UNIT-I

SIMPLE STRESSES AND STRAINS

Introduction

When a material is subjected to external forces or loads, it tends to deform. However due to cohesion between the particles in the body, the material offers resistance to deformation. (If the force is increased, the resistance as well as the deformation also increases). The resistance by which the material of the body opposes the deformation is known as strength of the material.

When material is not capable of offering necessary resistance against external forces, permanent deformation will be set and failure of the member may occur, if the load is further increased.

Strength: Ability of element or part to resist failure is known as strength.

Stiffness: Ability of element or part to resist deformation is known as stiffness.

Homogeneous: A body is said to be homogeneous if it has identical properties at all points in all Directions

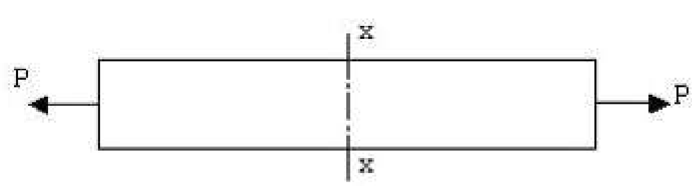
Isotropic : A body is said to be Isotropic if it has identical elastic properties at any point in all directions.

- **Stress (σ):-**

The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called load or force.

This resistance per unit area is known as "stress".

Mathematically stress is defined as the force per unit area.



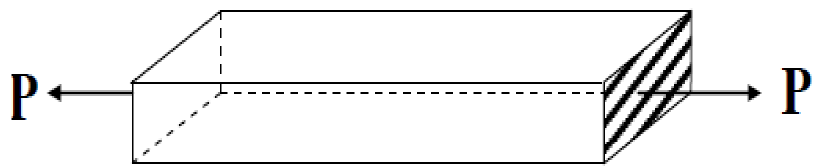


Fig. 1.1

In the above case the stress action on the body is given as

$$\sigma = P/A, \text{ Where,}$$

σ - Intensity of stress

A - Cross sectional area

P - Applied load

Units – N/mm^2, KN/m^2, N/cm^2 MPa etc

- **Types of Stresses:-**

(a)Tensile Stress (σ_t)

When a load applied on a body tends to pull the material away from each other, causing extension of the body in the direction of the applied load, the load is called a tensile load and the corresponding stress induced is called Tensile stress.

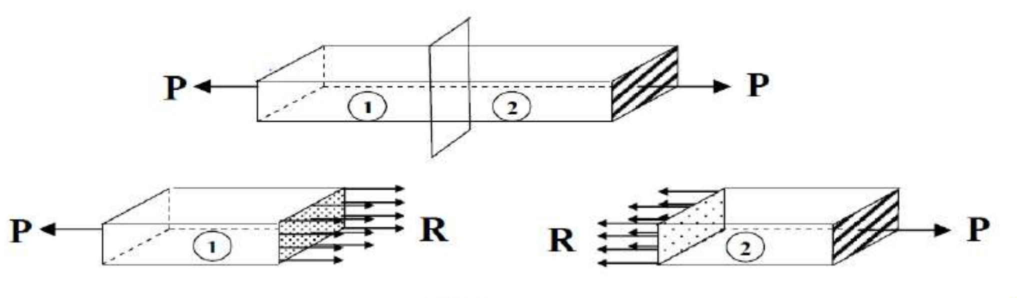


Fig.1.2

Let us consider a uniform bar of cross sectional area 'A' and subjected to an axial pull 'P' at both ends. Let us consider a section to divide the bar into two parts. For equilibrium the two portions of the bar at the sectional plane, a resisting force 'R' is developed due to equilibrium R= P.

Therefore tensile stress, $\sigma_t = R/A = P/A$

Units of stress – In SI system the units are expressed in N/mm^2, kN/m^2, N/cm^2

(It is also expressed in Pascal 1 Pascal = $1N/m^2$)

(b) Compressive Stress:- (σ_c)

When a load applied on a body tends to push the particles of the material closer to each other, causing shortening of the body in the direction of the applied forces, the applied force is known as compressive force and the corresponding stress is known as compressive stress.

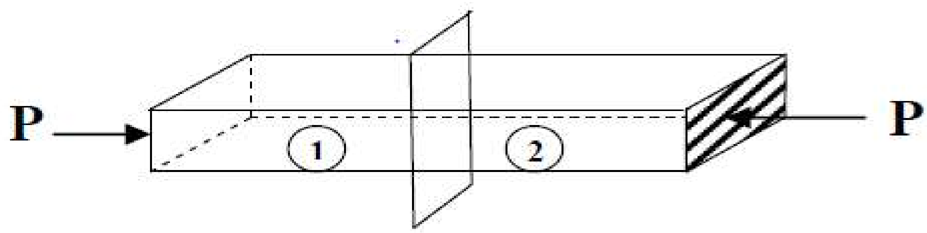


Fig. 1.3

The compressive stress at any section along the length of the load is given as

$$\sigma_c = \frac{Resisting\ force}{Cross\ sectional\ area}$$

$$\sigma_c = \frac{R}{A} = \frac{P}{A}$$

Units – N/mm^2, kN/m^2, N/cm^2etc

(c) Shear stress:- (τ or q)

When a load applied on a body causes one portion of the body to slide over the adjoining portion, such a force is known as Shear Force and the corresponding stress is known as Shear Stress.

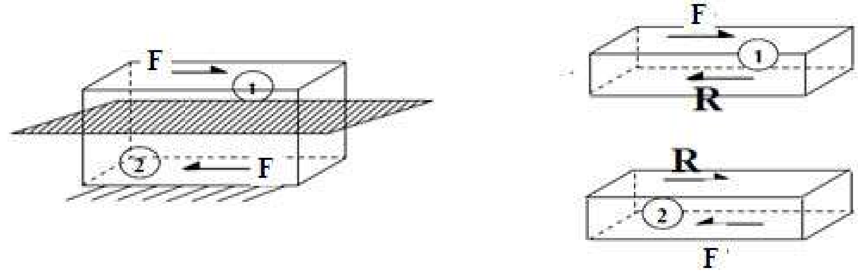


Fig. 1.4

As shown in the fig, the body might separate into two portions causing one portion to slide over another. At the plane of separation a resisting force ‘R’ is developed. Thus shear stress is given as

$$\tau = \frac{Resisting\ force}{Applied\ Area}$$

$$\tau = \frac{R}{A} = \frac{F}{A}$$

In the above, tensile stresses and compressive stresses are known as Direct stresses; whereas shear stresses are known as tangential stresses.

➢ **Strain:- (e or ε)**

When a force is applied on a body, there is some change in the dimension of the body. The ratio of change in dimension of the body to its original dimension is known as strain. It is a dimensionless quantity.

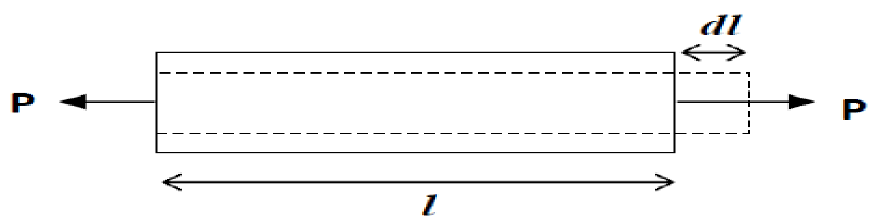


Fig. 1.5

Consider, a bar subjected to an axial pull P. Let *l* be the original length of the bar and (*l*+*dl*) be the final length of the bar, where dl represents change in length. Therefore, Strain

$$\epsilon = \frac{dl}{l}$$

The strain depends on the nature of the load acting on a body stresses are induced in the body and we can observe the following types of strains.

(a) Tensile strain:-

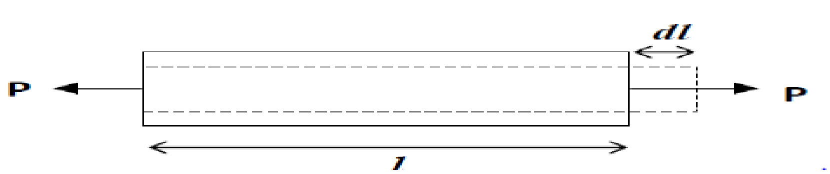


Fig. 1.6

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When a tensile force acts on a body, it elongates in the direction of force P by an amount of dl and then the strain is called Tensile Strain.Thus, tensile strain is given as $\epsilon = \frac{dl}{l}$

b) Compressive strain:-

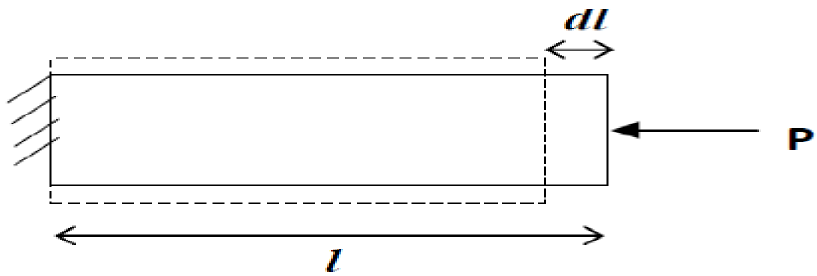


Fig. 1.7

when an axial compressive force acts on a body causing shortening of the body by an amount 'dl' in the direction of the applied force, the strain is called compressive strain and it is given as $\epsilon = \frac{dl}{l}$

(c) Shear strain:-

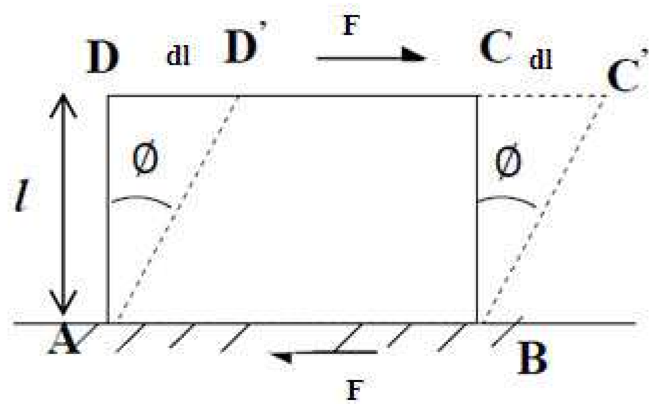


Fig. 1.8

Consider an element of unit thickness subjected to a shear force as shown in fig. The body deforms as shown (Point D shifts to D' and Point C shifts to C' with equal amount)

Let Φ represent the angular deformation of the vertical faces and dl represents the horizontal or transverse displacement of the upper face with respect to the lower face. This displacement occurs over a length 'l'. In such a case shear strain is defined as,

$$shear\ strain = \frac{transverse\ displacement}{vertical\ height\ from\ lower\ face(perpendicular\ height)}$$

From the triangle ADD[1], $tan\Phi = \frac{dl}{l}$

For very small values of Φ we have tanΦ = Φ

$$\Phi = \frac{dl}{l}$$

Where Φ is measured in radians

➤ **Mechanical properties of materials**

The following are considered as the most important properties of engineering materials

1) Elasticity 2) Plasticity 3) Ductility 4) Malleability

5) Brittleness 6) Toughness 7) Hardness

Any material cannot possess all the above properties because they are different from each other. Hence the important properties of materials can be classified as follows depending upon their mechanical properties

a) Elasticity:-This is the property of the material which enables the deformation due to application of external forces and once the external forces are removed the material regains its original shape and size. **Examples:** rubber,steel
* Steel is more elastic than rubber

b) Plasticity:-This is the property of the materialwhich enables the material do not regains their original shape and size even after the removal of external loads. **Example:** clay, lead

c) Ductility:-This is the property of the material whichenables considerable deformations without much increase in the load or in simple terms, these are materials that can be drawn into wires. **Examples:** mild steel, copper, aluminum, brass etc;

*It is tensile property of material.

d) Malleability:- This is the property of the material which can be extended in two directions easily or in simple terms, materials which can be drawn into thin sheets.

Examples: Aluminum, copper, tin, lead

* It is compressive property of material

e) Brittleness:-This is the property of the material which doesn't allow any deformation before failure when external forces act on them. **Examples:** cast iron, concrete and glass.

f) Toughness:-This is the property of the material which resists sudden loads or shock loads without showing any fracture on failure.**Example:** mild steel, brass, wrought Iron

g)Hardness:-This is the property of the material which has an ability to resist surface abrasion or indentation.**Example:**diamond, topaz, quartz.

➢ **Stress- Strain diagram for mild steel:-**

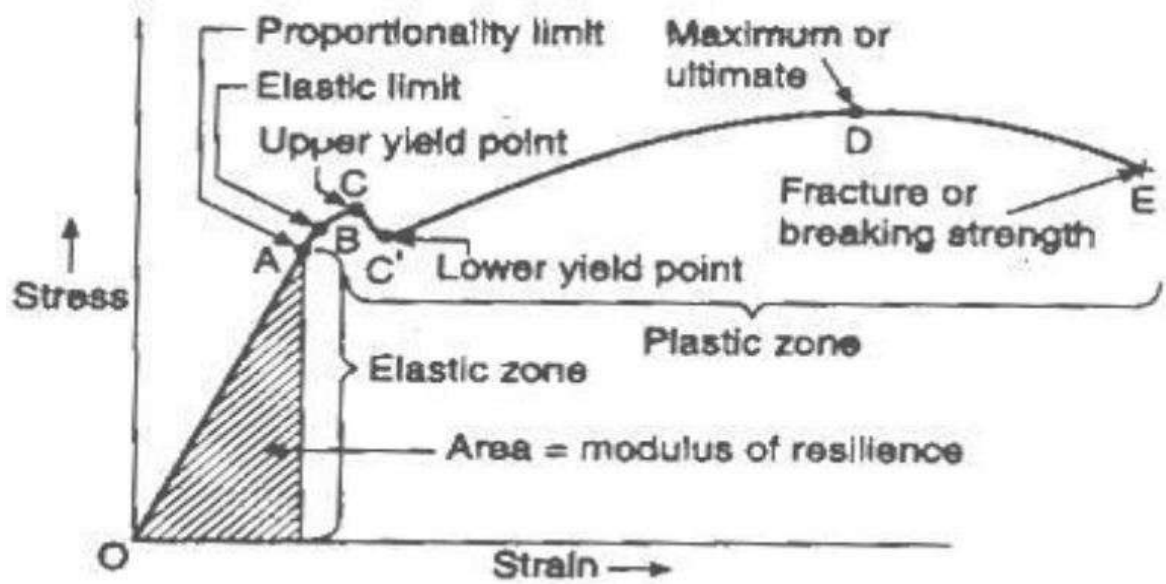


Fig. 1.9

Point A-Proportionality Limit: The stress is proportional to the strain within this limit."Hooke's Law" is valid in this region

Point B- Elastic Limit: The maximum stress upto which a specimen regains its original length on removal of applied load

Point C- Upper Yield point: The magnitude of the stress corresponding to C depends on the cross-sectional area, shape of the specimen and the type of the equipment used to perform the test.

Point C-Lower yield Point: The stress at C is the yield stress with a typical value of 250 MPa for mild steel. The yielding begins at this stress.

Point D- Ultimate Point: Ultimate load is defined as maximum load which can be placed prior to the breaking of the specimen. Stress corresponding to the ultimate load is known as ultimate stress. This is the maximum stress, the material can resist

Point E- Failure or Rupture or Breaking stress: This is stress at which the specimen fails or breaks or ruptures.

Engineers Stress-Strain curve : The curve drawn for stress considering original area of cross section of the bar while calculating stress at various points to respective strains.

True Stress strain Curve: The curve drawn for stress considering Instantaneous area of cross section of the bar while calculating stress at various points to respective strains.

- **Working stress :-**

  It is known as the maximum allowable stress that a material or object will be subjected to when in service. This is always lower than the yield stress.

  Working stress= Ultimate stress/Factor of safety

- **Factor safety :-**

The ratio of the ultimate stress to the working stress is known as "Factor of Safety".

In designing any engineering components stressing the material up to its ultimate strength is not advisable for following reasons

- Stressing the material till ultimate strength causes the deformation or the failure of the member.
- The material may not be 100% reliable.
- The material may contain minor defects.

- **Hooke's Law (Sir Robert hooke in 1678)**

It states that "stress is directly proportional to strain within the elastic limit".(Strictly speaking upto proportionality limit)

i,e. Stress α strain (for uniaxial force)

$$\frac{stress}{strain} = constant$$

This constant up to limit of proportionality is called Elastic modulus or Modulus of Elasticity or Young's Modulus and it is denoted as E

$$E = \frac{\sigma}{e}$$

i.e. Young's modulus has same unit as stress

Consider $E = \frac{\sigma}{e}$

But $\sigma = \frac{P}{A}$ and $e = \frac{dl}{l}$

$$E = \frac{\frac{P}{A}}{\frac{dl}{l}}$$

$$E = \frac{P.l}{A.dl}$$

$$dl = \frac{Pl}{AE}$$

Whenever a force is acting on a body whose cross sectional area is A and length l and E is its elastic modulus, the change in length is given by

$$dl = \frac{Pl}{AE}$$

- **Principle of Superposition:-**

When a number of loads are acting on a body the resulting strain, according to the principle of superposition, will be the algebraic sum of the strains caused by the individual forces.

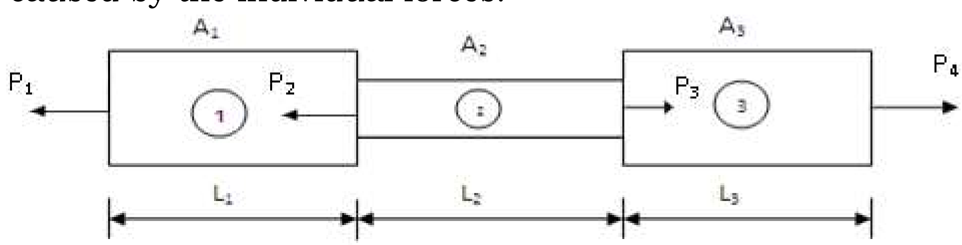


Fig.1.10

$dl = dl_1 + dl_2 + dl_3$

- **Deformation of uniform bars:-**

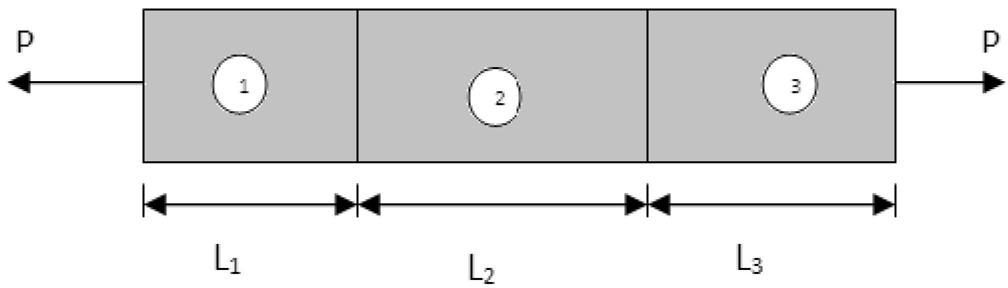


Fig.1.11

Procedure:

STEP 1: Check whether the system of forces is in equilibrium or not. If there is any unknown force in the system, determine it using equilibrium condition.

STEP 2: Separate each part and find the force acting on each part.

STEP 3: The total deformation is given by the algebraic sum of deformation of each part.

$$dl = (dl_1 \pm dl_2 \pm dl_3) = \frac{P}{AE}(L_1 \pm L_2 \pm L_3)$$

- **Deformation of Bars of Varying Cross section:-**

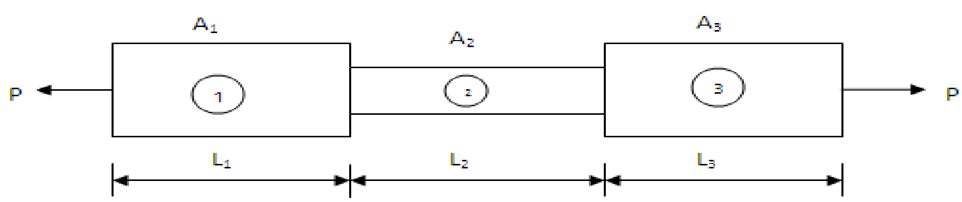


Fig.1.12

Procedure:

STEP 1: Check whether the system of forces is in equilibrium or not. If there is any unknown force, determine it using equilibrium condition.

STEP2: Separate each part and find the force acting on each part.

STEP 3: The total deformation is given by the algebraic sum of deformation of each part.

$$dl = (dl_1 \pm dl_2 \pm dl_3) = \frac{P}{E}\left(\frac{L_1}{A_1} \pm \frac{L_2}{A_2} \pm \frac{L_3}{A_3}\right)$$

If the Young's Modulus of the material is different, then the change in length is given by

$$\boldsymbol{dl} = (\boldsymbol{dl_1} \pm \boldsymbol{dl_2} \pm \boldsymbol{dl_3}) = \boldsymbol{P}\left(\frac{\boldsymbol{L_1}}{\boldsymbol{A_1E_1}} \pm \frac{\boldsymbol{L_2}}{\boldsymbol{A_2E_2}} \pm \frac{\boldsymbol{L_3}}{\boldsymbol{A_3E_3}}\right)$$

- **Deformation of Tapering Bars of Uniform Thickness:-**

Let us consider a tapering bar of constant thickness 't' and width varying from 'B' to 'b' over a length 'L' subjected to direct load 'P'. Let us consider an elemental strip of length 'dx' at a distance 'x' as shown in fig. The elemental strip can be considered to be uniform.

We have deformation of elemental strip = $\frac{PL}{AE} = \frac{P\,dx}{(b_1 t)\,E}$

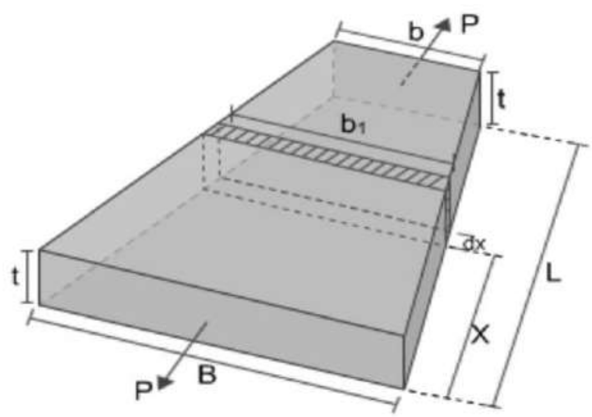


Fig.1.13

Deformation of entire bar $= \int_0^l \frac{P}{t\,E\,b_1}\,dx$

To express b_1 in terms of x or as a function of x :-

b_1 = B – decrease in width over the length x

Decrease in width over the length L= B – b

Decrease in width over the length x = $\frac{B-b}{L}\,x = kx$

b_1 = B – kx

Elongation of the smallest element is $\delta l = \frac{Pl}{AE} = \frac{P}{E}\frac{dx}{(B-kx)t}$

Total elongation $dl = \int_0^l \frac{P}{E}\frac{dx}{(B-kx)t} = \frac{P}{Et}\int_0^l \frac{dx}{(B-kx)}$

$= \frac{P}{Et}\left[-\frac{1}{k}\log_e(B-kx)\right]_0^l$

$= -\frac{P}{Et}\frac{l}{(B-b)}\left[\log_e b - \log_e B\right)$

$$\boldsymbol{dl} = \frac{\boldsymbol{Pl}}{\boldsymbol{tE(B-b)}}\boldsymbol{\log_e\frac{B}{b}}$$

- Tapering bar of **circular cross-section** whose diameter changes from d_1 at one end to d_2 at the other end $dl = \frac{4PL}{\pi E d_1 d_2}$

- **Composite Bars:-**

A bar, made up of two or more bars of equal lengths but of different material rigidly fixed with each other and behaving as one unit for extension or compression when subjected to an axial tensile or compressive loads is called a composite bar.The conditions to analyse the problems are

(i) The change in length in each bar and the corresponding strain is equal and

(ii) The external load is shared by the bars based on their material property (i.e., on the modulus of elasticity).

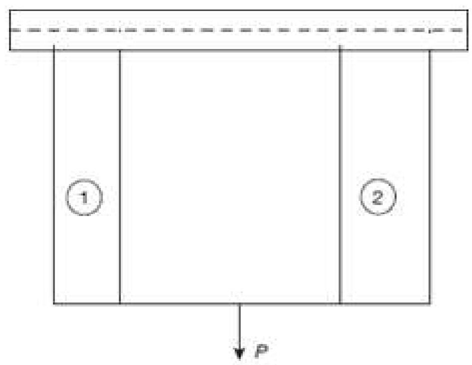


Fig.1.14

Considering figure1.14, the load P is shared by the bar ①as P_1 and bar ②as P_2. Thus,

$$P = P_1 + P_2 \quad – (a)$$

Stresses in bars ①and ②are, respectively, as

$$\sigma_1 = \frac{P_1}{A_1} \text{and} \sigma_2 = \frac{P_2}{A_2}$$

whereA_1 and A_2 are the areas of cross section of bar ① and bar ②, respectively.

$P_1 = \sigma_1 A_1 \qquad P_2 = \sigma_2 A_2$

Substituting for P_1 and P_2 in equation (a)

$P = \sigma_1 A_1 + \sigma_2 A_2 \quad$ __ $\quad$ (b)

Strain in bar①, e_1 and in bar②, e_2 are

$$e_1 = \frac{p_1}{E_1} \quad \text{and} \quad e_2 = \frac{p_2}{E_2}$$

Where E_1 and E_2 are modulus of elasticity of bar ① and bar ②, respectively.

As strain is equal in both the bars, $\boldsymbol{e_1 = e_2}$ - (c)
Thus, the load and the stress in bar ① and bar ② can be found out by using Equations (b) and (c)

- **Thermal stresses:-**

Temperature causes bodies to expand or contract. Thermal stresses are the stresses induced in a body due to change in temperature. Thermal stresses are set up in a body, when the temperature of the body is raised and the body is not allowed to expand or contract freely. Change in length due to increase in temperature can be expressed as

$$\boldsymbol{\Delta L = L.\alpha.\Delta t}$$

Where, L is the length, α (/°C) is the coefficient of linear expansion, and $\Delta t = t_2 - t_1$, where **Δt** is the temperature change, t_2 final temperature and t_1 initial temperature.

$$\text{Thermal Strain } \mathbf{e} = \frac{\Delta L}{L} = \boldsymbol{\alpha\Delta t}$$

If the temperature deformation is permitted to occur freely no load or the stress will be induced. When such deformations are prevented internal stresses will be developed and such stresses are known as thermal stresses.

If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive stress and strain will be set up in the rod.

When the temperature deformation is prevented, thermal stress developed due to temperature change can be given as: $\boldsymbol{\sigma = E.\alpha.\Delta t}$

If the ends of the rod yield by an amount δ, the total amount of expansion is given by

$$\boldsymbol{\Delta L^1 = L.\alpha.\Delta t - \delta}$$

- **Temperature stresses in Composite Bar:-**

If a compound bar made up of several materials is subjected to a change in temperature there will be tendency for the components parts to expand different amounts due to the unequal coefficient of thermal expansion. If the parts are constrained to remain together then the actual change in length must be the same for each. This change is the resultant of the effects due to temperature and stresses condition.

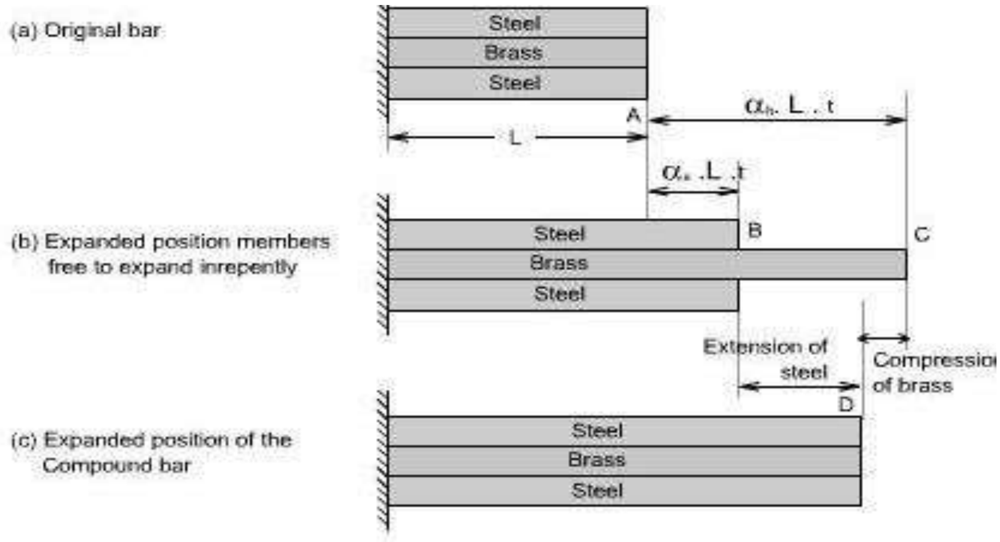


Fig.1.15

Now let σ_b= Stress in brass

e_b= Strain in brass

α_b= Coefficient of linear expansion for brass

A_b= Cross sectional area of brass bar

And σ_s, e_s, α_s, A_s= Corresponding values for steel.

Δ = Common expansion in the bar

As compressive load on the brass in equal to the tensile load on the steel, therefore

$$\sigma_b . A_b = \sigma_s . A_s \text{ ---- (a)}$$

(i) Free expansion of brass is more than free expansion of steel as $\alpha_b > \alpha_s$

(ii) Expansion of composite bar is less than free expansion of brass and it is more than free expansion of steel.

(iii) Hence compressive stresses will be produced in the brass as expansion is prevented and tensile stresses will be produced in steel due to extended expansion.

Prevented expansion in brass $e_b = L\,\alpha_b\,t - \Delta$ (i.e, $\Delta = L\,\alpha_b\,t - e_b$)

Extended expansion in steel $e_s = \Delta - L\,\alpha_s\,t$ (i.e, $\Delta = e_s - L\,\alpha_s\,t$)

$$L\,\alpha_b\,t - e_b = e_s - L\,\alpha_s\,t \text{ --------(b)}$$

From equations (a) and (b) stresses σ_b and σ_s can be obtained.

- **Longitudinal Strain:-**
  When a body is subjected to an axial tensile or compressive load, there is an axial deformation in the length of the body. The ratio of axial deformation (dL) to the original length(L) of the body is known as longitudinal or linear strain. The longitudinal strain is also defined as the deformation of the body per unit length in the direction of the applied load.

$$\text{Longitudinal strain} = \frac{dL}{L}$$

- **Lateral Strain:-**

Lateral strain of a deformed body is defined as the ratio of the change in length (breadth of a rectangular bar or diameter of a circular bar) of the body due to the deformation to its original length (breadth of a rectangular bar or diameter of a circular bar) in the direction perpendicular to the force.

$$\text{Lateral strain} = \frac{db}{b} \text{or} \frac{dd}{d}$$

- **Poisson's Ratio:-**

Any direct stress is accompanied by a strain in its own direction and called linear strain and an opposite kind of strain in every direction at right angles to it is lateral strain. This lateral strain bears a constant ratio with the linear strain. This ratio is called the Poisson's ratio and is denoted by (1/m) or μ.

Poisson's Ratio = Lateral Strain / Linear Strain.

The maximum value of the Poisson's ratio is 0.5

- **Volumetric Strain:-**

Volumetric strain of a deformed body is defined as the ratio of the change in volume of the body to the deformation to its original volume.

If V is the original volume and δV the change in volume occurred due to the deformation, the volumetric strain e_v induced is given by

$$\mathbf{e_v = \delta V/V}$$

Consider a uniform rectangular bar of length l, breadth b and depth d as shown in figure. Its volume V is given by,

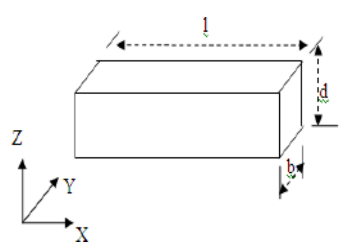


Fig.1.16

$V = lbd$

$\delta V = \delta lbd + \delta bld + \delta dlb$

$\delta V / V = (\delta l / l) + (\delta b / b) + (\delta d / d)$

$e_v = e_x + e_y + e_z$

This means that volumetric strain of a deformed body is the sum of the linear strains in three mutually perpendicular directions.

- **ELASTIC CONSTANTS :**

Elastic constants are those factors which determine the deformations produced by a given stress system acting on a material. These factors are constant within the limits for which Hooke's laws are obeyed. Various elastic constants are

(i) Modulus of elasticity (E)

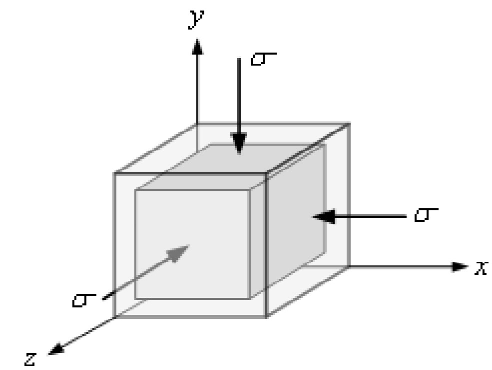
(ii) Poisson's ratio (μ or 1/m)

(iii) Modulus of rigidity (G or N or C)

(iv) Bulk modulus (K)

- **BULK MODULUS**

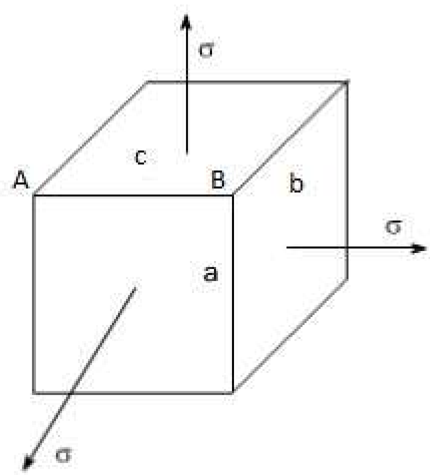
When a body is subjected to the mutually perpendicular like and direct stresses, the ratio of direct stress corresponding to the corresponding volumetric strain is found to be a constant for a given material when the deformation is within a certain limit. This ratio is known as bulk modulus and is usually denoted by K.



$$K = \frac{DirectStress}{VolumetricStrain} = \frac{\sigma}{(\frac{dV}{V})}$$

- **RELATION AMONG YOUNGS MODULUS (E) AND BULK MODULUS (K)**

Consider a cube of side L be subjected to three mutually perpendicular tensile stresses of equal intensity.



Volume of Cube, V = L^3 …..(eq1)

Strains on the one of sides of the cube (say AB) under the action of three perpendicular stresses are

1. Strain Of AB due to stresses on b & b' = $\frac{\sigma}{E}$
2. Strain Of AB due to stresses on c & c' = $-\mu\frac{\sigma}{E}$
3. Strain Of AB due to stresses on a & a' = $-\mu\frac{\sigma}{E}$

Total Strain of AB is

$$\frac{dL}{L} = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = \frac{\sigma}{E}(1-2\mu) \quad \text{.......(eq2)}$$

If dL is change in length, then dV is change in volume

Differentiate eq1, with respect to L,

$$dV = 3L^2 \text{ x } dL \quad \text{.....(eq3)}$$

Eq3/eq1 gives

$$\frac{dV}{V} = \frac{3L^2 \text{ x dL}}{L^3} = \frac{3dL}{L}$$

Substitute the value of dL/L in above equation gives

$$\frac{dV}{V} = \frac{3\sigma}{E}(1-2\mu)$$

Bulk Modulus is given by

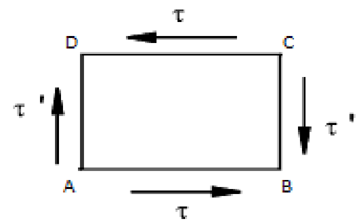
$$K = \frac{\sigma}{(\frac{dV}{V})} = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$= \frac{E}{3(1-2\mu)}$$

$$E = 3K(1-2\mu)$$

- **PRINCIPLE OF COMPLEMENTARY SHEAR**

A shear stress in a given direction cannot exist without a balancing shear stress of equal intensity in a direction at right angles to it.



Consider a rectangular block ABCD of unit thickness subjected to a set of shear stresses τon faces AB & CD

Force on AB = Stress x Area = τ x AB x 1 = τ.AB

Force on CD = Stress x Area = τ x CD x 1 = τ.CD = τ.AB (as CD=AB)

Forces on faces AB &CD are equal and opposite which forms a couple

Moment of couple = Force x perpendicular distance

=τ.AB x AD …… (eq1)

A restoring couple will act if the block is in equilibrium whose moment will be equal to the moment given by eq1

Let τ' is the shear stresses on faces AD & BC

Force on AD = τ' x AD x 1 =τ'.AD

Force on BC = τ' x BC x 1 = τ'.BC = τ'.AD (as BC=AD)

Forces on faces AD & BC are equal and opposite which also forms a couple

Moment of couple = Force x perpendicular distance

=τ'.AD x AB …… (eq2)

For equilibrium the moments in eq1 & eq2 should be equal

τ.AB x AD = τ'.AD x AB

$\tau = \tau'$

It is clear from the above equation that set of shear stresses is always accompanied by a transverse set of shear stresses of the same intensity.

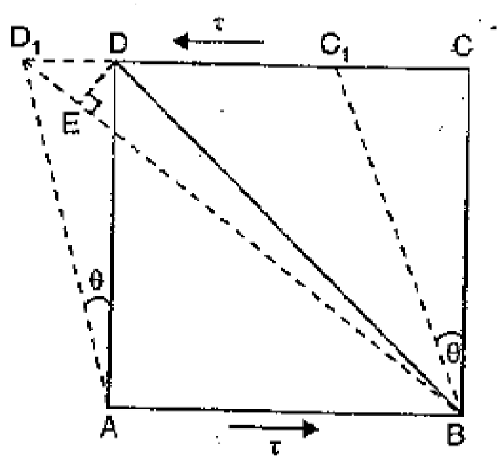
The stress τ' is known as complementary shear stress and the two stresses (τ and τ') at right angles together constitute a state of simple shear.

Due to the formation of couples, the diagonal BD will be in tension and diagonal AC will be in compression.

- **DIRECT (TENSILE AND COMPRESSIVE) STRAINS OF THE DIAGONALS**

From the Principle of complementary shear stresses, we came to know that diagonal BD will be in tension & diagonal AC will in compression.

Due to this, the diagonal BD will be elongated and the diagonal AC will be shortened.



Considering the joint effect of the stresses (tensile & compression) on the diagonal BD.

Tensile strain in diagonal BD due to tensile stress τ along BD

$$= \frac{\text{Tensile stress along BD}}{E} = \tau/E$$

Tensile strain in diagonal BD due to compressive stress τ along AC

$= \mu \times \tau/E$

Total tensile strain along diagonal BD

$= \tau/E + \mu \times \tau/E$

$= (\tau/E) \times (1+ \mu)$

Similarly the total compressive strain in diagonal AC is

$= (\tau/E) \times (1+ \mu)$

The total tensile strain in the diagonal BD is equal to half the shear strain

Tensile strain in diagonal = 1/2 x Shear strain.

- **RELATION BETWEEN YOUNGS MODULUS (E) & MODULUS OF RIGIDITY (G)**

From the previous article, the diagonal strain due to shear stress τ is given by

$= (\tau/E) \times (1+ \mu)$

And we have total tensile strain in diagonal BD

$= \frac{1}{2}$ x Shear strain $= \frac{1}{2}$ x Shear stress/G

$= \frac{1}{2} \times \tau/G$ (Shear stress/shear strain = G) & (Shear stress = τ)

Equating two tensile strains along BD gives

$$(\tau/E) \times (1+ \mu) = \frac{1}{2} \times \tau/G$$

$$\frac{1}{E}(1+\mu) = \frac{1}{2G}$$

$$G = \frac{E}{2(1+\mu)}$$

$$E = 2G\,(1+\mu)$$

- **RELATION BETWEEN E, G AND K**

The relationship between E, G and K can be easily determined by eliminating μ from the derived relations

E = 2 G (1 +μ) and E = 3 K (1 - 2μ)

Thus, the following relationship may be obtained

$$E = \frac{9GK}{(3K+G)}$$

UNIT – II

SHEAR FORCE AND BENDING MOMENT

Beam: A beam is a structural member used for bearing loads. It is typically used for resisting vertical loads, shear forces and bending moments.

- **Types of Beams:**

Beams can be classified into many types based on four main criteria. They are as follows:

1. Based on geometry:
   1. Straight beam – Beam with straight profile
   2. Curved beam – Beam with curved profile
   3. Tapered beam – Beam with tapered cross section
2. Based on the shape of cross section:
   1. I-beam – Beam with 'I' cross section
   2. T-beam – Beam with 'T' cross section
   3. C-beam – Beam with 'C' cross section
   4. Rectangular Beam
3. Based on equilibrium conditions:
   1. Statically determinate beam – For a statically determinate beam, equilibrium conditions alone can be used to solve reactions.
   2. Statically indeterminate beam – For a statically indeterminate beam, equilibrium conditions are not enough to solve reactions. Additional deflection equations are needed to solve reactions.
4. Based on the type of support:

   1. Simply supported beam 2. Cantilever beam 3.Overhanging beam

   4. Continuous beam 5. Fixed beam

Classification of beams based on the type of support is discussed in detail below:

1. Simply supported beam:

A simply supported beam is a type of beam that has pinned support at one end and roller support at the other end. Depending on the load applied, it undergoes shear and bending. It is the one of the simplest structural elements in existence.

At roller support: It has no Bending moment, free to rotate, horizontal displacement is possible, only vertical reaction.

The following image illustrates a simply supported beam.

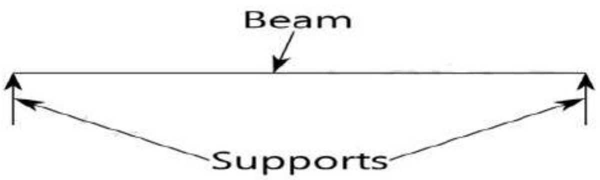


Fig.3.1 Simply Supported Beam (SSB)

2. Cantilever beam:

A cantilever beam is fixed at one end and free at other end. It can be seen in the image below.

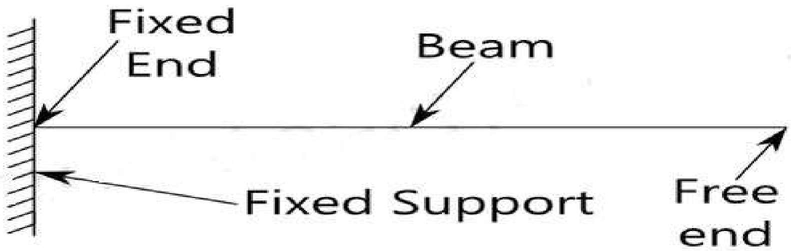


Fig.3.2 Cantilever Beam

3. Overhanging beam:

A overhanging beam is a beam that has one or both end portions extending beyond its supports. It may have any number of supports. If viewed in a different perspective, it appears as if it is has the features of simply supported beam and cantilever beam.

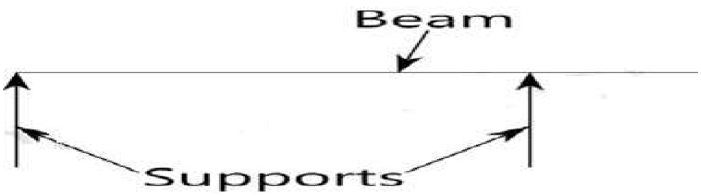


Fig.3.3 Overhanging Beam

4. Continuous beam:

A continuous beam has more than two supports distributed throughout its length. It can be understood well from the image below.

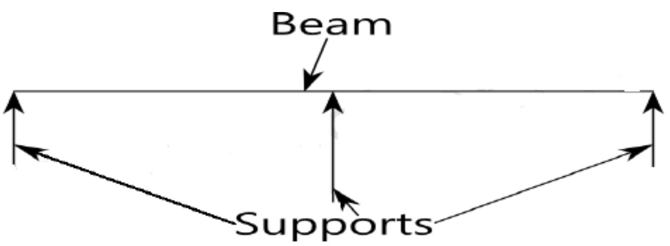


Fig.3.4 Continuous Beam

5. Fixed beam: As the name suggests, fixed beam is a type of beam whose both ends are fixed.

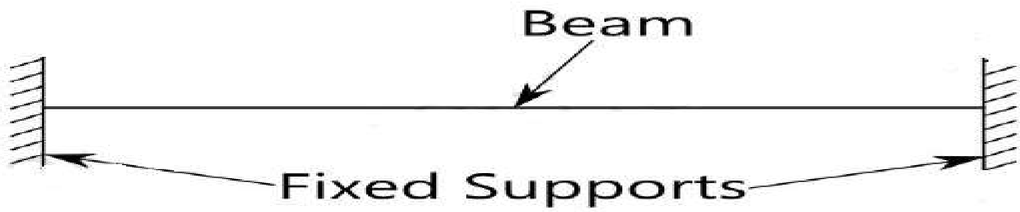
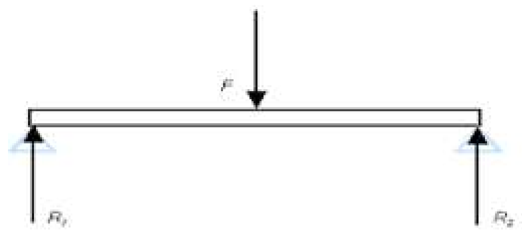


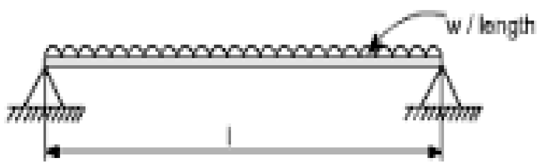
Fig.3.5 Fixed Beam

➢ **Types of Loads:**

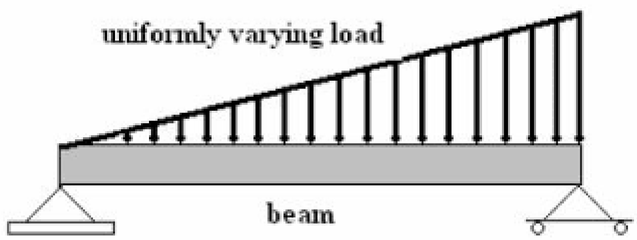
Concentrated or Point load:- A concentrated load is one which is considered to act at a Point.



Uniformly Distributed Load:-It is a type of load which is distributed uniformly over the entire length of the beam

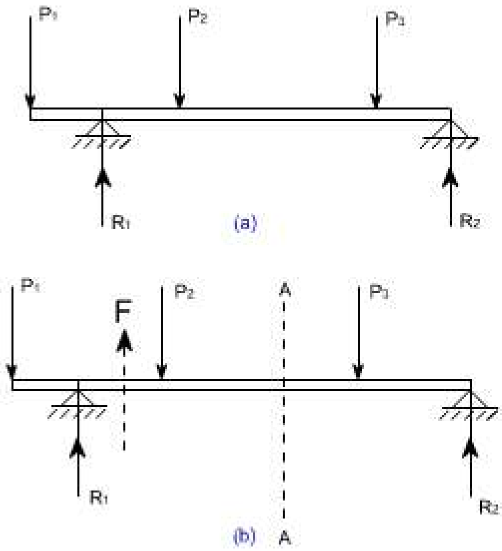


Uniformly Varying Load:- These are the loads varying uniformly from zero to a particular value and spread over a certain length of the beam. Such load is also called as Triangular Load.



- **Concept of Shear force and Bending moment**

When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the terms shear force and bending moments come into pictures which are helpful to analyze the beam further. Let us define these terms

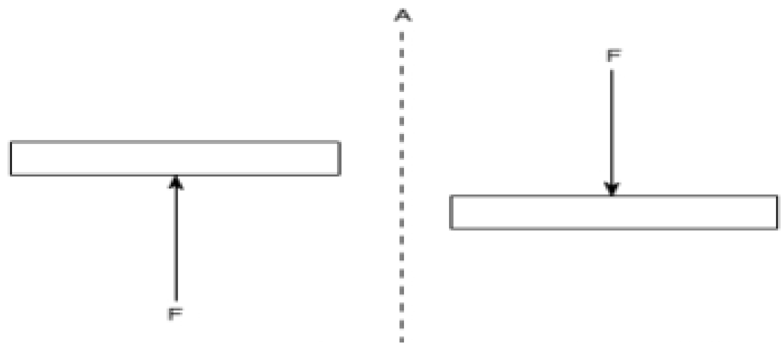


Now let us consider the beam as shown in fig(a) which is supporting the loads P_1, P_2 P_3 and is simply supported at two points creating R_1 and R_2 respectively. Now let us assume that the beam is to divided into or imagined to be cut into two portions at a section AA. Now let us assume the resultant of loads and reactions to the left of AA is F vertically upwards, and Since the entire beam is to remain in equilibrium, thus the resultant of forces to the right of AA must also be F, acting downwards. This force F is as a Shear force. The shearing force at any X-section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

Therefore, Shear force F is define as

At any X-section of a beam, the shear force F is the algebraic sum of all the lateral components of the forces acting on either side of the X-section.

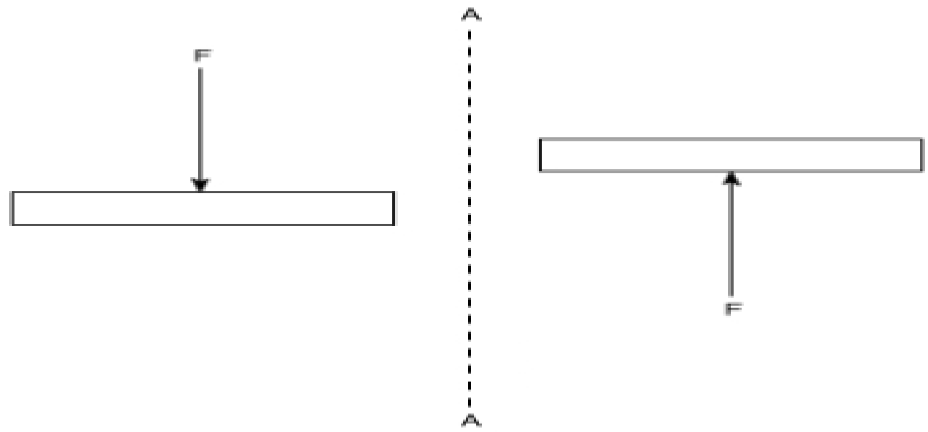
Sign convention for shear force:-



The resultant force which is in upward direction and is towards the L.H.S of the X-section is +ve shear force

The resultant force which is in the downward direction and is towards the R.H.S of the X-section is +ve shear force

Positive shear force



The resultant force which are in the downward direction and is on the L.H.S of the X-section is -ve shear force

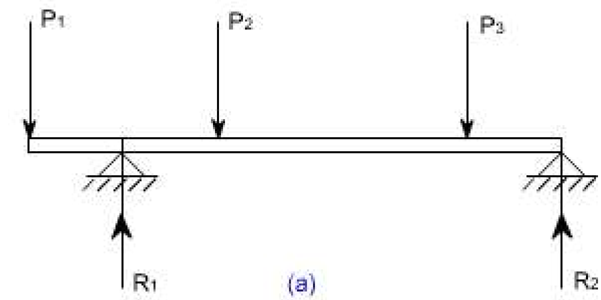
The resultant force which are in the upward direction and is on the R.H.S of the X-section is -ve shear force

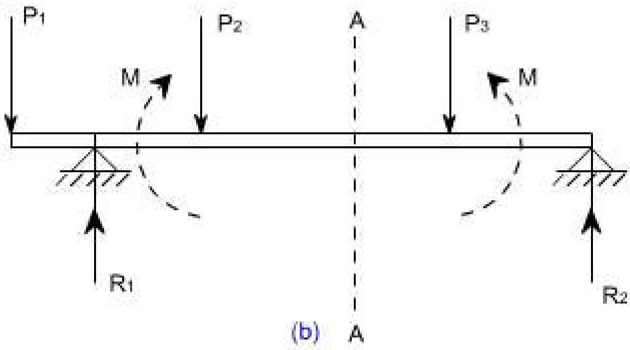
Negative shear force

Bending Moment

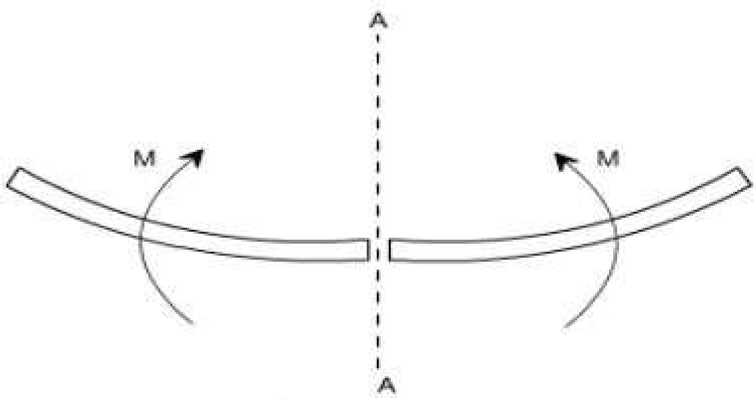
Let us again consider the beam which is simply supported at the two prints, carrying loads P1, P2 and P3 and having the reactions R1 and R2 at the supports Fig 4. Now, let us imagine that the beam is cut into two potions at the x-section AA. In a similar manner, as done for the case of shear force, if we say that the resultant moment about the section AA of all the loads and reactions to the left of the x-section at AA is M in C.W direction, then moment of forces to the right of x-section AA must be 'M' in C.C.W. Then 'M' is called as the Bending moment and is abbreviated as B.M.

Bending moment can be defined as the algebraic sum of the moments about an x-section of all the forces acting on either side of the section



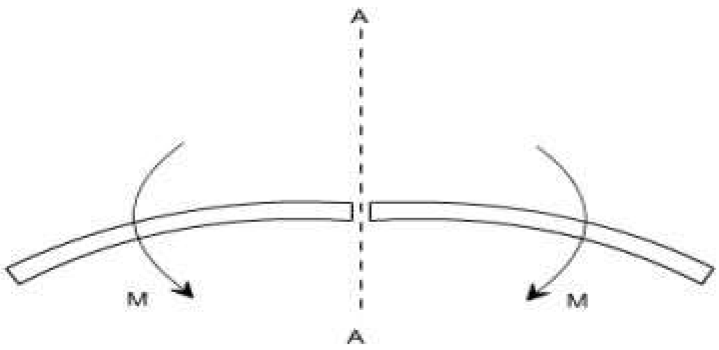


Sign convention for Bending Moment:-



Resultant moment on the L.H.S of the X-section is C.W, then it is a Positive bending moment	Resultant moment on the R.H.S of the X-section is C.C.W, then it may be as considered as positive B.M

Positive Bending Moment



Resultant moment on the L.H.S of the X-section is C.C.W, then it is a negative B.M

Resultant moment on the R.H.S of the X-section is C.W, then it is a negative B.M

Negative Bending Moment

Some times, the terms 'Sagging' and Hogging are generally used for the positive and negative bending moments respectively.

- **Bending Moment and Shear Force Diagrams:**

The diagrams which illustrate the variations in B.M and S.F values along the length of the beam for any fixed loading conditions would be helpful to analyze the beam further.

Thus, a shear force diagram is a graphical plot, which depicts how the internal shear force 'F' varies along the length of beam. If x dentotes the length of the beam, then F is function x i.e. F(x).

Similarly a bending moment diagram is a graphical plot which depicts how the internal bending moment 'M' varies along the length of the beam. Again M is a function x i.e. M(x).

Cantilever beam with point load at the end:

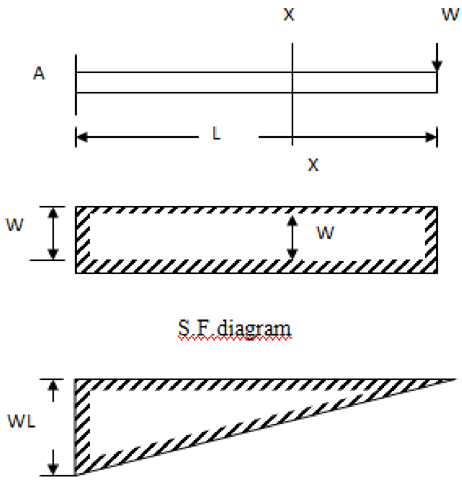
Take a section X-X from B.

S.F at distance x, $F_x = +w$

B.M at distance x, $M_x = -wx$.

At x=0, $M_B = 0$

At x=l, $M_A = -wl$



S.F diagram

B.M diagram

Cantilever beam with u.d.l :

Take a section X-X from B.

S.F at distance x, F_x= +wx

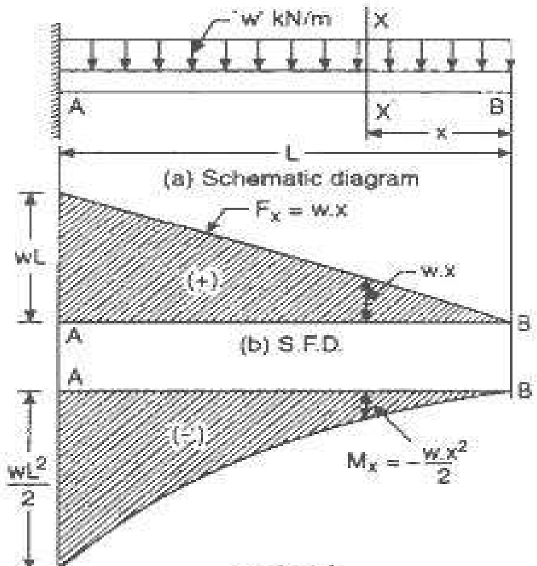
At x=0, F_B= **0**

At x=l, F_A= **wl**

B.M at distance x, $M_x = -\mathbf{wx}.\frac{x}{2}$

At x=0, M_B= **0**

At x=l, M_A= **-wl.** $\frac{l}{2} = \frac{wl^2}{2}$



Simply supported beam with point load:

Reaction at each support=$\frac{w}{2}$

$$R_A = R_B = \frac{w}{2}$$

For any section between A and C

$$\text{S.F } S_x = +\frac{w}{2}$$

For any section between C and B

$$\text{S.F } S_x = -\frac{w}{2}$$

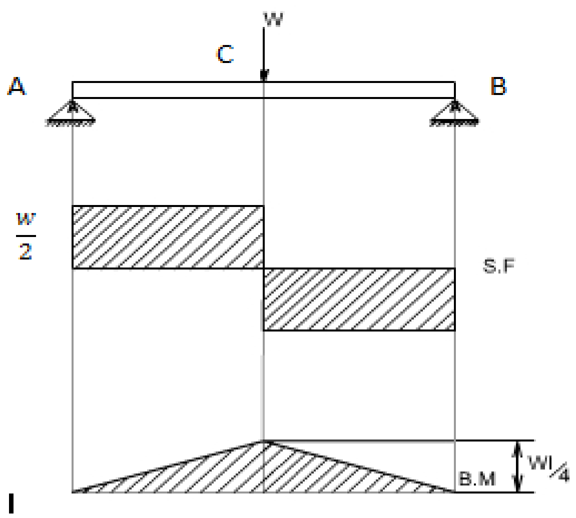
At the section C the S.F changes from $+\frac{w}{2}\ to\ -\frac{w}{2}$

The B.M is given by

$M_x = +\frac{w}{2}x$

At x=0, $M_x = 0$

At $x=\frac{l}{2}$, $M_x = +\frac{w}{2}\times\frac{l}{2} = \frac{wl}{4}$



Simply supported beam with U.D.L:

$$R_A = R_B = \frac{wl}{2}$$

Consider any section x at a distance x from the left end A.

S.F $S_x = R_a - wx = \frac{wl}{2} - \text{wx}$

At x=0, $S_x = \frac{wl}{2}$

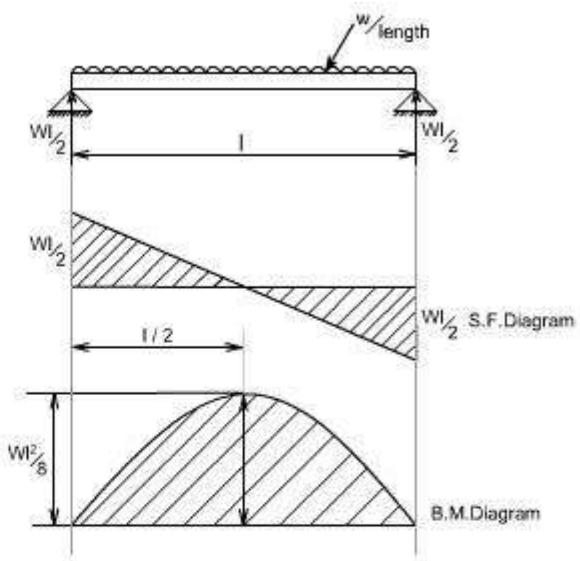
At x=l, $S_x = -\frac{wl}{2}$

B.M $M_x = \frac{wl}{2}x - \frac{wx^2}{2}$

At x=0, $M_x = 0$

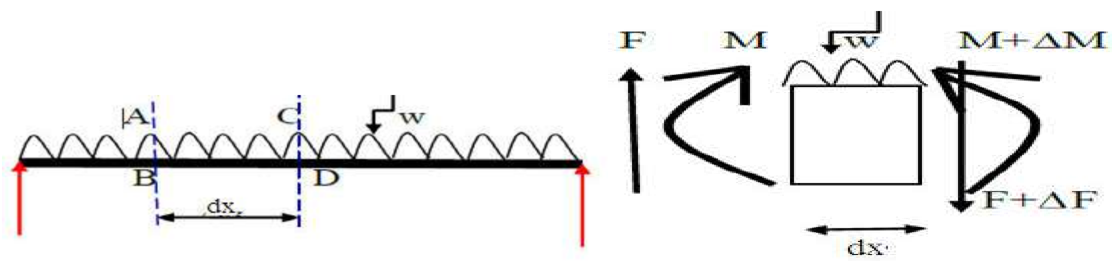
At x=l, $M_x = 0$

At $x=\frac{l}{2}$, $M_x = \frac{wl^2}{8}$



➢ **Relations between load, Shear force and Bending moment:-**

A simply supported beam carrying a uniformly distributed load of w/unit length. Consider the equilibrium of the portion of the beam between the sections AB and CD. This portion is at a distance of x from left support and is of length dx.



Let F= Shear force at section AB.

F+dF= Shear force at section CD.

M = Bending moment at section AB.

M+dM= Bending moment at section CD.

The forces and moments acting on the length dx of the beam are:

(i) The force F acting vertically up at section AB.

(ii) The force F+dF acting vertically downwards at CD.

(iii) The load wdx acting downwards.

(iv) The moment M and (M+dM) acting at section AB and CD respectively.

The portion of the beam of length dx is in equilibrium.

Hence resolving the forces acting on this part vertically.

$$F - wdx - (F + dF) = 0$$

$$-dF = wdx \qquad \boxed{\frac{dF}{dx} = -w}$$

The above equation shows that the rate of change of shear force is equal to the rate of loading.

Now taking the moments of the forces and couples about the section CD.

$$M - wdx\frac{dx}{2} + Fdx = M + dM$$

$$-\frac{wdx^2}{2} + Fdx = dM \quad \text{(Neglecting the higher power of small quantities)}$$

$$F\,dx = dM$$

$$\boxed{\frac{dM}{dx} = F}$$

The above equation shows that the rate of change of bending moment is equal to the shear force at the section.

Variation of SF and BM along the beam

S.No	Type of Loading	Variation of SF	Variation of BM
1	Point load	Rectangle	Inclined
2	UDL	Linear	parabola
3	UVL or Triangle load	Parabola	cubic
4	Parabolic	Cubic	Fourth degree Polynomial
5	couple	No shear variation	Vertical step at the point of application

Note: 1. At internal hinge Bending Moment is Zero

2. The Point where the bending moment changes its sign is known as point of contra-flexure.

Max. SF and Max BM values

S No	Type of loading and type of beam	Max SF value	Location	Max BM	Location
1	**Cantilever with point load W at its free end**	**W**	**At support**	**Wl**	**At support**
2	**Cantilever with UDL w kN/m over the entire length**	**wl**	**At support**	$Wl^2/2$	**At support**

3	**Simply supported beam W at its center**	**W/2**	**At support**	**Wl/4**	**At mid span**
4	**Simply supported beam w kN/m over the entire length**	**Wl/2**	**At support**	**$Wl^2/8$**	**At mid span**

UNIT –III
FLEXURE, DIRECT AND BENDING STRESSES

Objective: To familiarize with flexural stresses and Direct & bending stresses developed for different sections of beams.

Syllabus: FLEXURE, DIRECT AND BENDING STRESSES

<u>Flexure stresses:</u>

Concept of theory of simple bending - Assumptions

Derivation of bending equation, $M / I = \sigma / y = E / R$

Neutral axis, determination of bending stress - section modulus – rectangular , circular (solid and hollow), I, L , T and channel sections

Simple problems on bending stresses

<u>Direct and Bending stresses :</u>

Stresses under the combined action of direct loading and bending moment.

Kern of a section

Determination of stresses under eccentric loads

Conditions of stability – Middle third rule – Application to retaining walls – Intensity of pressure diagram below foundation

Introduction:

A beam is a structural member whose length is large compared to its cross sectional area which is loaded and supported in the direction transverse to its axis. Lateral loads acting on the beam cause the beam to bend or flex, thereby deforming the axis of the beam into a curved line. The bending is resisted by the internal resistance set up by the cross-section of the beam. The process of bending will stop when it has set up full resistance to the bending moment and the shearing force. The stresses produced at the section to resist the bending moment are known as the **bending stress** or **longitudinal stress** or **flexural stress**. The bending moment at any section represents the resultant moment, called the moment of resistance, of internal stresses distributed over the section.

Pure Bending Assumptions

1. Material of beam is homogenous and isotropic.
2. Young's modulus is constant in compression and tension.
3. Transverse sections which are plane before bending remain plane after bending.
4. Beam is initially straight and all longitudinal filaments bend in circular arcs.
5. Radius of curvature is large compared with dimension of cross sections.
6. Each layer of the beam is free to expand or contract.

Concept of pure bending:

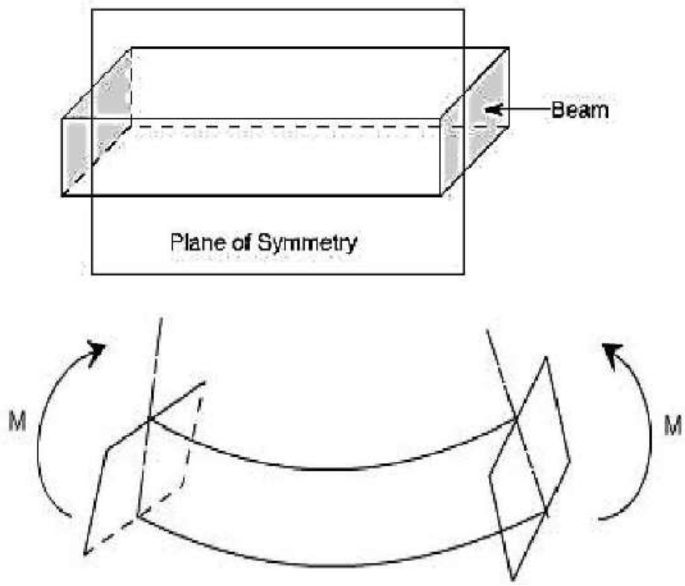
Loading restrictions:

The internal reactions developed on any cross-section of a beam may consists of a resultant normal force, a resultant shear force and a resultant couple. In order to ensure that the bending effects alone are investigated, we shall put a constraint on the loading such that the resultant normal and the resultant shear forces are zero on any cross-section perpendicular to the longitudinal axis of the member.

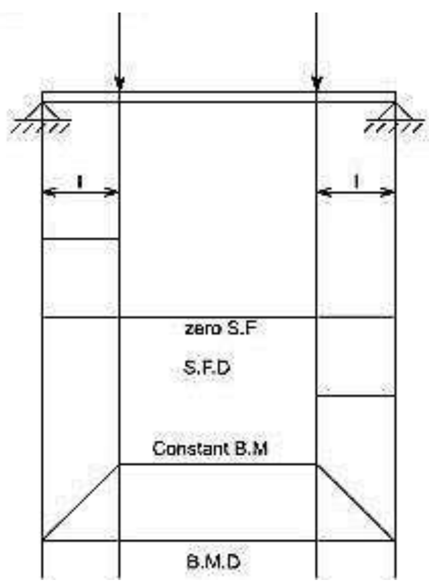
That means F = 0

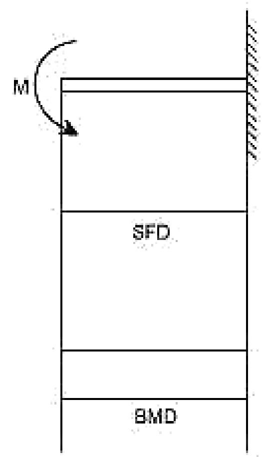
Since $\frac{dM}{dx} = F = 0$ or M= constant

Thus, the zero shear force means that the bending moment is constant or the bending is same at every cross-section of the beam. Such a situation may be visualized when the beam or some portion of the beam has been loaded only by pure couples at its ends. It must be recalled that the couples are assumed to be loaded in the plane of symmetry.



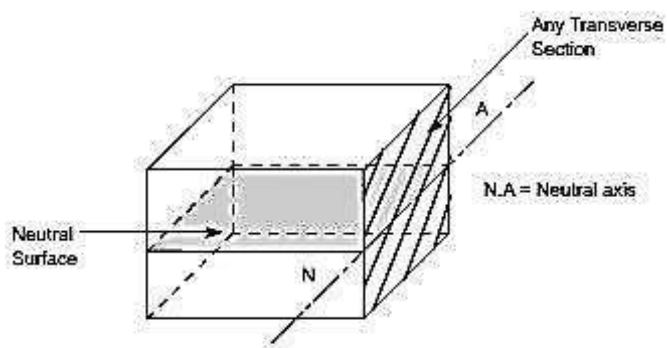
When a member is loaded in such a fashion it is said to be in **pure bending.** The examples of pure bending have been shown below





When a beam is subjected to pure bending are loaded by the couples at the ends, certain cross-section gets deformed and we shall have to make out the conclusion that,

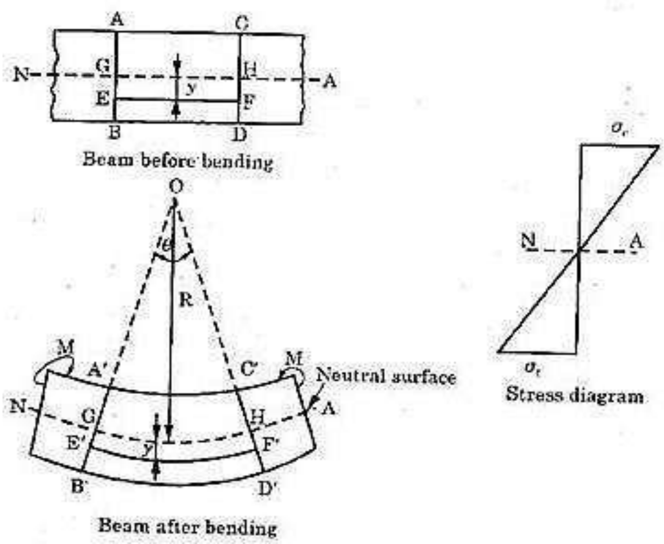
- Plane sections originally perpendicular to longitudinal axis of the beam remain plane and perpendicular to the longitudinal axis even after bending , i.e. the cross-section do not get warped or curved.
- In the deformed section, the planes of this cross-section have a common intersection i.e. any time originally parallel to the longitudinal axis of the beam becomes an arc of circle.



We know that when a beam is under bending the fibres at the bottom will be lengthened while at the top will be shortened provided the bending moment M acts at the ends. In between these there are some fibres which remain unchanged in length that is they are not strained, that is they do not carry any stress. The plane containing such fibres is called neutral surface. The line of intersection between the neutral surface and the transverse exploratory section is called the Neutral axis **(N A)**.

Bending Stresses in Beams or Derivation of Elastic Flexural formula :

In order to compute the value of bending stresses developed in a loaded beam, let us consider the two cross-sections of a beam **AB** and **CD**, originally parallel as shown in fig.



when the beam is to bend it is assumed that these sections remain parallel i.e. **A'B'** and **C'D'** , the final position of the sections, are still straight lines, they then subtend some angle θ.

Consider now fiber EF in the material, at a distance y from the N.A, when the beam bends this will stretch to E'F'

Therefore,

Strain in the layer EF=$\frac{change\ in\ length}{original\ length} = \frac{y\times\theta}{EF} = \frac{y\times\theta}{R\times\theta} = \frac{y}{R}$

The above equation shows the variation of strain along the depth of the beam.

Stress variation

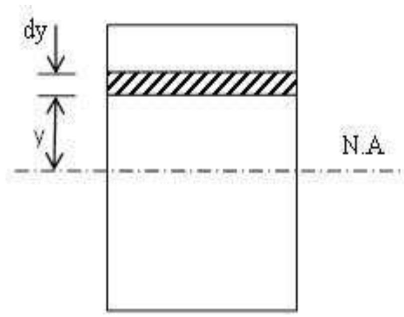
E= $\frac{stress\ in\ the\ layer\ EF}{Strain\ in\ the\ layer\ EF} = \frac{\sigma}{\frac{y}{R}}$

$$\sigma = E \times \frac{y}{R}$$

The above equation can also be written as

$$\frac{\sigma}{y} = \frac{E}{R}$$

Now, Consider any arbitrary cross-section of beam, as shown below.



Now the stress on a fibre at a distance 'y' from the N.A, is given by the expression

$$\sigma = E \times \frac{y}{R}$$

if the shaded strip is of area 'dA' then the force on the strip is

F= $\sigma \times dA = E \times \frac{y}{R} \times dA$

Moment about the neutral axis = F. y = $E \times \frac{y}{R} \times dA \times y$

The total moment for the whole cross-section is therefore equal to

$$M = \int \frac{E}{R} \times dA \times y^2$$

But the expression $\int dA \times y^2$ represents the moment of inertia of the area of the section about the neutral axis. Let this moment of inertia be I.

$$M = \frac{E}{R} \times I$$

$$\frac{M}{I} = \frac{E}{R}$$

From equation we have

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

The above equation is known as bending equation.

Section modulus (Z)

Another property used in beam design is **section modulus (Z)**.The section modulus of the cross-sectional shape is of significant importance in designing beams. It is a direct measure of the **strength** of the beam. A beam that has a larger section modulus than another will be stronger and capable of supporting greater loads.

It is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outer most layer from the neutral axis.

$$Z = I/y_{max}$$

The stress will be maximum when y is maximum

Hence the above equation can be written as

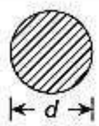
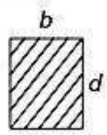
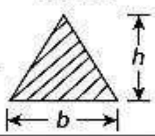
$$\frac{\sigma_{max}}{y_{max}} = \frac{M}{I}$$

$$M = \sigma_{max} \times \frac{I}{y_{max}}$$

$$M = \sigma_{max} \times Z$$

In the above equation, M is the maximum bending moment. Hence moment of resistance offered by the section is maximum when section modulus is maximum. Hence section modulus represents the strength of the section.

- **Section modulus of different sections**

Section	I_{NA}	y_1	y_2	Z_1	Z_2
Circular 	$\frac{\pi d^4}{64}$	$\frac{d}{2}$	$\frac{d}{2}$	$\frac{\pi d^3}{32}$	$\frac{\pi d^3}{32}$
Rectangular 	$\frac{bd^3}{12}$	$\frac{d}{2}$	$\frac{d}{2}$	$\frac{bd^2}{6}$	$\frac{bd^2}{6}$
Triangular 	$\frac{bh^3}{36}$	$\frac{h}{3}$	$\frac{2h}{3}$	$\frac{bh^2}{12}$	$\frac{bh^2}{24}$

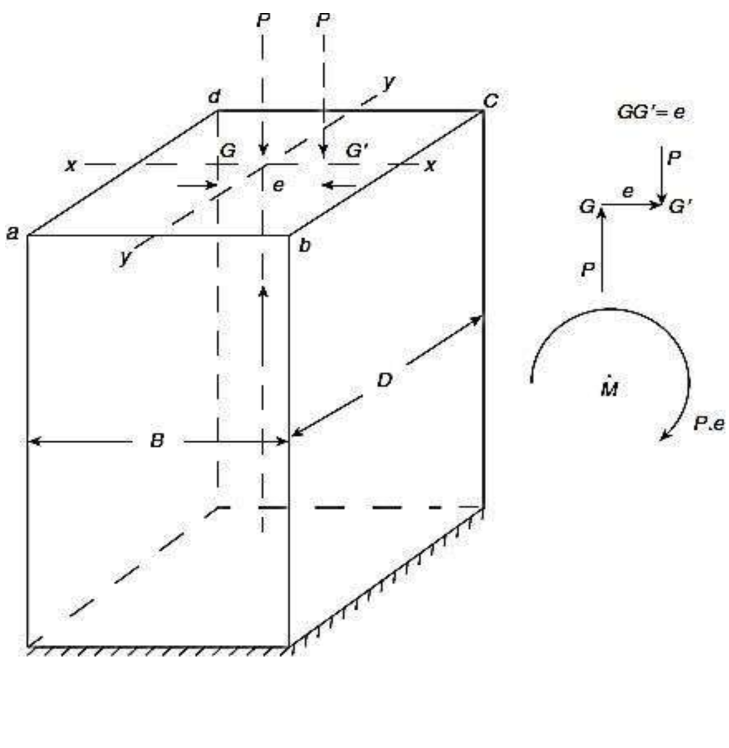
Combined Bending and Direct Stresses

Direct stress alone is produced in a body when it is subjected to an axial tensile or compressive load. Bending stress is produced in the body, when it is subjected to a bending moment. But if a body is subjected to axial loads and bending moments, then both the stresses will be produced in the body. Both these stresses act normal to a cross-section, hence two stresses may be algebraically added into a single resultant stress.

Eccentric Axial Thrust on a Column

A column of rectangular section $B \times D$ is shown in figure. G is the centroid of the section *abcd.* A vertical load P is applied at G along xx axis such that $GG' = e$, eccentricity.

If this load acts at the CG of the section, then a direct compressive stress is developed in section. Effect of the eccentric load is to produce bending moment, $M = Pe$, on the section producing tensile and compressive stresses in the section. Along the edge *ad*, there will be maximum tensile stress due to bending and along edge *bc*, there will be maximum compressive stress due to bending.



Say, I_{yy} = moment of inertia of section about yy axis = $\frac{DB^3}{12}$

Direct compressive load = P

Direct stress, $\sigma_d = \frac{P}{BD}$ (compressive)

Section modulus, $Z_y = \frac{I_{yy}}{\frac{B}{2}} = \frac{DB^2}{6}$

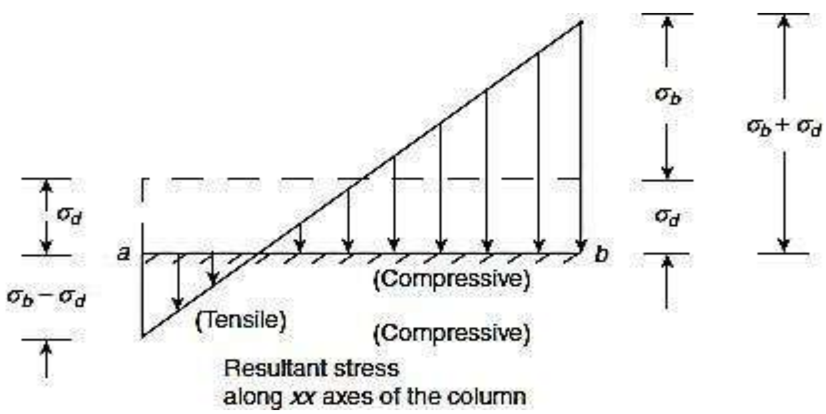
Maximum bending stresses developed in section,

$$\sigma_b = \pm \frac{Pe}{Z_y} = \pm \frac{6Pe}{DB^2}$$

Resultant stress at edge bc = $\frac{P}{BD} + \frac{6Pe}{DB^2}$ (compressive)

Resultant stress at edge ad = $\frac{P}{BD} - \frac{6Pe}{DB^2}$

If $\sigma_b > \sigma_d$, then resultant stress at edge *ad* will be tensile. Resultant stress distribution along *xx* axis is shown in the figure.



Along edge bc

$$\sigma_b + \sigma_d = \frac{P}{BD} + \frac{6Pe}{DB^2}$$

Along edge ad

$$\sigma_b - \sigma_d = \frac{6Pe}{DB^2} - \frac{P}{BD}$$

If $\sigma_b - \sigma_d = 0$, no tensile stress is developed along edge ad

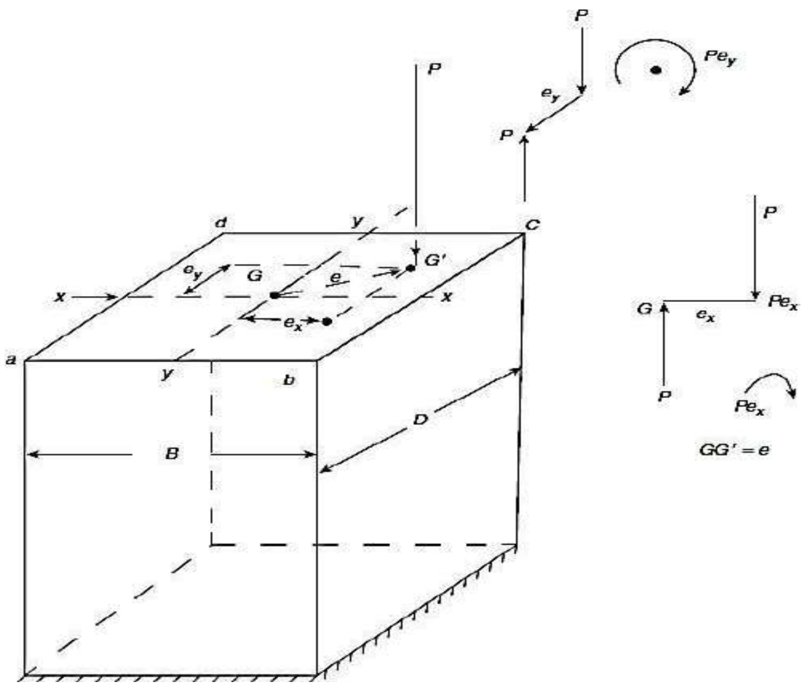
Therefore,

$$\frac{6Pe}{DB^2} < \frac{P}{BD}$$

$e < \frac{B}{6}$, eccentricity must be less than $\frac{B}{6}$ for no tensile stress to develop in section, when load is applied along *xx* axis.

Load Eccentric to Both Axes (Rectangular Section)

Consider a column of rectangular section B × D as shown in figure. Centroid of the section abcd is at G but a vertical load P is applied at G′ such that GG′ = e, eccentricity. There are two components of this eccentricity along *xx* and *yy* axes, that is, e_x and e_y as shown is the figure.



Direct vertical load on column = P

Bending moment in *xx* plane, $M_x = Pe_x$

Bending moment in *yy* plane, $M_y = Pe_y$

Section modulus, $Z_y = \frac{DB^2}{6}$

Section modulus, $Z_x = \frac{BD^2}{6}$

Bending stresses along *xx* axis, edges *ad* and *bc* , $= \pm\sigma_{b1} = \pm\frac{Pe_x}{Z_y} = \pm\frac{6Pe_x}{DB^2}$

(Compressive along edge *bc* and tensile along edge *ad*)

Bending stresses along *yy* axis, $\pm\sigma_{b2} = \pm\frac{6Pe_y}{BD^2}$

(Compressive along edge *cd* and tensile along edge *ab*)

Direct compressive stress, $\sigma_d = \frac{P}{BD}$

Resultant stresses at corners

$$\sigma_a = \sigma_d - \sigma_{b_1} - \sigma_{b_1} = \frac{P}{BD} - \frac{6Pe_x}{DB^2} - \frac{6Pe_y}{BD^2}$$

$$\sigma_b = \sigma_d + \sigma_{b_1} - \sigma_{b_1} = \frac{P}{BD} + \frac{6Pe_x}{DB^2} - \frac{6Pe_y}{BD^2}$$

$$\sigma_c = \sigma_d + \sigma_{b_1} + \sigma_{b_1} = \frac{P}{BD} + \frac{6Pe_x}{DB^2} + \frac{6Pe_y}{BD^2}$$

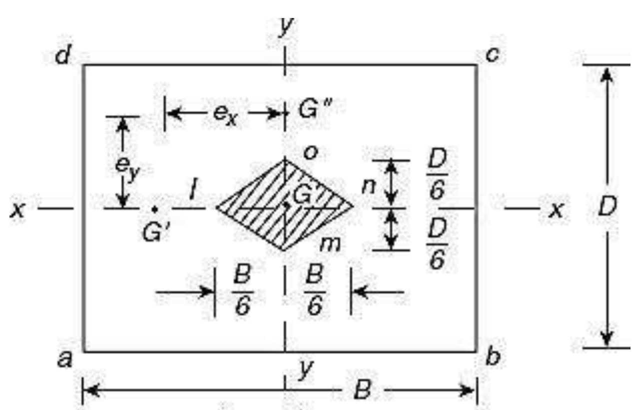
$$\sigma_d = \sigma_d - \sigma_{b_1} + \sigma_{b_1} = \frac{P}{BD} - \frac{6Pe_x}{DB^2} + \frac{6Pe_y}{BD^2}$$

➢

Core of Rectangular Section (Middle Third rule)

The core of a rectangular section is the part of the column section in which the load can be applied without causing tensile stress anywhere in the section. Since the columns are made of cast iron, brick work, concrete etc., which are not supposed to withstand tensile stresses because tensile stress may develop cracks in column leading to its failure.

Consider a rectangular section $B \times D$ of a column. Say a load P is applied at point G', such that $GG' = e_x$



Imno-Core

Bending moment, $M_x = Pe_x$

Section modulus, $Z_y = \frac{DB^2}{6}$

Stress due to bending, $\sigma_{bx} = \frac{Pe_x}{z_y} = \pm \frac{6Pe_x}{DB^2}$

Direct stress, $\sigma_d = \frac{P}{BD}$ (compressive)

If the resultant stresses $\sigma_d \pm \sigma_{bx}$ has to be only compressive and no tensile stress is permitted, then

$$\sigma_{bx} < \sigma_d$$

$$\frac{6Pe_x}{DB^2} < \frac{P}{BD}$$

Or $e_x < \frac{B}{6}$

Similarly, load can be considered on the other side of G, and $e_x < \frac{B}{6}$

In the figure, dimension ln = $\frac{B}{6} + \frac{B}{6} = \frac{B}{3}$

If the load is applied within the line ln along the x axis, then there will not be any tensile stress anywhere in the setion.

Now, Eccentricity along *yy* axis = e_y

Bending moment, $M_y = Pe_y$

Stress due to bending, $Z_x = \frac{BD^2}{6}$

Bending stress $\sigma_{by} = \pm\frac{6Pe_y}{BD^2}$

Resultant stress $\sigma_d \pm \sigma_{by} = \frac{P}{BD} \pm \frac{6Pe_y}{BD^2}$

If no tensile stress is permitted in the section, then

$$\sigma_{by} < \sigma_d$$

$$\frac{6Pe_y}{BD^2} < \frac{P}{BD}$$

$$e_y < \frac{D}{6}$$

Line mo shows that mo = $\frac{D}{6} + \frac{D}{6} = \frac{D}{3}$

If load is applied anywhere within line *mo*, no tensile stress is produced in the section. Joining the ends *l, m, n* and *o* makes a rhombus of diagonals *B*/3 and *D*/3.*This rhombus is termed as core or kernal* of a rectangular section

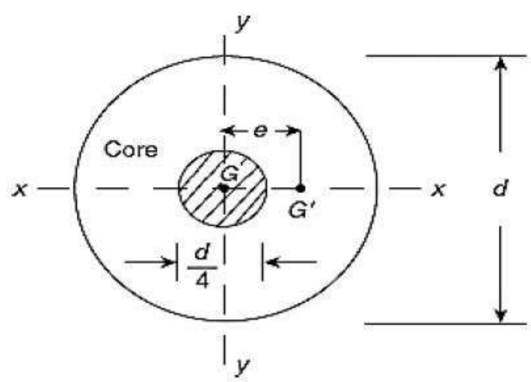
Core of circular section

Let us consider that the section of a column is circular of diameter d as shown in fig xx and yy are centroidal axes, with G as centroid of the section. Say load P applied along xx axis at G', such that G G'=e, eccentricity

Bending moment M_x= Pe

Section modulus $Z_y = \frac{\pi d^3}{32}$

Direct stress $\sigma_d = \frac{4P}{\pi d^2}$



Bending stress $\sigma_b = \pm \frac{32Pe}{\pi d^3}$

If the resultant stress is not to be tensile anywhere in the section, then

$$\sigma_b < \sigma_d$$

$$\frac{32\text{Pe}}{\pi \text{d}^3} < \frac{4\text{P}}{\pi \text{d}^2}$$

Or $e < \frac{D}{8}$

Similarly load can be considered along yy axis and on the other side of the yy axis, the

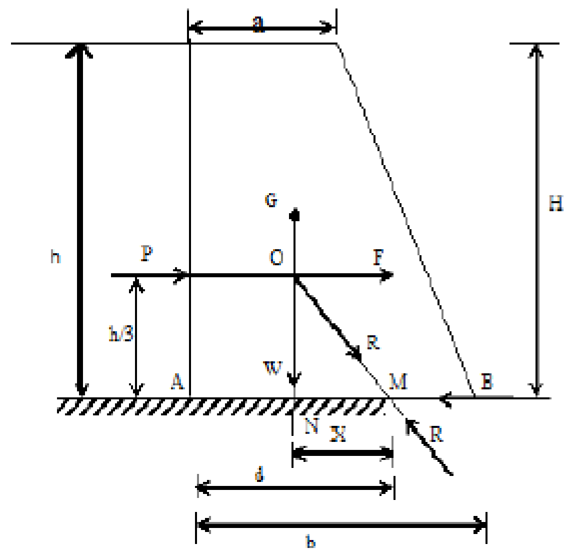
$$e < \frac{D}{8}$$

Core of the circular section is a circle of radius d/8 or diameter d/4 as shown in fig. Area covered by a circle of diameter d/4 is called core or kernel of the cicular section. If a load on column applied within the core, then no tensile stresses will be developed anywhere in the section.

Retaining wall

A retaining wall is a structure that holds or retains soil behind it. The earth, retained by a retaining wall, exerts pressure on the retaining wall.

A retaining wall is a structure designed and constructed to resist the lateral pressure of soil when there is a desired change in elevation that exceeds the angle of repose of the soil.



Let a = top width of the wall

b = bottom width of the wall

w=weight density of earth

wo = weight density of masonry

P= lateral thrust of the soil

W = weight of the wall

The thrust of earth on the vertical face of the retaining wall $P = \frac{1}{2} wh^2 \frac{1-sin\emptyset}{1+sin\emptyset}$

Where Ø= Angle of repose

The horizontal force P acts at a height of h/3 above the base.

The horizontal force P acts at a height of h/3 above the base.

Weight of the wall $W = w_o \frac{a+b}{2} h$

The weight W will be acting through the C.G. of the trapezoidal section. The distance of the C.G. of the trapezoidal section from the point A is

$\text{AN} = \frac{a^2+ab+b^2}{3(a+b)}$

The horizontal distance x between the line of action of W and the point at which the resultant force R cuts the base, $x = \frac{P}{W} \times \frac{h}{3}$

Distance between A and the point M where the resultant cuts the base d= AN + x

Eccentricity of the vertical component W is equal to distance NM

$e = d - b/2$

Total stress across the base of the wall at point B, $\sigma_{max} = \frac{W}{A}\left(1 + \frac{6e}{b}\right)$

Total stress across the base of the wall at point A, $\sigma_{min} = \frac{W}{A}\left(1 - \frac{6e}{b}\right)$

Dams

A dam is a structure which is constructed to store water. A retaining wall is constructed to retain the earth in hilly area.

Types of Dams

Rectangular dams

Trapezoidal dams

a. Water face vertical

b. Water face inclined

Rectangular dams

Consider a rectangular dam as shown in figure

Let H=height of dam

b = width of dam

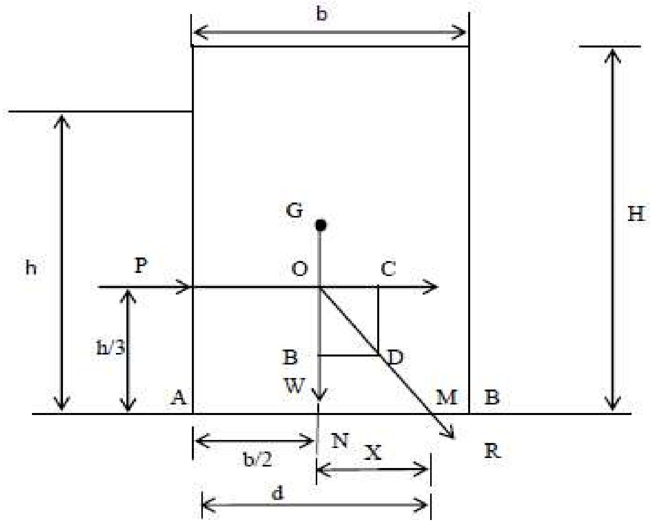
h = height of water

P = Force exterted by water on dam

wo = weight density of dam

Consider 1m length of dam

The forces acting on dam are



1) The force P due to water in contact with the side of dam.

P = w.A.h

= w x(hx1)x h/2

$= \frac{wh^2}{2}$

This force P acts at a distance h/3 from the base.

2) Weight of dam, w= $w_o \times A \times L$

$= w_o$xbxhx1 [length=1m]

$= w_o$b H

This force will be acting downwards through the C.G of the dam.

The resultant 'R' can be determined by considering

parallelogram law of forces.

Let OC = F and OB = W

Now, diagonal OD is the resultant R and is given as,

Resultant, R= $\sqrt{P^2 + W^2}$

Therefore angle made by resultant with vertical is

$$\tan\theta = \frac{BD}{OB} = \frac{P}{W}$$

The horizontal distance between the line of action W and the point through which resultant cuts the base

Let OD is extended to cut the base of the dam at M and OW is extended to cut the base at point N. The distance MN is the horizontal distance between the line of action W and the point through which resultant cuts the base.

Let MN= x

Taking moments about

M P. h/3 = W x

$x = \frac{P}{W} \times \frac{h}{3}$

Stresses across section of the dam

Let d = distance between A and point M, where the resultant cuts the base

= AM = AN+NM

$= \frac{b}{2} + \left(\frac{F}{W} \times \frac{h}{3}\right)$

The resultant meeting at point M can be resolved into vertical and horizontal components.

Let W = vertical component

F = horizontal component

Thus the vertical component 'W' at point M is an eccentric load, as it is not acting at the middle of the base.

We know that the eccentric load produces both direct and bending stresses.

Eccentricity of vertical component 'W' is equal to the distance NM and is equal to 'X' in this case.

Now, e = NM = AM-AN = d-b/2.

Now, Moment at base of the section due to eccentricity, M = W.e

$\sigma_b = \frac{MY}{I}$

I = $\frac{db^3}{12} = \frac{1b^3}{12}$

y = $\pm b/2$

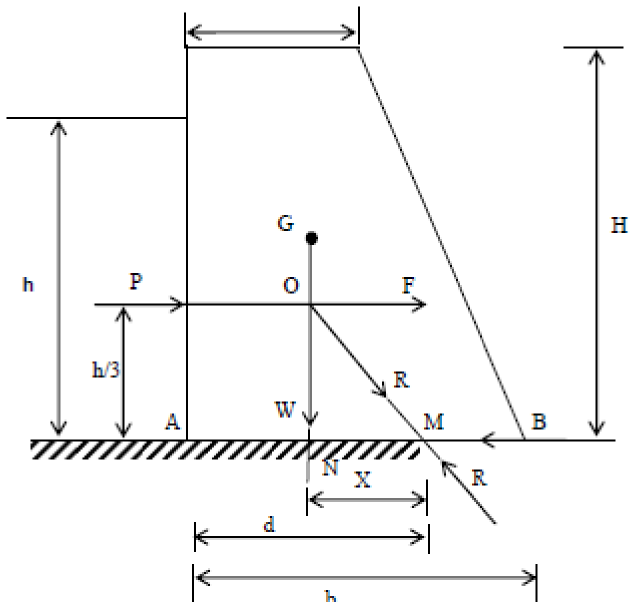
$$\sigma_b = \frac{We}{\frac{1b^3}{12}} \times \left(\pm\frac{b}{2}\right) = \pm\frac{6We}{b^2}$$

$\sigma max = \sigma_d + \sigma_b = \frac{W}{A} + \frac{6We}{b^2} = \frac{W}{b\times1} + \frac{6We}{b^2} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$ occurs at B)

$\sigma_{min}= \sigma_d-\sigma_b=\frac{W}{A}-\frac{6We}{b^2}=\frac{W}{b}\left(1-\frac{6e}{b}\right)$ (occurs at A)

Trapezoidal dam having water face vertical

Consider a dam as shown in figure



We know

Force excerted by water $P=\frac{wh^2}{2}$ acting at $h/3$

Weight of dam, $W=w_o \times A \times L = w_o \frac{a+b}{2} H \times 1 = w_o \frac{a+b}{2} h$

acting vertically downwards through C.G of dam

Distance C.G can be obtained from the formula

Distance C.G = AN = $\frac{a^2+ab+b^2}{3(a+b)}$

Now, x = horizontal distance between line of action of weight of dam and the point where resultant cuts the base.

= Distance MN

x = $\frac{P}{W} \times \frac{h}{3}$

distance d is given by

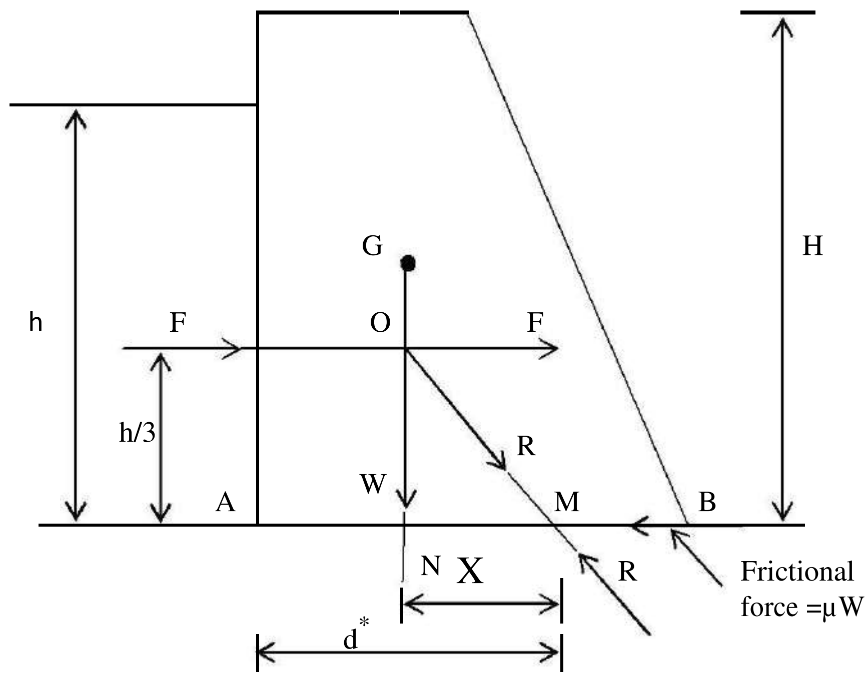
$d=AM=AN+NM$

eccentricity, e= d-b/2.

Total stress across the base of the dam at point B, $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

at point A, $\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

Stability conditions of a dam



A dam should be stable under all conditions, but the dam may fail

1) By sliding on the soil on which it rests

2) By over turning

3) Due to tensile stresses developed

4) Due to excessive compressive stresses

Condition to prevent the sliding of the dam

The figure shows a dam of trapezoidal section of height H and having water upto a depth of h.

The forces acting on the dam are

1) Force to water pressure F acting horizontally at a height of h/3 above the base.

2) Weight of dam W acting vertically downwards through the C.G of the dam.

The resultant R of the forces F and W is passing through the point M. The dam will be in equilibrium if a force R^* equal to R is applied at the point M in the opposite direction of R.

The R^* can be resolved into two components. The vertical component of R^* will be equal to W, where as the horizontal component will be equal to frictional force at the base of the dam. The maximum force of friction is given by $F_{max} = \mu w$

If the force of friction, F_{max} is more than the force due to water pressure, F, the dam will be safe against sliding.

$$F_{max} > F$$

Condition to prevent the overturning of the dam

If the resultant R of the weight W of the dam and the horizontal force F due to water pressure, strikes the base within its width i.e., the point M lies within the base AB, then there will be no overturning of the dam.

$$NB > NM$$

Condition to avoid tension in the masonry of the dam at its base

The masonry of dam is week in tension and hence the tension in the masonry of the dam should be avoided.

If the maximum distance between A and the point through which resultant force R meets the base (i.e., distance d^*) is equal to or less than two third of the base width, then there will be no tension at the base of the dam.

$$d^* \leq \frac{2b}{3}$$

Condition to avoid the excessive compressive stresses at the base of the dam

If the maximum stress in the masonry is less than the permissible stress in the masonry, then the excessive compressive stress in the base of the dam can be avoided

MECHANICS OF SOLIDS

Unit-IV

SHEAR STRESSES AND TORSION

Objectives:

- To Impart the knowledge on pure torsion
- To familiarize with the different types of springs
- To familiarize with the Shear stress distribution across various beam and shaft cross sections

Syllabus:

Torsion:
Theory of pure torsion – Derivation of Torsion equation - Assumptions made in theory of pure torsion – Torsional moment of resistance – Polar section Modulus – Power transmitted by shafts. Types of springs – springs in series and parallel - Deflection of closely coiled helical springs under axial pull.

Shear Stresses:
Shear stress at a section – Derivation for shear stress at a section – Shear stress distribution across various beam sections like rectangular, circular, T, I and channel sections.

Learning Out Comes:

- understand the concept of pure torsion
- calculate the power transmitted by a shaft
- Identify the types of springs and analyze the springs
- Illustrate the shear stress across various beam and shaft cross sections

Torsion

Torsion refers to twisting of a straight member under the action of a turning moment or torque which tends to produce a rotation about the longitudinal axis.

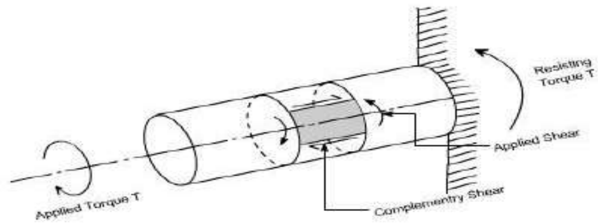
Examples: steering rods, propeller shafts

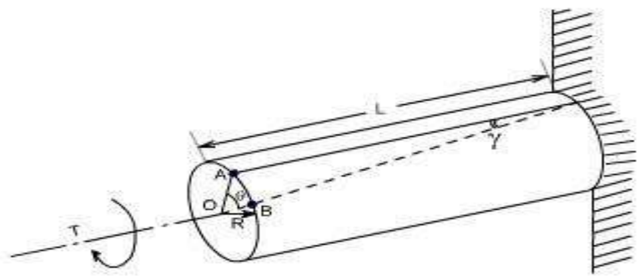
Pure Torsion

A shaft of circular section is said to be in pure torsion when it is subjected to equal and opposite end couples whose axis coincide with the axis of the shaft.

Theory of pure torsion:

Consider a solid cylindrical shaft of radius R and length L subjected to a couple or a twisting moment T at one end, while its other end is held or fixed by the application of a balancing couple of the same magnitude.





Shear strain at outer surface = Distortion per unit length

$$= \frac{AB}{L} = tan\emptyset \approx \emptyset$$

from fig AB =Rθ

therefore $\emptyset = \frac{R\theta}{L}$

modulus of rigidity (C) of the material of the shaft is given as

C $= \frac{shear\ stress\ induced}{shear\ strain\ produced} = \frac{\tau}{\left(\frac{R\theta}{L}\right)} = \frac{\tau x L}{R\theta}$

$$\frac{C\theta}{L} = \frac{\tau}{R}$$

Now for a given shaft subjected to a given torque (T), the values C, θ and L are constant. Hence shear stress produced is proportional to the radius R.

$\tau \propto R\ or\ \frac{\tau}{R}$ =constant

If q is the shear stress induced at a radius r from the centre of the shaft then

$$\frac{\tau}{R} = \frac{q}{r}$$

But

$$\frac{C\theta}{L} = \frac{\tau}{R}$$

$$\frac{q}{r} = \frac{C\theta}{L} = \frac{\tau}{R}$$

The shear stress is maximum at the outer surface and shear stress is zero at the axis of the shaft.

Assumptions in the theory of pure torsion

1. The material of the shaft is uniform throughout
2. The twist along the shaft is uniform.
3. The shaft is of uniform circular section throughout.
4. Cross-sections of the shaft, which are plane before twist remain plane after twist.
5. All radii which are straight before twist remain straight after twist.

Torsional moment of resistance

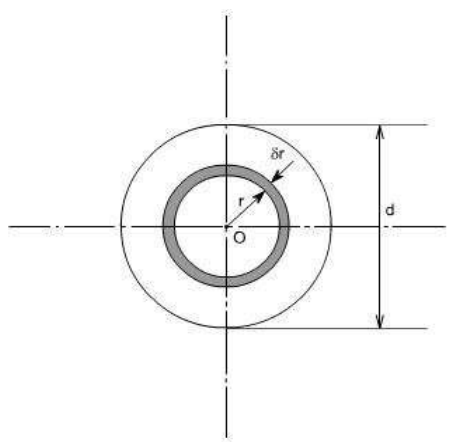
The maximum torque transmitted by a circular shaft, is obtained from the maximum shear stress induced at the outer surface of the solid shaft. Consider a shaft subjected to a torque T.

Let τ = Maximum shear stress induced at the outer surface

R = Radius of the shaft

q = Shear stress at a radius r from the centre.

Consider an elementary circular ring of thickness dr at a distance r from the centre as shown in fig.



Then the area of the ring, dA= 2πrdr

we have $\frac{\tau}{R} = \frac{q}{r}$

Shear stress at the radius r, $q = \tau \frac{r}{R}$

Turning force on the elementary circular ring

= Shear stress acting on the ring x Area of ring

= q x dA

$= \tau\ x\ \frac{r}{R}\ x\ 2\pi r dr$

Now turning moment due to the turning force on the elementary ring,

dT = Turning force on the ring x Distance of the ring from the axis
$= \frac{\tau}{R}\ x\ 2\pi r^3 dr$

The total turning moment is obtained by integrating the above equation between the limits 0 and R

$$T = \int_0^R dT = \int_0^R \frac{\tau}{R}\ x\ 2\pi r^3 dr$$

$$T = \frac{\pi}{16}\tau D^3$$

Power transmitted by shafts

Once the expression for torque (T) for a solid shaft is obtained, power transmitted by shafts can be determined.

Let N = r.p.m of the shaft

T= Mean torque transmitted in N-m

ω = Angular speed of shaft.

Then $power = \frac{2\pi NT}{60}\ watts$

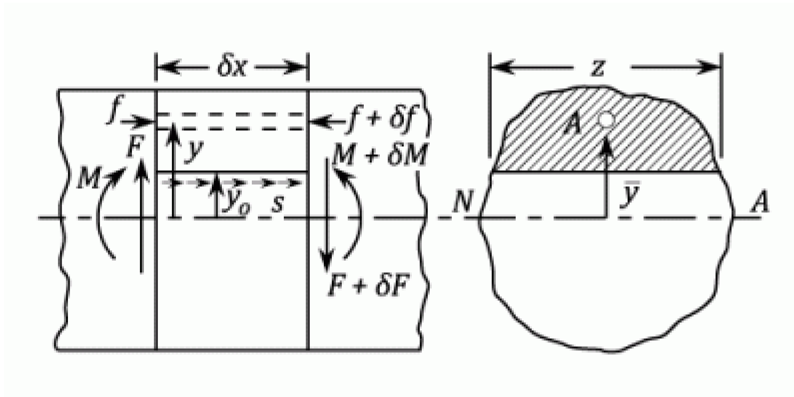
= ω x T

Variation of Shear Stress.

When a beam is in a state of pure bending, the only stresses in the beam are due to bending moments. In the practical cases, the beams carry loads which produce both bending moments and shear force. In these cases, both normal and shear stresses are induced in the beam.

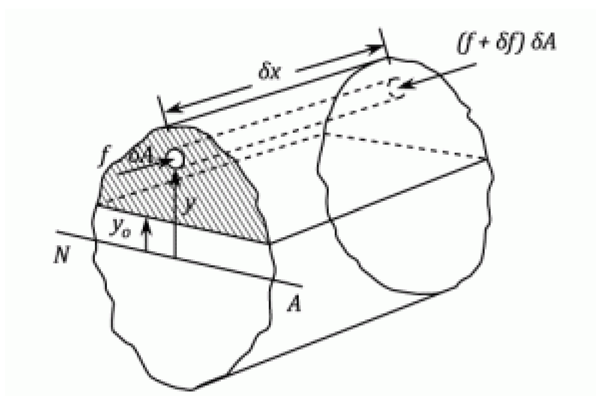
Any transverse section of abeam is subjected to a bending moment and a shear force.

Let at any section AB the bending moment and shear force be M and f respectively. Let at another section CD distant dx from the section AB the B.M and S.F. be (M+dM) and (f+df) respectively.



Let

- *s* be the value of the complementary shear stress and hence the transverse shear stress at a distance $y_0$from the Neutral Axis.
- *z* be the width of the cross section at y_0
- *A* be the area of cross section cut off by a line parallel to the Neutral Axis.
- $\bar{y}$be the distance of the centroid of *A* from the Neutral Axis.



$f$and are the Normal Stresses on an element of Area δA. There is a difference in Longitudinal forces equal to and this summed over the area *A* must be in equilibrium with the transverse Shear Stress *s* on the longitudinal plane of area

Thus, $s\ z\ \delta x = \int \delta f \quad \delta A$---------(2)

But $f = \frac{M\ y}{I}$ and $f + \delta f = \frac{(M+\delta M)\ y}{I}$

Therefore, $\delta f = \delta M\ \frac{y}{I}$

Substituting in equation (2)

$s\ z\ \delta x = \left(\frac{\delta M}{I}\right) \int y\ \delta A$ (or) $s = \left(\frac{\delta M}{\delta x}\right) \frac{A\ \bar{y}}{z\ I} = F.\frac{A\ \bar{y}}{z\ I}$**---------**(3)

Note:$\left(\frac{\delta M}{\delta x}\right) = F$

It should also be noted that Z is the actual width of the section at the position where s is being calculated and that I is the Total Moment of Inertia about the neutral axis. In some applications it is advantageous to calculate as several parts.

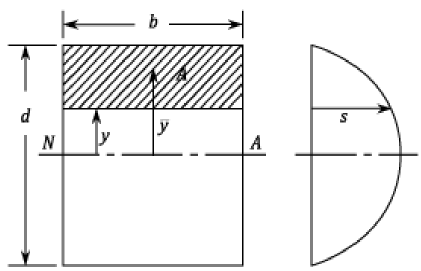
Shear Stress Distribution for Beam Sections of Various Shapes

Rectangular Sections.

Consider rectangular section of width b and depth d. Let the section be subjected to shear force S.

consider a level at any distance y from the Neutral Axis :

$$a = b\left(\frac{d}{2} - y\right) \quad I = \frac{bd^3}{12}\, \bar{y} = \frac{1}{2}\left(\frac{d}{2} + y\right)$$



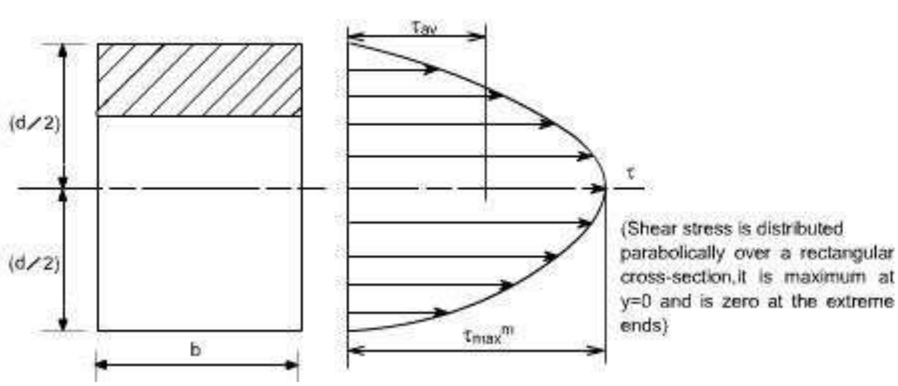
$$a\bar{y} = \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

$$q = \frac{Sa\bar{y}}{Ib}$$

$$q = \frac{6S}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

This shows that there is a parabolic variation of Shear Stress with y. The maximum Shear Force occurs at the Neutral Axis and is given by $q = \frac{3S}{2\,bd}$

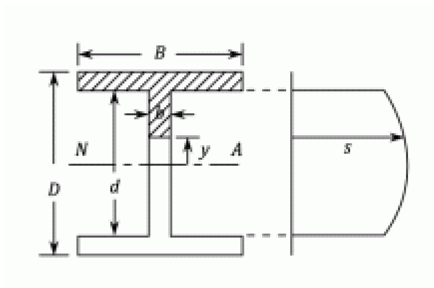
Average shear stress or mean shear stress $q_{avg} = \frac{S}{bd}$



$$q_{max} = \frac{3S}{2\,bd} = 1.5\,q_{avg}$$

Hence the maximum shear stress intensity for a rectangular section is 1.5 times the average shear stress.

<u>I-section</u>



The dimensions are shown in the diagram. It is required to find an expression for the **Shear Stress in the Web.**

is made up of two parts as follows:

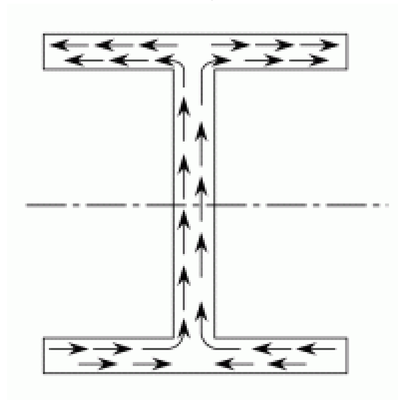
- for the flanged area

- for the web part

As with the rectangular section, the maximum transverse Shear Stress is at the neutral axis.

At the top of the web,

Since the Shear Stress has to follow the direction of the boundary, the distribution must be of the

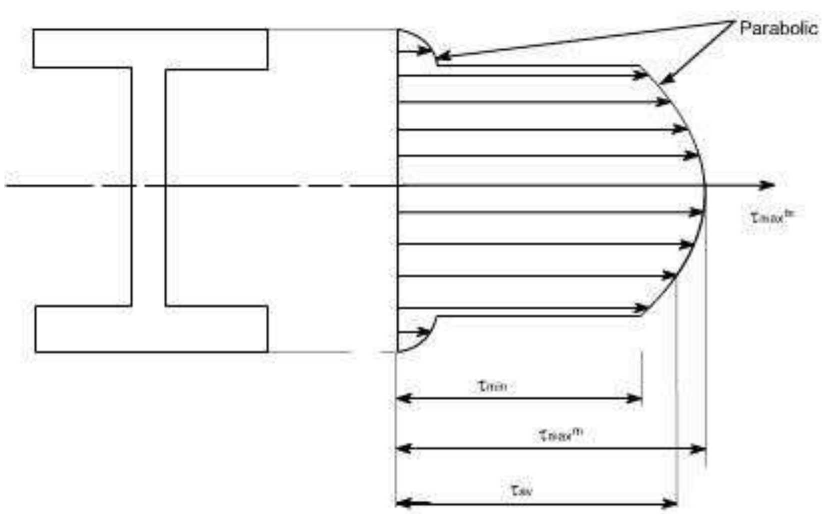


form shown becoming horizontal at the flanges.

Consequently the complementary Shear Stress in the flanges is on longitudinal planes perpendicular to the neutral axis and the "width z" is replaced by the flange thickness

Then,

Showing that the Shear Stress in the flanges varies from a maximum at the top web to zero at the outer tips.



As the variation over the web is comparatively small (about 25%) it is convenient for design purposes and also in calculating deflection due to Shear, to assume that all the Shearing Force is carried by the Web and is uniformly distributed. Similarly it is normal practice to assume that, as a first approximation, the Bending Moment is carried wholly by the flanges.

Assignment-Cum-Tutorial Questions

A. Questions testing the remembering / understanding level of students

I) Objective Questions

1. Shear stress in a beam is maximum at the
   (a) Centeroidal axis (b) Extreme fibres(c) Geometric axis (d) None

2. Shear stress in a beam is zero at the
   (a) Centeroidal axis (b) Extreme fibres(c) Geometric axis (d) None

3. Variation of shear stress in a beam has
   (a) Parabolic variation (b) Linear variation (c) Cubical variation (d) None

4. The direction of shear stress in a loaded beam is
   (a) Horizontal (b) Horizontal as well as vertical (c) Vertical(d) None

5. Shear stress in the beam acting on the cross section is
   (a) Normal to the cross section (b) Tangential to the cross section
   (c) Neither normal nor tangential (d) None

6. For a beam of rectangular cross section, the ratio τ_{max}/ τ_{avg} is
   (a) 2 (b) 1 (c) 1.5 (d) None

7. Shear stress in a I-section beam is maximum at the
   (a) Outermost fiber (b) At the junction of web and flange
   (c) Central fiber (d) None

8. For a beam of circular cross section, the ratio τ_{max}/ τ_{avg} is
   (a) 2/3 (b) 5/3 (c) 4/3 (d) None

9. Shear stress causes
   (a) Deformation(b) Distortion(c) Deformation as well as distortion (d) None

10. Variation of shear stress in a shaft is
    (a) Parabolic (b) Linear (c) Cubical (d) None

11. Power in watts in a shaft having N RPM is given by the equation
    (a) Power = 2π NT/60 (b) Power = 2π N T
    (c) Power = 2000 π N T/60 (d) None

II) Descriptive Questions

1. Derive the expression $\tau = \frac{FAY}{Ib}$

2. Show that for a rectangular section, the maximum shear stress is 1.5 times the average stress.
3. Draw the shear stress distribution diagrams for the following sections: rectangle, triangle, circle, I – section, T – section and channel section.
4. Derive the expression $T / J = f_s / R = G\theta / L$.
5. Define the terms: Torsion, Torsion rigidity.What are the assumptions made in the derivation of torsion equation?
6. What is a spring? Name the two important types of spring.
7. Obtain the shear stress distribution for a rectangular cross section 230 × 400mm subjected to a shear force of 40 KN. Calculate the maximum shear stress, average shear stress and shear stress at a distance of 25mm above neutral axis.
8. The maximum shear stress in a beam of circular section of diameter 150mm is 5.28 N/mm^2. Find the shear force to which the beam is subjected.
9. Find the maximum shear stress induced in a solid circular shaft of diameter 15cm when the shaft transmits 150kN power at 180 r.p.m.
10. In a hollow circular shaft of outer and inner diameters of 30 cm and 20 cm respectively, the shear stress is not to exceed 40 N/mm^2. Find the maximum torque which the shaft can safely transmit.

B. Question testing the ability of students in applying the concepts.

I) Multiple Choice Questions:

1) A solid shaft has diameter 80 mm. It is subjected to a torque of 4 KNm. The maximum shear stress induced in the shaft would be
(a) $75/\pi$ N/mm^2 (b) $250/\pi$ N/mm^2(c) $125/\pi$ N/mm^2 (d) $150/\pi$ N/mm^2

2) A shaft turns at 150 rpm under a torque of 1500 N-m. Power transmitted is
(a) 15π kW(b) 10π kW (c) 7.5π kW (d)5π kW
3) Two shafts in torsion will have equal strength if
(a) Only diameter of the shafts is same
(b) Only angle of twist of the shaft is same
(c) Only material of the shaft is same
(d) Only torque transmitting capacity of the shaft is same

4) A closed helical spring under axial load is designed on the basis of

(a) Shear (b) Compression (c) Bending (d)None

5) A closed coil helical springs has 15 coils. If five coils of this spring are removed by cutting, the stiffness of the modified spring will
(a)increases to 2.5 times (b) increases to 1.5 times
(c)reduce to 0.66 times (d) remain unaffected

6) A circular shaft subjected to torsion undergoes a twist of 10 in a length of 120 cm. If the maximum shear stress induced is limited to 1000 kg/cm2 and if modulus of rigidity G = 0.8×10^6 then the radius of the shaft should be
(a) $\pi/8$ (b) $\pi/27$ (c) $18/\pi$ (d) $27/\pi$

7) Two shafts having the same length and material are joined in series. If the ratio of the diameter of the first shaft to that of the second shaft is 2, then the ratio of the angle of twist of the first shaft to that of the second shaft is
(a) 16 (b) 8 (c) 4 (d) 2

8) When a close coil helical spring is compressed its wire is subjected to
(a) Compression (b) Shear (c) Tension (d) Bending

II) Problems:

1. A timber beam of rectangular section is simply supported at the ends and carries a point load at the centre of the beam. The maximum bending stress is 12 N/mm^2 and maximum shearing stress is 2 N/mm^2, find the ratio of the span to the depth.
2. A beam of I-section is having overall depth as 500 mm and overall width as 190 mm. The thickness of flanges is 25 mm whereas the thickness of web is 15 mm. If the section carries a shear force of 40 kN, calculate the maximum shear stress. Also sketch the shear stress distribution across the section.
3. The shear force acting on a section of a beam is 100KN. The section of the beam is of T-shaped with 200mm flange width and overall depth 250mm. The flange thickness and web thickness are 50mm. Moment of inertia about its horizontal neutral axis is 1.134×10^8 mm^4. Find the shear stress at neutral axis and at the junction of the web and flange.
4. A timber beam of 150mm wide and 260mm deep supports u.d.l of intensity 'w' KN/m over a span of 2.5m. If the safe stresses are 27MPa in bending and 2.0MPa in shear, calculate the safe intensity of load which can be supported by the beam.
5. Two shafts of the same material and of same lengths are subjected to the same torque, if the first shaft is of a solid circular section and the second shaft is of hollow circular section,

whose internal diameter is 0.7 times the outside diameter and the maximum shear stress developed in each shaft is the same, Compare the weights of the shafts.

6. A solid shaft has to transmit 112.5 kW at 250 r.p.m. Taking allowable shear stress as 70 N/mm^2, find suitable diameter for the shaft, if the maximum torque transmitted at each revolution exceeds the mean by 20%.
7. A hollow shaft, having an internal diameter 50% of its external diameter transmits 600 kW at 150 r.p.m. Determine the external diameter of the shaft if the shear stress is not to exceed 65 N/mm^2 and the twist in a length of 3 m should not exceed 1.4 degrees. Assume maximum torque is 20% of mean torque and modulus of rigidity= 1×10^5 N/mm^2
8. A closely coiled helical spring is to have a stiffness of 10 N/cm of compression under a max. load of 50N and max. shearing stress of 125N/mm^2. The solid length of spring is 45mm. Find the dia. of wire, mean dia of spring and the number of coils required. Take N = 7×10^4 N/mm^2

C. Questions testing the analyzing / evaluating ability of students

1. A hallow steel shaft of external diameter 120mm and internal diameter 80mm is 1.3m long. Find the maximum torque required to produce a twist of 0.5 degree over the length of the shaft. Take C = 8×10^4 N/mm^2.
2. A solid aluminium shaft 1 meter long and 50mm diameter is to be replaced by a tubular steel shaft of the same length and the same outside diameter such that each of the two shafts could have the same angle of twist per unit torsional moment over the total length. What must the inner diameter of the tubular steel shaft be? Modulus of rigidity of steel is three times that of aluminium.
3. A composite spring has two close coiled helical steel springs in series. Each spring has a mean coil diameter of 8 times diameter of its wire. One spring has 20 coils and wire diameter of 2.5mm. find the diameter of the wire in the pther spring if it has 15 coils and the stiffness of the composite spring is 1.25N/mm. find the greatest axial load that can be applied to the spring and the corresponding extension for a maximum shearing stress of 300N/mm^2. Take C = 7.89×10^4 N/mm^2.
4. A closely- coiled helical spring of round steel wire 8 mm in diameter having 10 complete turns with a mean diameter of 10 cm is subjected to an axial load of 250 N. Determine: (i) the deflection of the spring, (ii) maximum shear stress in the wire and (iii) stiffness of the spring. Take C= 8×10^4 N/mm^2.
5. In a tension test, a metal test specimen of 30cm diameter, 240mm gauge length stretched 0.1018mm under a pull of 60KN. In torsion test, an identical specimen of same material

twisted 0.0208 radian over a length of 240mm when a torque of 575N-m was applied. Determine the three elastic constants for the material.

D. Gate or Competitive Questions:

1. A timber beam is 100mm wide and 150mm deep. The beam is simply supported and carries a central load W. if the maximum stress in shear is 2 N/mm^2, what would be the corresponding load W on the beam?

2. A rectangular beam of c/s 10cm wide, is subjected to a maximum shear force of 5000N, the corresponding maximum shear stress is being $3N/mm^2$. The depth of the beam is…………..

3. A beam has triangular c/s, having base ‘b’ and altitude ‘h’. if a section of the beam is subjected to a shear force F, the shear stress at the level of neutral axis in the c/s is….

4. The torsion applied to a circular shaft results in a twist of one degree over a length of 1m. the maximum shear stress induced is $120N/mm^2$ and the modulus of rigidity of the shaft material is $0.8x10^5N/mm^2$. What is the radius of the shaft?

5. What is the ratio of torsional moments of resistance of a solid circular shaft of diameter D to that of a hallow shaft with external diameter D and internal diameter‘d’?

UNIT-5
PRINCIPAL STRESSES AND STRAINS

Objective:

To impart the knowledge on the concept of principal stresses & principal strains, Various Energy methods and theories of failure.

Syllabus:

Introduction, stresses on an inclined section of a bar under axial loading.

Normal and tangential stresses on an inclined plane for biaxial stresses.

Two perpendicular normal stresses accompanied by a state of simple shear.

Mohr's circle of stresses, Principal stresses and strains, Analytical and graphical solutions.

Learning Outcomes:

1. To determine the principal stresses and strains for different cases using analytical solutions
2. To determine the principal stresses and strains for different cases using graphical solutions

Principal planes and principal stresses:

- The planes which have no shear stress, are known as principal planes. For any strained body there will be three planes mutually perpendicular to each other , on one plane maximum principle stress will act ,which is known as major principal plane ,on other plane minimum principal stress is acting known as minor principal plane and the other plane is intermittent plane on which intermittent stress will act . These planes will carry only normal stresses.
- The normal stresses acting on a principal plane are known as principal stresses.

Methods for determining the stresses on oblique section:

The stresses on oblique section are determined by the following methods

- Analytical Method
- Graphical method

Analytical method for determining the stresses on oblique section

The following three cases will be considered

- A member subjected to a direct stress in one plane
- The member is subjected to like direct stresses in two mutually perpendicular directions.
- The member is subjected to like direct stresses in two mutually perpendicular directions accompanied by a simple shear stress.

Case 1: Member subjected to a direct stress in one plane:

The bar subjected to principal tensile stresses on the faces AD and BC.

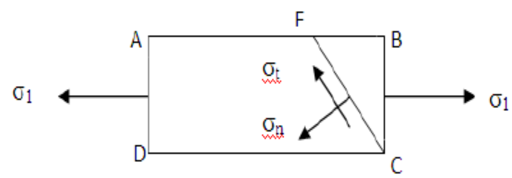


Fig:1

Let the stresses on the oblique plane FC are to be calculated.

Tensile stress on face BC = σ_1

Now tensile force on BC = $\sigma_1 \times BC \times 1$

The above tensile force is also acting on inclined section FC and it is resolved into two components.

P_n = component of the force P_1, normal to the section FC

$P_n = P_1 \cos\theta$

$P_n = \sigma_1 \times BC \times 1 \times \cos\theta$

P_t = component of the force P_1, along the section FC

$P_n = P_1 \sin\theta$

$P_n = \sigma_1 \times BC \times 1 \times \sin\theta$

Normal stress on the section FC, $\sigma_n = \dfrac{Force\ normal\ to\ the\ section\ FC}{Area\ of\ the\ section\ FC}$

$$= \frac{Pn}{FC \times 1}$$

$$= \frac{\sigma 1 \times BC \times 1 \times \cos\theta}{FC}$$

$$\sigma_n = \sigma_1 \times \cos^2\theta$$

Tangential or shear stress on Fc, $\sigma_t = \dfrac{Force\ along\ to\ the\ section\ FC}{Area\ of\ the\ section\ FC}$

$$= \frac{Pt}{FC \times 1}$$

$$=\frac{\sigma 1}{2}\times sin2\theta$$

Case2: A member subjected to like direct stresses in two mutually perpendicular directions.

Consider a body under the action of biaxial stresses σ_x and σ_y as shown in fig:2

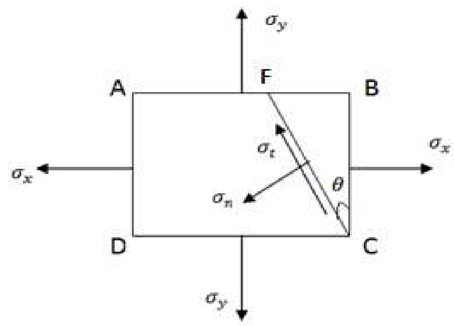


Fig:2

Let the oblique plane be at angle θ with the plane BC. Let σ_n and σ_t be the normal and tangential stresses on the plane BC.

The forces acting at any point on the plane BC are shown in fig:3

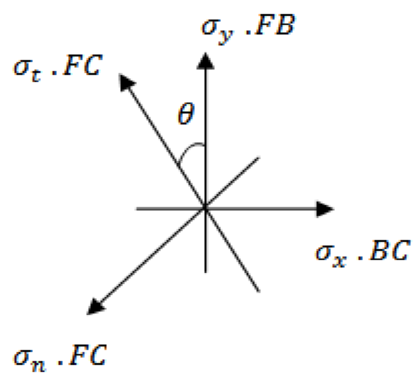


Fig:3

Resolving the forces perpendicular to the plane CF we get,

$$\sigma_n \times FC = \sigma_x \times BC \times cos\theta + \sigma_y \times FB \times sin\theta$$

$$\sigma_n = \sigma_x \times \frac{BC}{FC} \times cos\theta + \sigma_y \times \frac{FB}{CF} \times sin\theta$$

$$= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$$

$$= \sigma_x \frac{1+cos\theta}{2} + \sigma_y \frac{1-cos2\theta}{2}$$

$$\boxed{\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}cos2\theta}$$

Similarly resolving forces along the plane CF, we get

$$\sigma_t X\ FC = \sigma_x \times BC \times sin\theta - \sigma_y \times FB \times cos\theta$$

$$= \sigma_x \times \frac{BC}{CF} \times sin\theta - \sigma_y \times \frac{FB}{FC} \times cos\theta$$

$$= \sigma_x cos\theta \times sin\theta - \sigma_y cos\theta \times sin\theta$$

The tangential stress is $\boxed{\sigma_t = \frac{\sigma_x - \sigma_y}{2} sin2\theta}$ 5° and its maximum value is

Maximum Shear stress $\boxed{\sigma_{t,max} = \frac{\sigma_x - \sigma_y}{2}}$

when $\theta = 45°$, normal stress $\sigma_n = \frac{\sigma_x + \sigma_y}{2}$

Resultant stress $\sigma_r = \sqrt{(\sigma_n{}^2 + \sigma_t{}^2)}$

$$\sigma_r = \left[\sigma_x^2 \times cos^2\theta + \sigma_y^2 \times sin^2\theta\right]^{\frac{1}{2}}$$

If σ_r is inclined at an angle $\emptyset$ with σ_n, then

$$\tan\emptyset = \frac{\sigma_t}{\sigma_n}$$

$$\emptyset = \tan^{-1}\left(\frac{\sigma_y - \sigma_x}{\sigma_y tan\theta + \sigma_x cot\theta}\right)$$

Case3: A member is subjected to like direct stresses in two mutually perpendicular directions accompanied by a simple shear stress.

Consider a rectangular bar of uniform cross-section and of unit thickness.

The bar is subjected to tensile stresses σ_x and σ_y and a simple shear τ on face AB and DC as shown in fig:4

From principle of complementary shear stress, the faces BC and AD will also be subjected to a shear stress τ

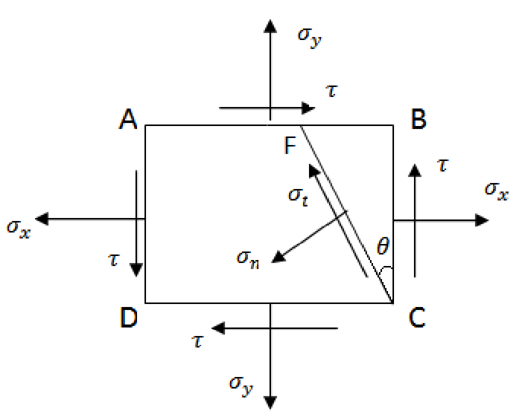


Fig:4

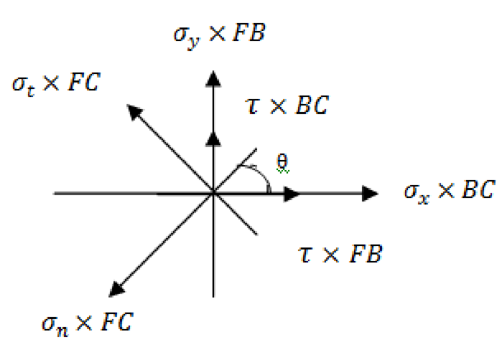


Fig:5

Let σ_n & σ_t be the normal and tangential stresses on oblique section CF, which inclined at an angle θ with the plane CB.

Resolving the forces perpendicular to the plane CF we get,

$$\sigma_n \times FC = \sigma_x . BC . cos\theta + \sigma_y . FB . sin\theta + \tau . FB . cos\theta + \tau . BC . sin\theta$$

$$\sigma_n = \sigma_x \times \frac{BC}{FC} \times cos\theta + \sigma_y \times \frac{FB}{CF} \times sin\theta + \tau . \frac{FB}{FC} . cos\theta + \tau . \frac{BC}{FC} . sin\theta$$

$$= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau \sin\theta \cos\theta + \tau \sin\theta \cos\theta$$

$$= \sigma_x \frac{1+cos\theta}{2} + \sigma_y \frac{1-cos2\theta}{2} + 2\tau\, cos\theta\, sin\theta$$

$$\boxed{\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2} cos2\theta + \tau\, sin2\theta}$$

Similarly resolving forces along the plane CF, we get

$$\sigma_t X\, FC = \sigma_x \times BC \times sin\theta - \sigma_y \times FB \times cos\theta + \tau . FB . sin\theta - \tau . BC . cos\theta$$

$$\sigma_t = \sigma_x \times \frac{BC}{CF} \times sin\theta - \sigma_y \times \frac{FB}{FC} \times cos\theta + \tau . \frac{FB}{FC} . sin\theta - \tau . \frac{BC}{FC} . cos\theta$$

$$= \sigma_x cos\theta \times sin\theta - \sigma_y cos\theta \times sin\theta + \tau\, sin^2\theta - \tau . cos^2\theta$$

$$\boxed{\sigma_t = \frac{\sigma_x-\sigma_y}{2} sin2\theta - \tau\, cos2\theta}$$

Position of principal planes:-

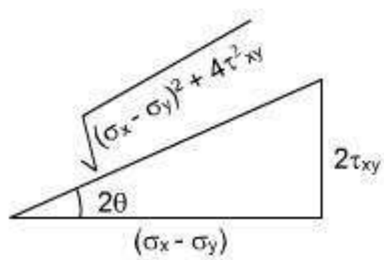
$$\sigma_t = 0$$

$$\frac{\sigma_x - \sigma_y}{2} sin2\theta - \tau\, cos2\theta = 0$$

$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}$$

There are two values of 2θ differing by 180°

1st case $\quad \sin 2\theta = \frac{2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} \quad \cos 2\theta = \frac{\sigma_x-\sigma_y}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$



To get principal stresses, substitute the values of 2θ in the expression for the normal stress σ_n

Major Principal stress $= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2} cos2\theta + \tau\, sin2\theta$

$$= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2} X \frac{\sigma_x-\sigma_y}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} + \tau\, \frac{2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

$$\boxed{\text{Major Principal Stress } = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2}}$$

2nd ca

$$\sin 2\theta = \frac{-2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} \qquad \cos 2\theta = \frac{-(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

Minor Principal Stress $= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2} cos2\theta + \tau\, sin2\theta$

$$= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2} X \frac{-(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} + \tau\, \frac{-2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

$$\boxed{\text{Minor Principal Stress } = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2}}$$

Maximum shear stress

$$\frac{d}{d\theta}(\sigma_t) = 0$$

$\tan 2\theta = \frac{-(\sigma_x-\sigma_y)}{2\tau}$ (Condition for maximum or minimum shear stress)

If $\tan 2\theta = \frac{-(\sigma_x-\sigma_y)}{2\tau}$ then $\sin 2\theta = \frac{\pm(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$, $\cos 2\theta = \frac{\pm 2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$

Substituting the above values in σ_t, we get the maximum and minimum shear stress

$$\sigma_{t,max} = \frac{\sigma_x-\sigma_y}{2} sin2\theta - \tau\, cos2\theta$$

$$= \frac{\sigma_x-\sigma_y}{2} \frac{\pm(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} - \tau\, \frac{\pm 2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

$$= \pm\frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}$$

$$\boxed{\sigma_{t,max} = \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

Construction for Mohr's circle for a member subjected to like direct stresses in two mutually perpendicular directions:

Steps:

1. Using some suitable scale, measure OL &OM equal to σ_x and σ_y respectively on the axis OX.
2. Bisect LM at N.
3. With N as centre & NL or NM radius, draw a circle.
4. At the centre N draw a line NP at an angle 2θ, in the same direction as the normal to the plane make with direction of σ_x. The NP is drawn in the anticlockwise direction.
5. From P, drop a perpendicular PQ on the axis OX. PQ will represent τ & OQ is σ_n.

$$NP=NL = \frac{\sigma_x - \sigma_y}{2}$$

$$PQ = NP\sin 2\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \tau$$

$$OQ= ON+NQ = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta = \sigma_n$$

From stress circle, τ is maximum when 2θ =90° or θ =45°

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$$

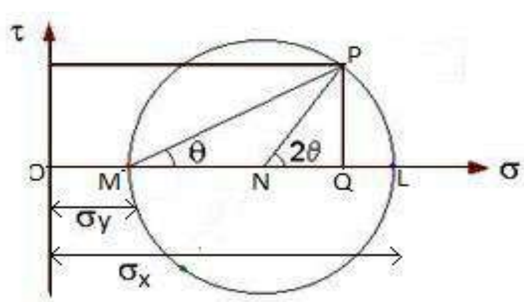


Fig:5

Construction of Mohr's circle for a member subjected to two mutually perpendicular direct stresses accompanied by a simple shear stress:

1. Using some suitable scale, measure OL= σ_2 and OM = σ_1 along the axis OX.

2. At L draw LT perpendicular to OX & equal to τ. LT has been draw downward because τ is acting up with respect to the plane across which σ_n is acting tending to rotate it in the anticlockwise direction & is –ve.
3. Similarly make MS perpendicular to OX & equal to τ ,but above OX.
4. Join ST to cut the axis in N
5. With N as centre & NS or NT as radius, draw a circle.
6. At N make NP at angle 2θ with NT in the anticlockwise direction.
7. Draw PQ perpendicular to the axis. PQ will give τ while OQ will give σ_n & OP will give $\sigma_{r.}$

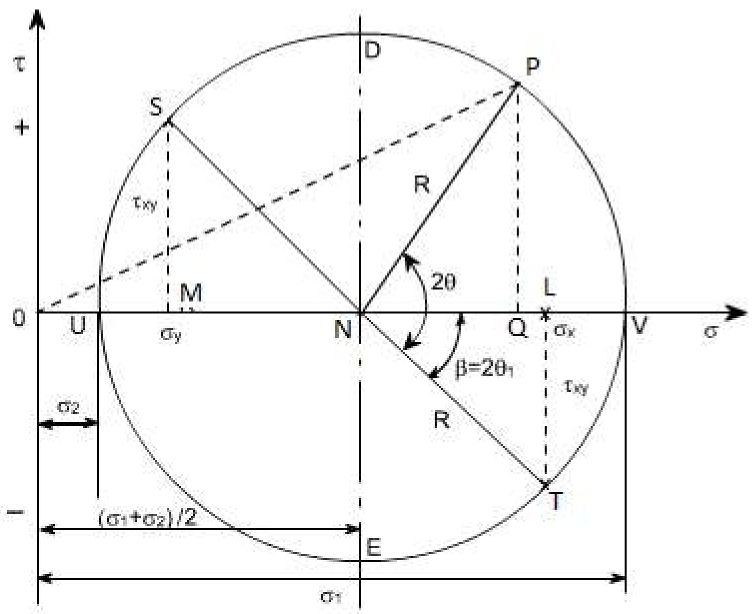


Fig:6

Note:1. To calculate principal planes equate Tangential stress on an oblique plane to zero find out and by substituting in the normal stress expression we may get major principal stress and minor principal stress

2. To calculate principal strain divide principal stress with young's modulus

ENERGY METHODS

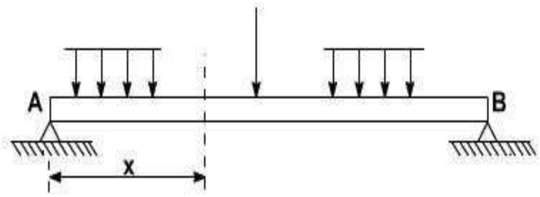
- **Introduction**

  To determine the deflections of beams the following methods are used

  1. **Strain energy/Real work method**
  2. **Virtual work method**
  3. **Castigliano's theorems.**

- **Strain Energy Expression due to Bending:**

  Consider a beam AB subjected to a given loading as shown in figure.



Let, M = Bending Moment at a distance x from end A.

From the simple bending theory,

Stress due to bending alone is expressed as $\sigma = \frac{My}{I}$

Strain, $e = \frac{\sigma}{E} = \frac{M}{EI}y$

Substituting the above relation in the expression of strain energy

$$\text{i.e., } U = \int \left(\frac{\sigma^2}{2E}\right) dv = \int \left(\frac{\left(\frac{My}{I}\right)^2}{2E}\right) dv$$

Substituting dv = dA x dx

Where, dA = elemental cross-sectional area

$$U = \int \left(\frac{M^2 y^2}{2EI^2}\right) dv = \int \left(\frac{M^2}{2EI^2} \int y^2 \, dA\right) dx$$

We known that $\int y^2 \, dA$ represents the moment of inertia

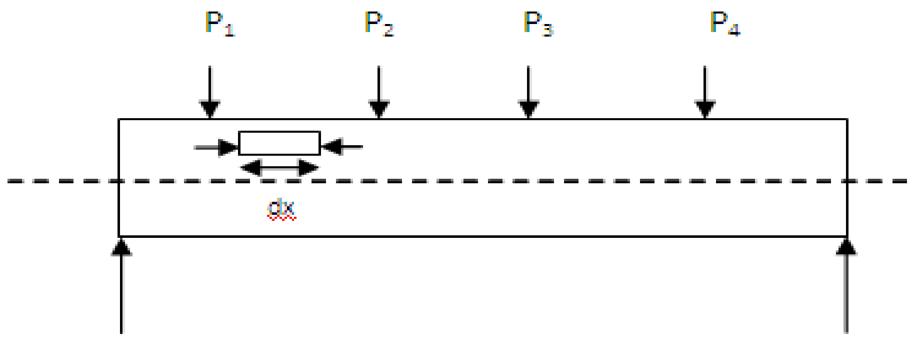
$$U = \int_0^L \frac{M^2}{2EI} dx$$

Note: to calculate deflection at any point equate total strain energy due to internal forces to total work done due to external loads

- **Principle of virtual work**:

  If a system in equilibrium under the action of a set of forces is given a virtual displacement, the virtual work done by the forces will be zero.
- **Unit load method**

Consider the beam subjected to a system of forces. Now, Consider an element of length dx and area be da and distance from the neutral axis be y.



Stress in the element at distance y from N.A is $P = \frac{M}{I}y$

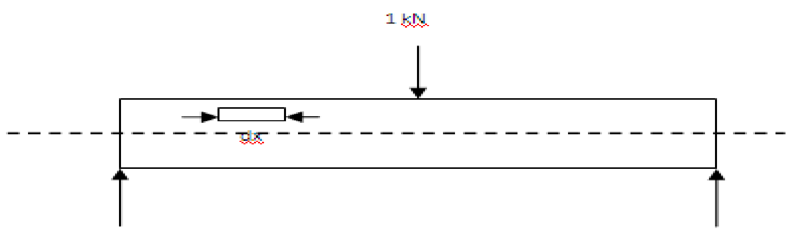
Where, M is the moment acting at the section

Strain in the element $e = \frac{M}{IE}y$

Now apply unit load on the beam and consider an element of length dx at a distance y from N.A.

Let m be the moment at the section

Stress in an element due to unit load $P^1 = \frac{m}{I}y$



Deflection at point where unit load is applied and measured in the direction of unit load

$$\Delta = \int P^1\, e\, dv$$

$$\Delta = \int \frac{m}{I}y\left(\frac{M}{IE}y\right) dv$$

$$\Delta = \int \frac{m}{I}y\left(\frac{M}{IE}y\right) dA.dx$$

$$\Delta = \int_0^L \frac{Mm}{EI^2}\left(\int_0^A y^2 dA\right) dx \qquad \textbf{Since, } \int_0^A y^2 dA = I$$

$$\Delta = \int_0^L \frac{Mm}{EI}\, dx$$

Where M is bending moment due to external load and m is bending moment due to unit load applied at a point where deflection is to be determined.

- **Castigliano's Theorems**

Castigliano's Theorems mainly depends on strain energy

- **Castigliano's Theorem-1**

  For linearly elastic structures, the partial derivative of the strain energy with respect to an applied force is equal to the displacement of the force along its line of action.

$$\delta i = \frac{\partial U}{\partial Pi} = \frac{\partial \int_0^L \frac{M^2}{2EI} dx}{\partial Pi}$$

For simplification δi = $\int_0^L \frac{M}{EI} \frac{\partial M}{\partial P} dx$

Where δi is the deflection at the point of application of force Pi in the direction of P, and U is the strain energy.

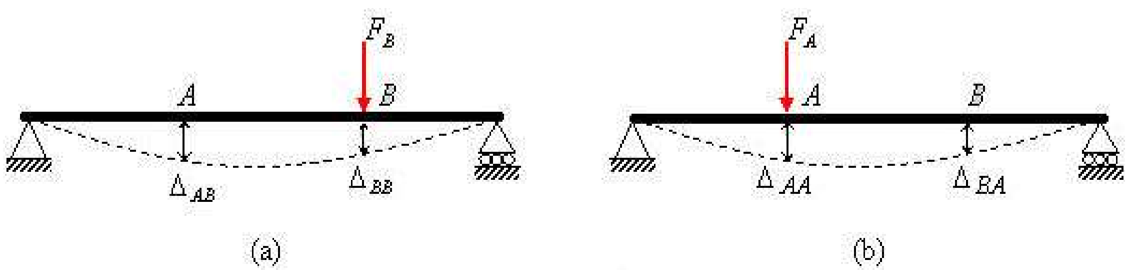
- **Castigliano's Theorem-2** For linearly elastic structures, the partial derivative of the strain energy with respect to couple or moment is equal to the rotation of the couple or moment at that point

$$\theta = \frac{\partial U}{\partial M}$$

θ is the rotation at the point of application of the couple M in the direction of M, and U is the strain energy.

- **Maxwell's Reciprocal Theorem:-**

Statement: Displacement at point A due to the load at Point B is same as displacement of point B due to the same load acting at point A, and its measured in the direction of the loads



$$\Delta_{AB} = \Delta_{BA}$$

Proof:

When load F is acting at B, the displacement at A be Δ_{AB} and the displacement at A be Δ_{BB}

Work done $= \frac{1}{2} F\Delta_{BB}$ -------------(1)

When load F is acting at A, the displacement at A be Δ_{AA} and the displacement at B be Δ_{BA}

Work done $= \frac{1}{2} F\Delta_{AA}$ -------------(2)

Now imagine that load F is applied first at B and then at A.

$$\text{External work done} = \frac{1}{2} F\Delta_{BB} + F\Delta_{BA} + \frac{1}{2} F\Delta_{AA} \text{-------------(3)}$$

If load F is applied first at A and then at B.

$$\text{work done} = \frac{1}{2} F\Delta_{AA} + F\Delta_{AB} + \frac{1}{2} F\Delta_{BB} \text{-------------(4)}$$

Equating 3 and 4 equations, we get

$$\Delta_{AB} = \Delta_{BA}$$

Applications of Castigliano's Theorems :

I. to determine the deflections of the beams: in this apply fictitious (imaginary) point load P at a point where deflection is to be calculated or already point load is acting at that point treat it as P. calculate Total strain energy U due to bending of the beam including load P. partial derivative of U with respect to P will give deflection of the beam at that point after substituting P value either Zero or actual value.

II. to determine the deflections of theTruss: in this apply fictitious (imaginary) point load P at a point where deflection is to be calculated or already point load is acting at that point treat it as P. calculate forces in all the members using any method . then $\Sigma\ p^2\ L/\ 2AE$

will give deflection of the truss at that point after substituting P value either Zero or actual value.

Unit-I
PRINCIPAL STRESSES & STRAINS
Assignment-Cum-Tutorial Questions

A. Questions testing the remembering / understanding level of students

I) Objective Questions

1. A principal plane is a plane of
   (a) Zero tensile stress (b) Zero compressive stress (c) Zero shear stress (d) None
2. On the planes of maximum shear, there are
   (a) Normal stresses (b) Bending stresses (c) Bucking stresses (d) None
3. Maximum shear stress is
   (a) Average sum of principal stresses (b) Average difference of principal stresses
   (c) Average sum as well as difference of principal stresses (d) None
4. When a body is subjected to a direct stress tensile stress (σ) in one plane, then tangential stress on an oblique section of body inclined at an angle to the normal section is equal to___________.
5. What is the value of shear stress acting on a plane of circular bar which is subjected to axial tensile load of 100 KN? (Diameter of bar = 40 mm , $\theta = 42.3^o$)
6. Mohr' s circle is a graphical method to find
   (a) Bending stresses (b) Bucking stresses (c) Maximum shear stresses (d) None
7. When the body is subjected to the mutually perpendicular stresses (σ_x & σ_y) then the centre of the Mohr's circle from y-axis is taken as _________________.
8. The radius of Mohr's circle gives the value of
   (a) Maximum normal stress (b) Minimum normal stress
   (c) Maximum shear stress (d) Minimum shear stress
9. Under uniaxial loading the maximum shear stress is equal to
   (a) Double of uni-axial stress (b) 1.5 times of uni-axial stress
   (c) Equal to uni axial stress (d) half of the uni-axial stress

II) Descriptive Questions

1. Define principal planes and principal stresses.
2. Derive an expression for the stresses on an oblique section of a rectangular body, when it is subjected to a direct stress in one plane.
3. Derive an expression for the stresses on an oblique section of a rectangular body, when it is subjected to direct stresses in two mutually perpendicular directions.
4. Obtain an expression for the major and minor principal stresses on a plane, when the body is subjected to direct stresses in mutually perpendicular directions accompanied by a shear stress.
5. A bar is subjected to a tensile stress of 150 MPa, Determine the normal and tangential stresses on a plane making an angle of 60^o with the direction of the tensile stress.

6. A point in a strained material is subjected to a tensile stress of 120 MPa and a clock wise shear stress of 40 MPa. What are the values of normal and shear stresses on a plane inclined at 45^o with the normal to the tensile stress.
7. A point in a strained material, the principal stresses are 100 MPa tensile and 50 MPa compressive. Find the normal and shear stresses on a plane inclined at 30^o with the normal to the tensile stress.
8. Find the diameter of a circular bar which is subjected to an a axial pull of 160kN, if the maximum allowable shear stress on any section is 65 kN/mm^2.

B. Question testing the ability of students in applying the concepts.

I) Multiple Choice Questions:

1. When a body is subjected to a direct stress tensile stress (σ) in one plane, then normal stress on an oblique section of body inclined at an angle to the normal section is equal to
   (a) $\sigma \sin\theta$ (b) $\sigma \cos\theta$ (c) $\sigma \sin^2\theta$ (d) $\sigma \cos^2\theta$
2. When a component is subjected to axial stress the normal stress σ_n is maximum, if cos θ is _______. ($\sigma_n = \sigma_x Cos^2\theta$)
   1. maximum 2. Minimum 3. always one 4. always zero
   (a) 1 and 4 (b)1 and 3 (c)2 and 3 (d) 2 and 4
3. The maximum tangential stress $\sigma_t = (\sigma_x \sin 2\theta)/2$ is maximum if, θ is equal to
   (a) 45^o (b) 90^o (c) 270^o (d) all of the above
4. The angle between normal stress and tangential stress is known as angle of
   (a) Declination (b) orientation (c) obliquity (d) rotation
5. In a state of simple shear, tensile and compressive stresses occur on planes inclined at 45^o to the shear stress, whose ratio to the shear stress is
   (a) 2 (b) 1.5 (C) 1.25 (d) 1
6. In a two dimensional stress system the normal stresses on two planes at right to one another are 150MPa (tensile) and 50MPa (compressive). The magnitude of the maximum shear stress at the point is 150MPa. The largest principal stress at the point is
   (a) 200 MPa (compressive) (b) 200 MPa (tensile)
   (c) 100 MPa (compressive) (d) 100 MPa (tensile)
7. The graphical method of Mohr's circle represents shear stress (τ) on ______
   (a) X-axis (b) Y-axis (c) Z-axis (d) None of the above
8. Principal stress is the magnitude of ________ stress acting on the principal plane.

II) Problems:

1. The state of stress at a point in a stressed material is given by σ_x =30 MPa, σ_y =20 MPa, τ_{xy}=25 MPa. Determine the direction and magnitude of the principal stresses in the material. Also locate the planes of maximum shearing stress and calculate the normal and shearing stress on these planes.
2. A bar is to carry an axial load of 70 MPa. Determine the stresses on a plane inclined at 30^o to the direction of loading as shown in fig. a. Also locate the plane of maximum shear stress and determine the stresses on it.

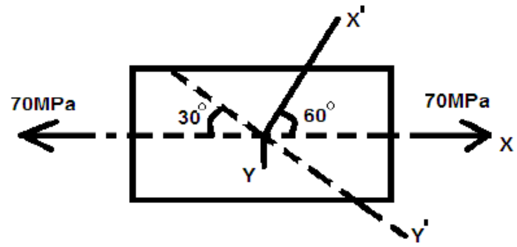


Fig: a

3. A rectangular block of material is subjected to tensile stress of 110 N/mm^2 on one plane and a tensile stress of 47 N/mm^2 on a plane at right angles, together with shear stresses of 63 N/mm^2 on the same planes. Calculate (a) the direction and magnitude of principal stresses. (b) The magnitude of greatest shear stress.
4. The stress components at a point are σ_x =100MPa, σ_y = -60MPa, τ_{xy}=40 MPa. Calculate (a) the principal stresses, (b) principal planes, (c) maximum shear stress and (d) Planes of maximum shear stress.
5. An elemental cube is subjected to tensile stresses of 80N/mm^2 and 30N/mm^2 acting on two mutually perpendicular planes and a shear stress of 20N/mm^2 on these planes. Draw the Mohr's circle of stresses and hence or otherwise determine the magnitudes and directions of principal stresses and also the greatest shear stress.
6. At a point in a strained material, the principal stresses are 160 N/mm^2 (tensile) and 50 N/mm^2 (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 45^o to the axis of the major principal stress. What is the maximum intensity of shear stress in the material at the point?
7. Direct stresses of +60 MPa and -30 MPa act on a plane lamina. Calculate the magnitude and direction of the resultant stress on a plane making an angle of 25^o with the plane of first principal stress.
8. At a point in a stressed element, the normal stresses in two mutually perpendicular directions are 45 MPa and 25 MPa both tensile. The complimentary shear stress in these directions is 15 MPa. By using Mohr's circle method, determine the maximum and minimum principal stresses.
9. At a point in a material, there is a horizontal tensile stress of 270 MPa, a vertical tensile stress of 130 MPa and shearing stress of 40 MPa downward on left. With the aid of Mohr's circle, determine the maximum and minimum Principal stresses and the plane on which they act. Determine also the shearing stress in magnitude and direction. Verify the results by calculations.

C. Questions testing the analyzing / evaluating ability of students

1. For a given loading conditions the state of stress in the wall of a cylinder is expressed as follows:

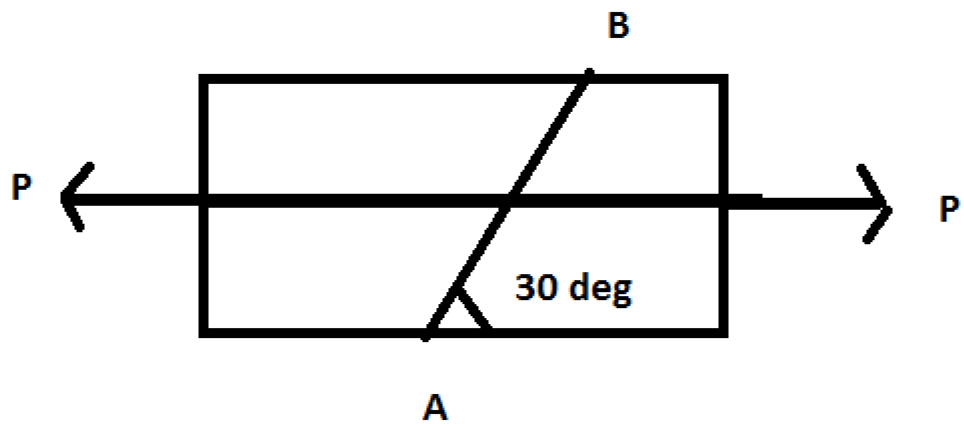
(a) 85 MN/m^2 tensile

(b) 25 MN/m^2 tensile at right angles to (a)

(c) Shear stresses of 60 MN/m^2 on the planes on which the stresses (a) and (b) act; the sheer couple acting on planes carrying the 25 MN/m^2stress is clockwise in effect.

Calculate the principal stresses and the planes on which they act. What would be the effect on these results if owing to a change of loading (a) becomes compressive while stresses (b) and (c) remain unchanged

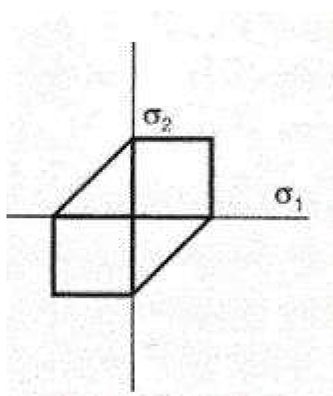
2. Two wooden pieces 10cmX10cm in cross section are gluet together along line AB as shown in the fig below. What maximum axial force P can be applied with the allowable shearing stress along AB is 1.2N/mm^2



3. An elemental cube is subjected to tensile stresses of 30 N/mm^2 and 10 N/mm^2 acting on two mutually perpendicular plains and a shear stress of 10 N/mm^2 on these planes. Draw the Mohr's circle of stresses and hence or otherwise determine the magnitude and direction of principal stress and also the greatest shear stress.

D. Gate Questions:

1.A failure theory postulated for metals is shown in a two dimensional stress plane. This theory is called.



a) Max distortion energy theory
b) Max shear stress theory
c)Max normal stress theory
d) Max strain theory

2. If an element of a stressed body is in a state of pure shear with a magnitude of 80Mpa,the magnitude of max principal stress at that location is

   a) 80Mpa b)113.14 Mpa c)120Mpa d)56.57 Mpa

.

3. Pick the in correct statement from the following four statements
   a) On the plane which carries maximum normal stress, the shear stress is zero
   b) Principal planes are mutually orthogonal
   c) On the plane which carries max shear stress, the normal stress is zero
   d) The principal stress axes and principal strain axes coincide for an isotropic material.

4. Two perpendicular axis X&Y of a section are called principal axes when
   a) moments of inertioal about the axes are equal(I_X=I_Y)
   b) product moment of inertial (I_{XY}) is zero
   c) product moment of inertial ($I_{X,}I_Y$) is zero
   d) moment of inertial about one of the axis is greater than the other

UNIT – VI
ANALYSIS OF PERFECT FRAMES

COURSE OBJECTIVES:

- To impart the knowledge on calculating forces in pin-jointed plane frames

LEARNING OUTCOMES:

Student will be able to

- Understand the methods of forming a Pin-jointed plane frame and its structural action.
- Explain the assumptions made in the analysis of pin-jointed plane frame
- Analyse the given pin-jointed plane frame and find the forces in the members using Method of joints and method of sections.

SYLLUBUS :

- Introduction
- Definition of a perfect frame and imperfect frames
- Simple Trusses
- Analysis of Trusses by the Method of Joints
- Analysis of Trusses by the Method of Sections
- Simple Problem on simply supported and cantilever type trusses

Introduction:

A structure is made up of several bars riveted or welded together is known as frame. If the frame is composed of such members which are just sufficient to keep the frame in equilibrium.

Types of Frames:

The different types of frames are

i) Perfect frames

ii) Imperfect frames

Perfect frame:

The frame which is composed of such members, which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load is known as perfect frame.

For a perfect frame the number of joints and number of members are given by

$$n=2j-3$$

Imperfect Frame:

A frame in which number of members in a frame will be more or less than (2j-3), is known as imperfect frame.

i) If the number of members in a frame are less than (2j-3), then the frame is known as **deficient frame.**

ii) If the number of members in a frame are more than (2j-3), then the frame is known as **redundant frame**.

Assumptions made in finding out the forces in a frame:

The assumptions made in finding out the forces in a frame are

i) The frame is a perfect frame

ii) The frame carries load at the joints

iii) All the members are pin jointed.

Reactions of supports of a frame:

The frames are generally supported

i) on roller support or

ii) on a hinged support

If the frame is supported on a roller support, then the line of action of the reaction will be at right angles to the roller base as shown in fig

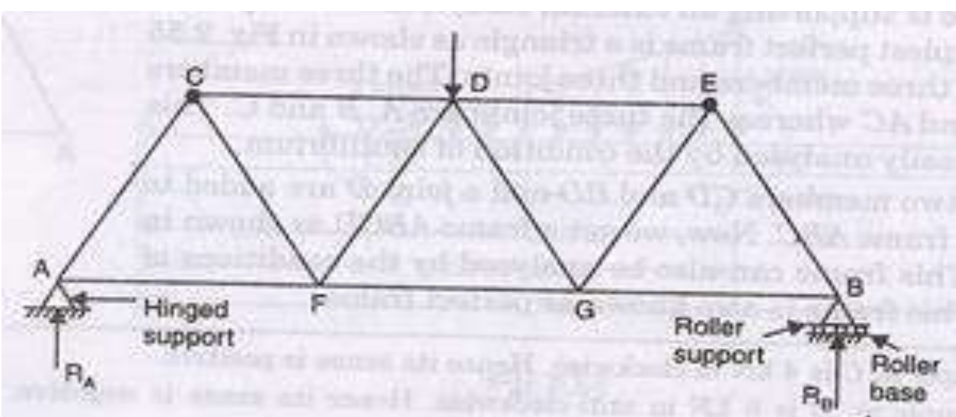


Fig. 2.57

If the frame is supported on a hinged support, then the line of action of the reaction will depend upon the load system on the frame.

The reactions at the support of a frame are determined by the conditions of equilibrium. The external load on the frame and the reactions at the supports must form a system of equilibrium.

Analysis of a frame

Analysis of a frame consists of

i) Determinations of the reactions at the supports and

ii) Determinations of the forces in the members of the frame.

The reactions are determined by the condition that the applied load system and the induced reactions at the supports form a system in equilibrium.

The forces in the members of the frame are determined by the condition that every joint should be in equilibrium and so, the forces acting at every joint should form a system in equilibrium.

A frame is analysed by the following methods

i) Method of joints

ii) Method of sections

iii) Graphical method

iv) Tension coefficient method

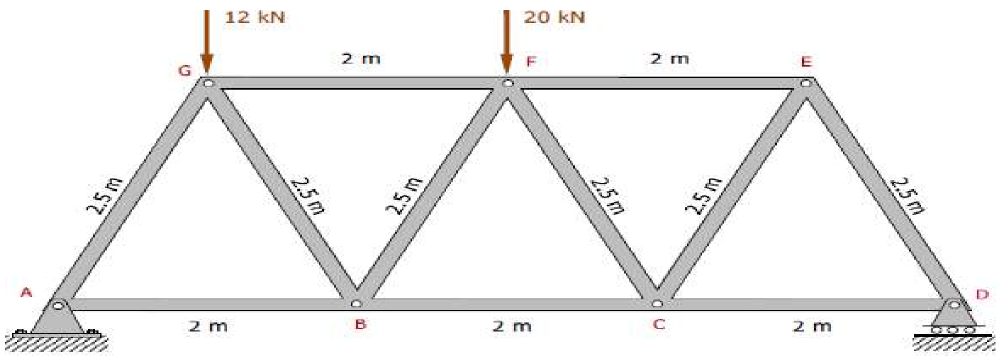
Method of joints:

The free-body diagram of any joint is a concurrent force system in which the summation of moment will be of no help. Recall that only two equilibrium equations can be written $\Sigma F_X = 0$ and $\Sigma F_Y = 0$

This means that to solve completely for the forces acting on a joint, we must select a joint with no more than two unknown forces involved. This can be started by selecting a joint acted on by only two members. We can assume any unknown member to be either tension or compression. If negative value is obtained, this means that the force is opposite in action to that of the assumed direction. Once the forces in one joint are determined, their effects on adjacent joints are known. We then continue solving on successive joints until all members have been found.

Problem on method of joints:

The structure shown in Fig. is a truss which is pinned to the floor at point A, and supported by a roller at point D. Determine the force to all members of the truss.

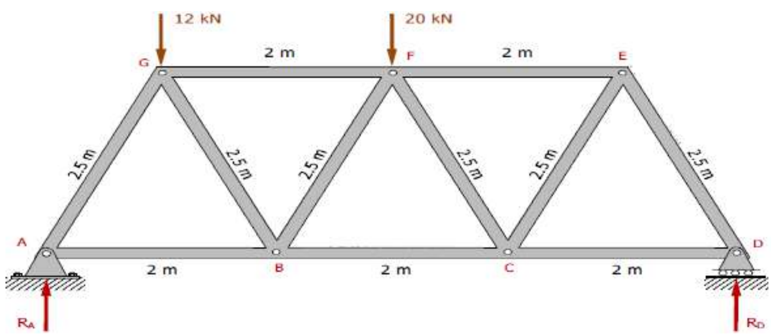


Solution:-

$\Sigma M_D=0$

$6R_A=5(12) + 3(20)$

$R_A=20$ kN

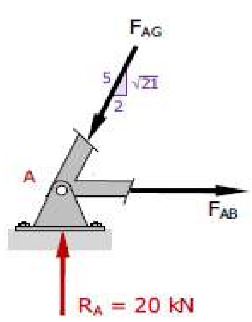


$\Sigma M_A=0$

$6R_D=1(12) + 3(20)$

$R_D=12$ kN

At joint A



FBD of Joint A

$\Sigma F_V = 0$

$21\sqrt{5}F_{AG}=R_A$

$21\sqrt{5}F_{AG}=20$

$F_{AG}=21.82$ kN compression

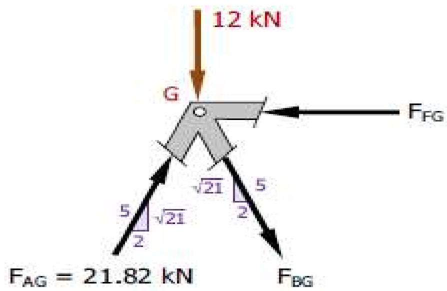
$\Sigma F_H = 0$

$F_{AB} = 25F_{AG}$

$F_{AB} = 25(21.82)$

$F_{AB}=8.73$ kN tension

At joint G



FBD of Joint G

$\Sigma F_V=0$

$21\sqrt{5}F_{BG}+12=21\sqrt{5}F_{AG}$

$21\sqrt{5}F_{BG}+12=21\sqrt{5}(21.82)$

$F_{BG}=8.73$ kN tension

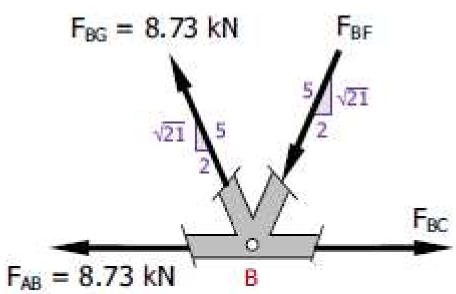
$\Sigma F_H=0$

$F_{FG}=25F_{AG}+25F_{BG}$

$F_{FG}=25(21.82)+25(8.73)$

$F_{FG}=12.22$ kN compression

At joint B



FBD of Joint B

$\Sigma F_V=0$

$21\sqrt{5}F_{BF}=21\sqrt{5}F_{BG}$

$F_{BF}=F_{BG}$

$F_{BF}=8.73$ kN compression

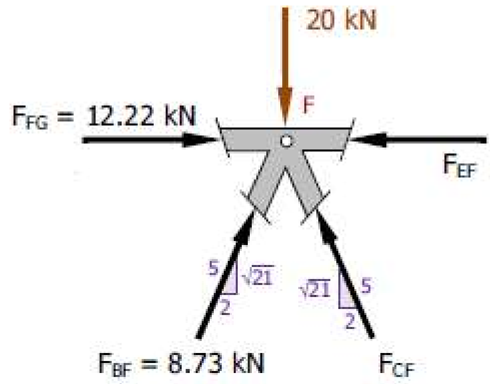
$\Sigma F_H=0$

$F_{BC}=F_{AB}+25F_{BG}+25F_{BF}$

$F_{BC}=8.73 + 25(8.73) + 25(8.73)$

$F_{BC}=15.71$kNtension

At joint F



FBD of Joint F

$\Sigma F_V=0$

$21\sqrt{5}F_{CF}+21\sqrt{5}F_{BF}=20$

$21\sqrt{5}F_{CF}+21\sqrt{5}(8.73)=20$

F_{CF}=13.09 kN compression

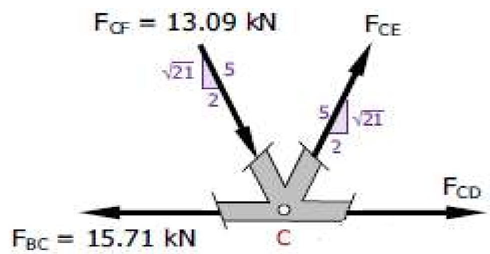
$\Sigma F_H=0$

$F_{EF}+25F_{CF}=25F_{BF}+F_{FG}$

$F_{EF}+25(13.09)=25(8.73)+12.22$

FEF=10.48 kN compression

At joint C



FBD of Joint C

$\Sigma F_V=0$

$21\sqrt{5}F_{CE}=21\sqrt{5}F_{CF}$

$F_{CE}=F_{CF}$

F_{CE}=13.09 kN tension

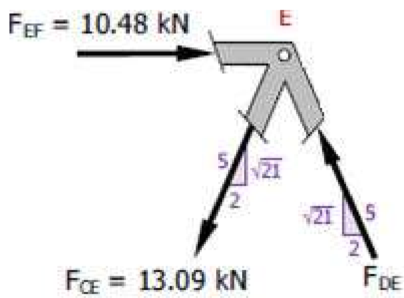
$\Sigma F_H=0$

$F_{CD}+25F_{CE}+25F_{CF}=F_{BC}$

$F_{CD} + 25(13.09) + 25(13.09)=15.71$

F_{CD} = 5.24 kN tension

At joint E



FBD of Joint E

$\Sigma F_V=0$

$21\sqrt{5}F_{DE}=21\sqrt{5}F_{CE}$

$F_{DE}=F_{CE}$

$F_{DE}=13.09$ kN compression

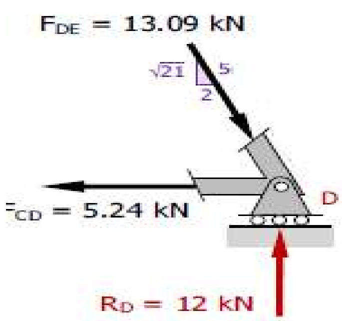
$\Sigma F_H=0$

$F_{EF} = 25F_{CE} + 25F_{DE}$

$10.48=25(13.09) + 25(13.09)$

$10.5=10.5$ *check*

At joint D



FBD of Joint D

$\Sigma F_V=0$

$R_D=21\sqrt{5}F_{DE}$

$12=21\sqrt{5}(13.09)$

$12=12$ *check*

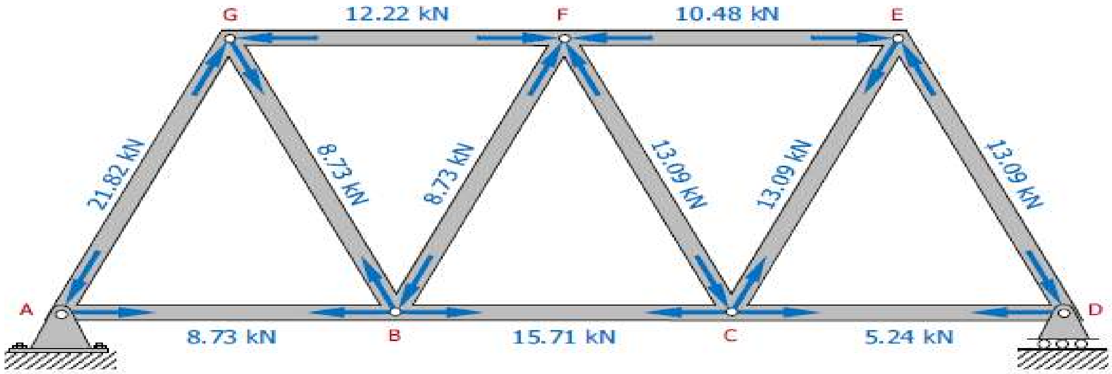
$\Sigma F_H=0$

$F_{CD}=25F_{DE}$

$5.24=25(13.09)$

$5.24=5.24$ *check*

Summary



F_{AB} = 8.73 kN tension

F_{AG} = 21.82 kN compression

F_{BC} = 15.71 kN tension

F_{BF} = 8.73 kN compression

F_{BG} = 8.73 kN tension

F_{CD} = 5.24 kN tension

F_{CE} = 13.09 kN tension

F_{CF} = 13.09 kN compression

F_{DE} = 13.09 kN compression

F_{EF} = 10.48 kN compression

F_{FG} = 12.22 kN compression

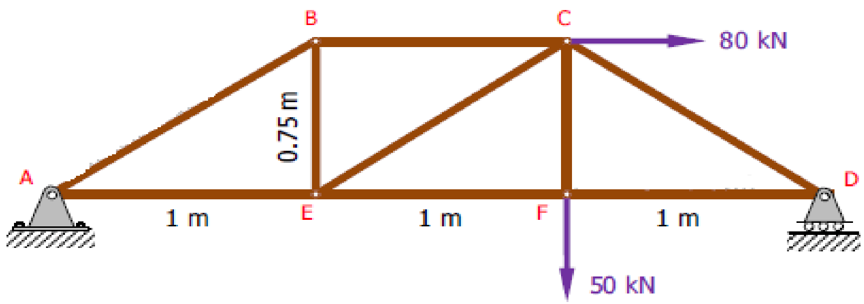
Method of Sections:

In this method, we will cut the truss into two sections by passing a cutting plane through the members whose internal forces we wish to determine. This method permits us to solve directly any member by analyzing the left or the right section of the cutting plane. To remain each section in equilibrium, the cut members will be replaced by forces equivalent to the internal load transmitted to the members. Each section may constitute of non-concurrent force system from which three equilibrium equations can be written. $\Sigma F_H = 0$ $\Sigma F_V = 0$ and $\Sigma M_O = 0$

Because we can only solve up to three unknowns, it is important not to cut more than three members of the truss. Depending on the type of truss and which members to solve, one may have to repeat Method of Sections more than once to determine all the desired forces.

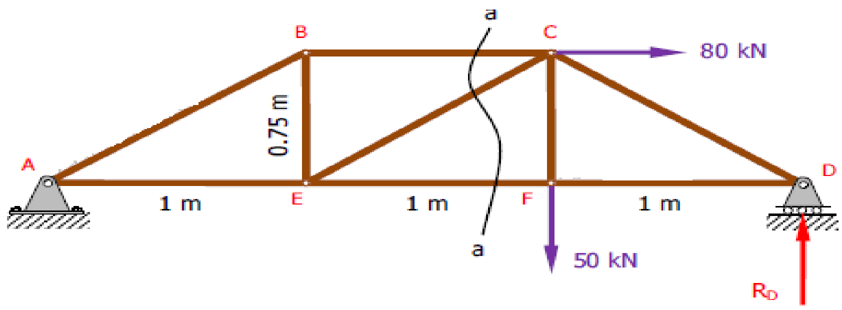
Problem on method of sections:

From the truss in Fig., determine the force in members BC, CE, and EF by using method of sections.



Figure

Solution:



$\Sigma M_A=0$

$3R_D=50(2)+80(0.75)$

$R_D=53.33$ kN

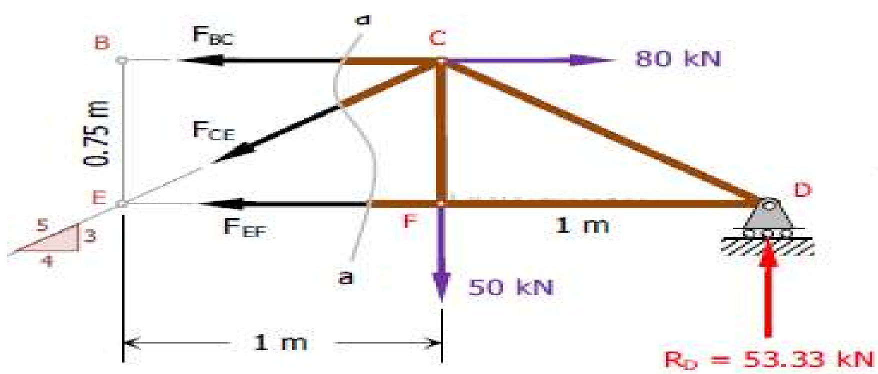
From the FBD of the section through a-a

$\Sigma M_E=0$

$0.75F_{BC}+2R_D = 0.75(80) +1(50)$

$0.75F_{BC}+2(53.33) = 60+50$

$F_{BC}=4.45$ kN tension



Section to the right of a-a

$\Sigma M_C=0$

$0.75F_{EF}=1(RD)$

$0.75F_{EF}=53.33$

$F_{EF}=71.11$ kN tension

$\Sigma F_V=0$

$35F_{CE}+50=R_D$

$35F_{CE}+50=53.33$

$F_{CE}=5.55$ kN tension

Assignment-Cum-Tutorial Questions

A. Questions testing the remembering / understanding level of students

I) Objective Questions

1. When using the method of joints, typically _________ equations of equilibrium are applied at every joint. ( )

   (a) Two (b) Three (c) Four (d) Six

2. In the method of sections for analysis of a plane truss, the maximum number of unknown member forces that can be found with a given section ( )

   (a) 1 (b) 2 (c) 3 (d) 4

3. Redundant truss is a type of ______ ` ( )

   (a) perfect truss (b) imperfect truss (c) stable truss (d) none of the above

4. Mathematical expression for perfect frame___________________

5. Shape of the simple basic frame______________

6. Define truss___________

7.Define Tie Member________________

II) Descriptive Questions

1. (a) Explain perfect and imperfect frames.

   (b) Find the forces in the members AB, AC and BC of the truss shown in figure 6.1

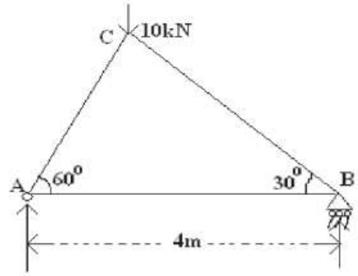


Fig 6.1

2. Analyse the frame shown in fig. 6.2 and find the forces in all the members using method of joints.

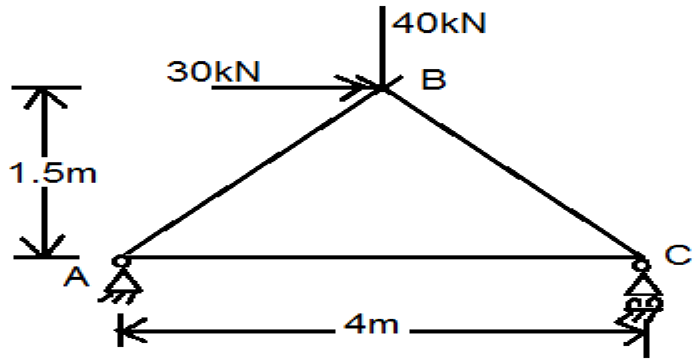


Fig. 6.2

3. Using the method of sections find the forces in all the members of the cantilever truss loaded as shown in figure 6.3.

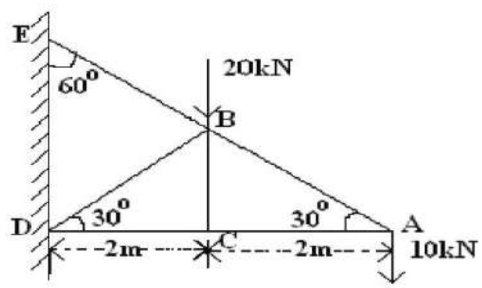


Figure 6.3

4. (a) What are the assumptions made in finding out the forces in a frame

   (b) A truss of span 7.5 m carries a point load of 500 N at joint D as shown in **Fig. 6.4.** Find the reactions and forces in the members of the truss.

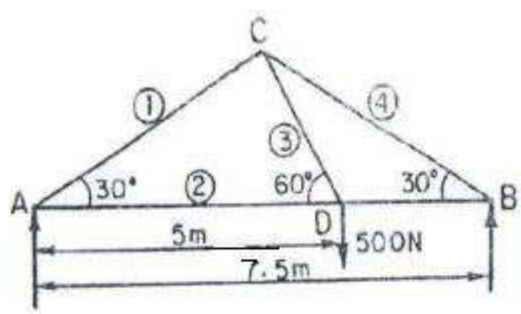
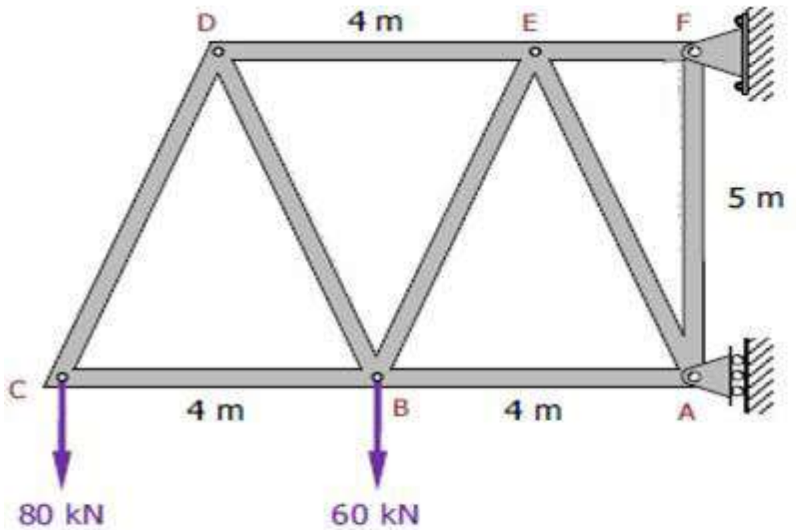


Figure 6.4

5. Find the force in each member of the truss shown in Fig.6.5.



B. Question testing the ability of students in applying the concepts.

I) Multiple Choice Questions:

1. If n > 2j – R, then the truss is called as _________. ( )

   (n = number of joints, j = number of members, R = number of reaction components)

   a. perfect truss b. redundant truss c. deficient truss d. none of the above

2. Which of the following statements is false about frame/truss? ( )

   a. Bent member is never used in a truss

   b. Internal hinges are used to connect members in a truss

   c. All members in the truss are two force members

   d. Multi force members can be used in a frame

3. The number of independent equations to be satisfied for static equilibrium in a plane structure is ( )

   a. 2 b. 3 c. 4 d. 6

4. Number of unknown internal forces in each member of a rigid jointed plane frame is ( )

   a. 1 b. 2 c. 3 d. 6

5. Independent displacement components at each joint of a rigid-jointed plane frame are ( )

   a. Three linear movements b. Two linear movements and one rotation

   c. One linear movement and two rotations d. Three rotations

6. Method of sections is more suitable when ( )

   a. only reactions at the supports are desired

   b. only forces in few of the members are desired

c. only forces in few of the members away from the supports are desired

d. forces in all members are desired

7. Define perfect frame__________

8. Define Imperfect frame_______________

9. The maximum Unknown force is allowed while considering the joint for analysis.

II) Problems:

1. Analyse the frame loaded as shown in fig.6.5 and find the forces in all the members using method of joints and method of sections.

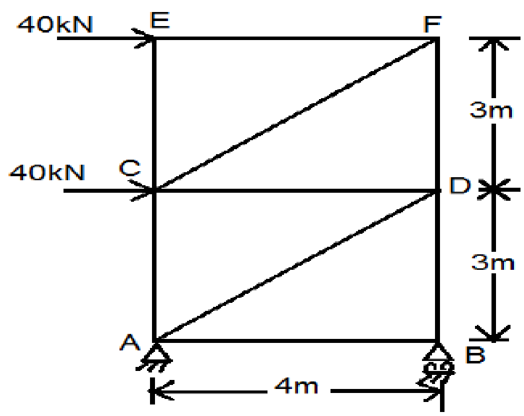


fig. 6.5

2. Using the method of joints find the forces in all the members of the cantilever truss loaded as shown in figure 6.6.

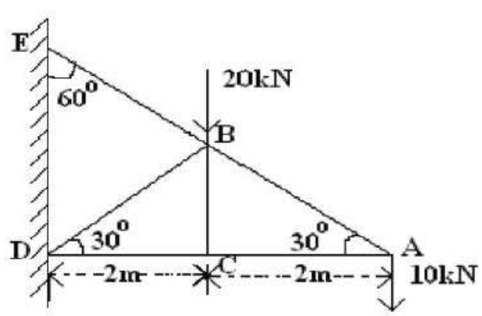


Figure 6.6

3. Find the forces in all the members of the simply-supported truss loaded as shown in fig. 6.7 by the method of joints and method of sections.

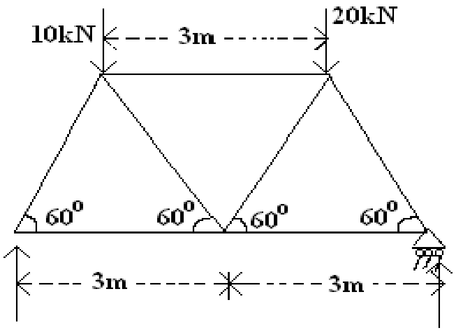


Figure 6.7

4. Analyse the frame loaded as shown in fig.6.8 and find the forces in all the members using method of joints and method of sections.

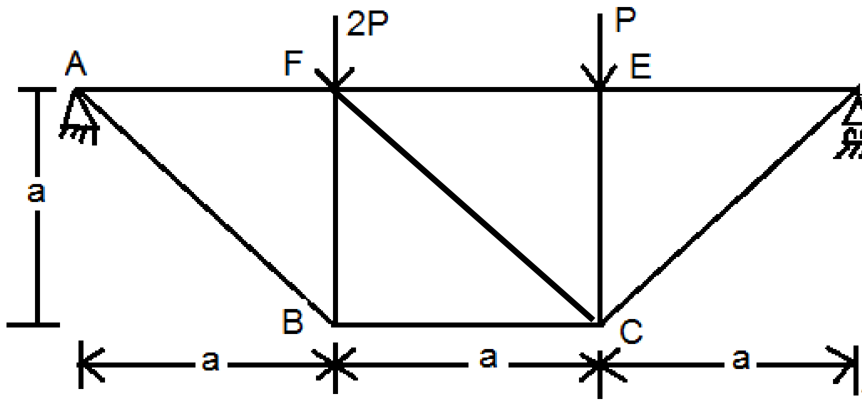


fig.6.8