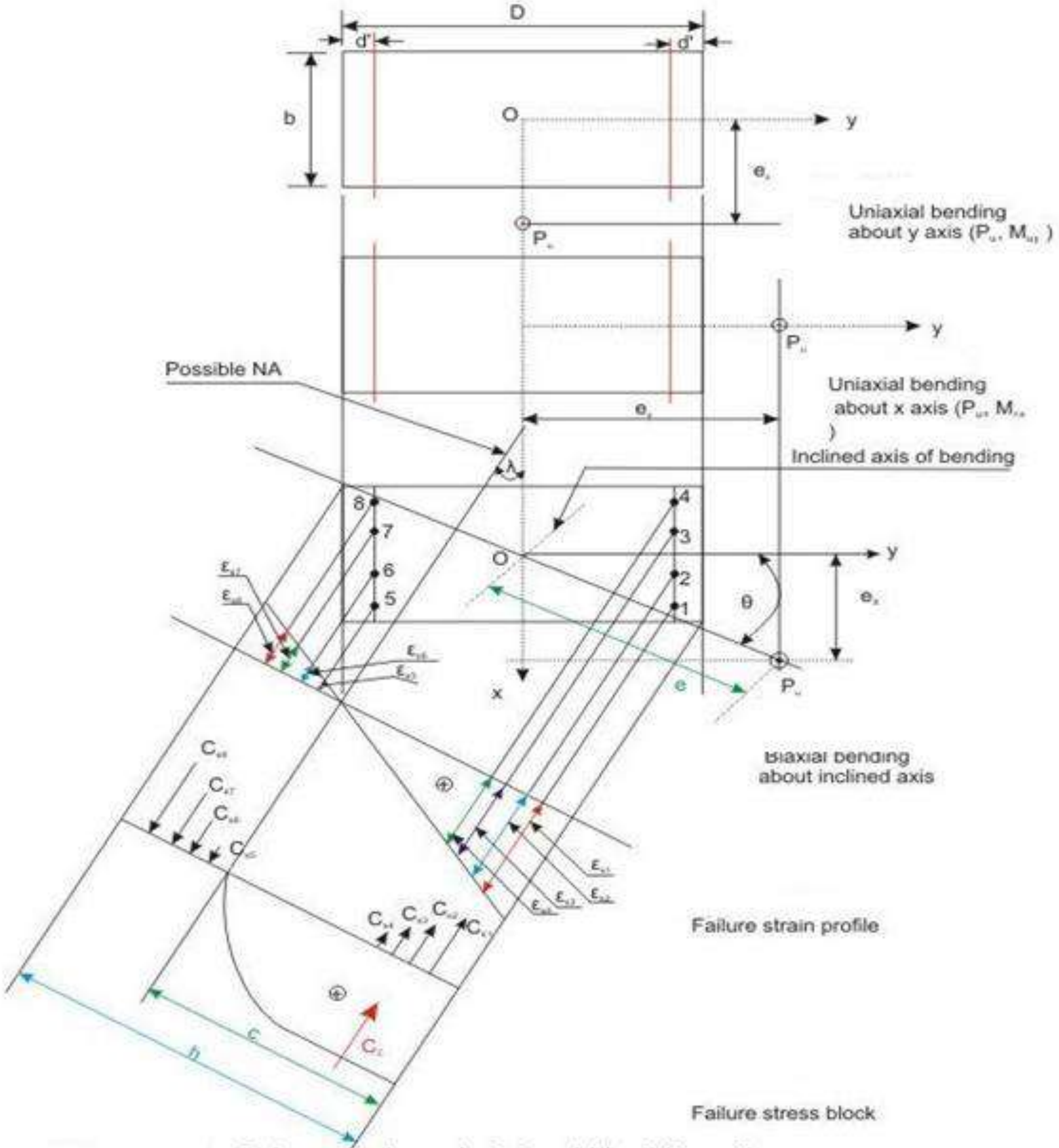
(iii) The third set of twelve charts, numbering from 51 to 62, are for circular columns having eight longitudinal steel bars of equal diameter and uniformly spaced circumferentially for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).

All the thirty-six charts are prepared for M 20 grade of concrete only. This is a justified approximation as it is not worthwhile to have separate design charts for each grade of concrete.

Short Compression Members under Axial Load with Biaxial Bending

Beams and girders transfer their end moments into the corner columns of a building frame in two perpendicular planes. Interior columns may also have biaxial moments if the layout of the columns is irregular. Accordingly, such columns are designed considering axial load with biaxial bending. This lesson presents a brief theoretical analysis of these columns and explains the difficulties to apply the theory for the design. Thereafter, simplified method, as recommended by IS 456, has been explained with the help of illustrative examples in this lesson.

Biaxial Bending



Column under uniaxial and biaxial bending

Column section under axial load and uniaxial bending about the principal axes x and y, respectively. The column section under axial load and biaxial bending. The eccentricities e_x and e_y of are the same as those of (for e_x) and (for e_y), respectively. Thus, the biaxial bending case (case c) is the resultant of two uniaxial bending cases a and b. The resultant eccentricity e, therefore, can be written as

$$e = (e_x^2 + e_y^2)^{1/2}$$

Designating the moments of cases a, b and c by M_{ux}, M_{uy} and M_u, respectively, we can write:

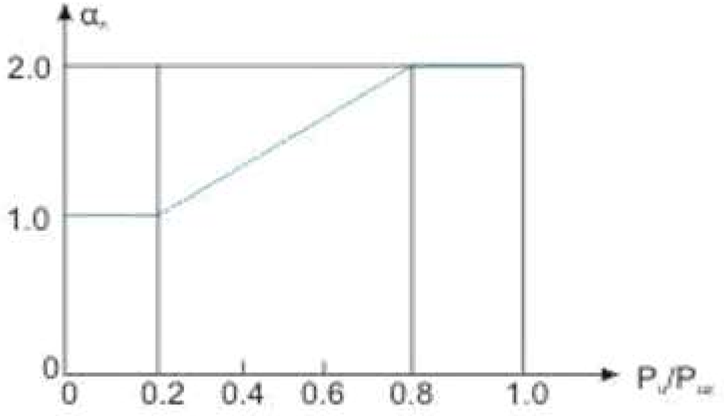
$$M_u = (M_{ux}^2 + M_{uy}^2)^{1/2}$$

and the resultant M_u is acting about an inclined axis, so that

$$\tan\theta = e_x/e_y = M_{uy}/M_{ux}$$

the angle of inclination θ is measured from y axis.

IS Code Method for Design of Columns under Axial Load and Biaxial Bending



IS 456 recommends the following simplified method, based on Bresler's formulation, for the design of biaxially loaded columns. The relationship between

M_{uxz} and M_{uyz} for a particular value of $P_u = P_{uz}$, expressed in non-dimensional form is:

$$(M_{ux} / M_{ux1})^{\alpha n} + (M_{uy} / M_{uy1})^{\alpha n} \leq 1$$

where M_{ux} and M_{uy} = moments about x and y axes due to design loads, and

α_n is related to P_u/P_{uz}, where

$P_{uz} = 0.45\, f_{ck}\, A_c + 0.75\, f_y\, A_{sc}$

$= 0.45\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$

where A_g = gross area of the section, and

A_{sc} = total area of steel in the section