GUDLAVALLERU ENGINEERING COLLEGE

(An Autonomous Institute with Permanent Affiliation to JNTUK, Kakinada)
Seshadri Rao Knowledge Village, Gudlavalleru – 521 356.

Department of Civil Engineering

HANDOUT

on

Mechanics of Fluids

Vision

"To provide quality education embedded with knowledge, ethics and advanced skills and preparing students globally competitive to enrich the civil engineering research and practice."

Mission

1. Aims at imparting integrated knowledge in basic and applied areas of civil engineering to cater the needs of industry, profession and the society at large.
2. to develop the faculty and infrastructure making the department a centre of excellence providing knowledge base with ethical values and transforming innovative and extension services to the community and nation.
3. To make the department a collaborative hub with leading industries and organizations, promote research and development and combat the challenging problems in civil engineering which leads for sustenance of its excellence.

Program Educational Objectives

1. Exhibit their competence in solving civil engineering problems in practice, employed in industries or undergo higher study.
2. Adapt to changing technologies with societal relevance for sustainable development in the field of their profession.
3. Develop multidisciplinary team work with ethical attitude &social responsibility and engage in life - long learning to promote research and development in the profession.

PROGRAM SPECIFIC OUTCOMES (PSOs)

Students will be able to

1. Survey, plot and prepare layout plans for buildings, dams, canals and highway alignments and conduct geotechnical and geological investigations of the project.
2. Test, analyze and design various substructures and superstructures by considering the environmental and societal issues.
3. Organize various construction projects considering modern construction techniques, equipment and management issues.

HANDOUT ON MECHANICS OF FLUIDS

Class& Sem. : II B.Tech – I Semester   Year : 2017-18
Branch : CE   Credits : 3

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1. **Brief History and Scope of the Subject**

   "Mechanics of fluids" is that branch of science which deals with the behavior of the fluids (liquids or gases) at rest as well as in motion. Fluid mechanics can be divided into fluid statics, the study of fluids at rest; fluid kinematics, the study of the fluids in motion without considering forces which causes it and fluid dynamics, the study on the effect of forces on fluid motion. Fluid mechanics, especially fluid dynamics, is an active field of research with many practical applications that are partly or wholly unsolved. Fluid mechanics can be mathematically complex, and can best be solved by numerical methods using computers.

2. **Pre-Requisites**
   - Mathematics I
   - Mathematics II
   - Engineering Mechanics

3. **Course Objectives:**
   - To familiarize with the static and dynamic aspects of fluids.
   - To impact knowledge on laminar, turbulent flows and dimensional analysis.
   - To introduce the concepts of the flow through closed conduits and measurement of flow.

4. **Course Outcomes:**

   Upon successful completion of the course, the students will able to

   CO1: analyze various fluid properties in the fluid flow problems and compute hydrostatic forces on submerged bodies

   CO2: identify the fluid flows and their behavior

   CO3: apply conservation laws to derive governing equations of fluid flow

   CO4: analyze the fluid properties using dimensional analysis and conduct model studies

   CO5: analyze flow in pipes, parallel plates and determine the losses in pipes

   CO6: Measure flow in tanks and canal by using various flow measuring instruments.

5. **Program Outcomes:**

   Graduates of the Civil Engineering Program will have

   a. An ability to apply knowledge of mathematics, science and engineering principles to civil engineering problems.
   b. An ability to analyze design and conduct experiments and interpret the resulting data.
   c. An ability to design a system, component or process to meet desired goals in civil engineering applications.
   d. An ability to function on multi disciplinary teams.
   e. An ability to identify, formulate and solve challenging engineering problems.

f. An understanding of professional and ethical responsibility.
g. An ability to communicate effectively through verbal, written and drawing presentations.
h. An ability to understand the impact of engineering solutions in a global, economical and social context with a commitment on environmental and safety issues.
i. An ability to recognize the need of engaging in lifelong learning and acquiring further knowledge in specialized areas.
j. Ability to excel in competitive examinations, advanced studies and become a successful engineer in construction industry.
k. An ability to use the techniques, skills and modern engineering tools and software for engineering design and practices.
l. An ability to understand basic finance & management techniques and construction practices including work procurement and legal issues.

6. Mapping of Course Outcomes with Program Outcomes: (Tick)

	a	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**
CO1	✓				✓							
CO2				✓	✓					✓		
CO3	✓	✓	✓									
CO4			✓							✓		
CO5			✓							✓		
CO6		✓			✓					✓		

7. Prescribed Text Books

a. Fluid Mechanics -P.N. Modi & S.M. Seth, Standard book house.
b. A text of Fluid Mechanics and Hydraulic Machines by Dr.R.K.Bansal – Laxmipublications(p)ltd., New Delhi.

8. Reference Text Books

a. Fluid Mechanics and Hydraulic machines, Rajput, S. Chand & Co.
b. Fluid Mechanics by J.F.Douglas, J.M. Gaserek and J.A.Swaffirld (Longman)
c. Fluid Mechanics by Frank.M.White (Tata Mc.Grawhillpvt. Ltd)
d. Fluid Mechanics by A.K.Mohanty, Prentice Hall of India Pvt.Ltd., New Delhi

9. URLs and Other E-Learning Resources

a. www.altavista.com
b. www.vivisimo.com
c. www.tatamegrawhill.com

10. Digital Learning Materials:

- Fluid Mechanics by Prof. S.K. SomIIT, Kharagpur.

11. Lecture Schedule / Lesson Plan

Topic	No. of Periods	
	Theory	**Tutorial**
UNIT –1: Introduction		
Physical Properties of Fluids - Specific gravity, Viscosity and problems	2	
Surface tension and Capillarity with problems	1	
Vapor pressure and their influences on fluid motion	1	
Hydro static law	1	2
Atmospheric, Gauge and Vacuum pressures – measurement of pressure	2	
Pressure gauges, Manometers –Differential and Inverted Manometers	2	
Hydrostatic forces on submerged plane-Horizontal, Vertical and Inclined surfaces	2	2
Center of pressure, derivations and problems	2	
Buoyancy-Introduction	1	
UNIT – 2: Fluid Kinematics		
Methods of describing fluid motion	1	
Classification of flows - Steady, Unsteady, Uniform, Non Uniform, Laminar, Turbulent, Rotational and Irrotational	1	2
Three , two and one dimensional flows	1	
Stream line, Path line, Streak line	2	2
Equation for acceleration, Convective and Local acceleration with problems	2	
Continuity Equation with problems	1	
Velocity Potential and Stream Function	1	2
Flow net-Introduction	1	
UNIT – 3: Fluid Dynamics		
Surface and body forces Euler's Equation of motion.	1	
Bernoulli's Equation from Euler's Equation with problems	2	2
Applications-Venturimeter and Orifice meter	2	
Momentum Principle	1	
Application of Momentum equations	1	
Force exerted on a Pipe Bend with problems	1	
UNIT – 4: Hydraulic Similitude		
Dimensional analysis-Rayleigh's method	1	2
Buckingham's pi theorem	1	
Study of Hydraulic models	2	
Hydraulic similitude	1	2
Dimensionless numbers	2	
Model laws	1	
UNIT – 5: Closed Conduit Flow		
Reynolds's experiment	1	
Characteristics of Laminar & Turbulent flows	1	
Flow through circular pipe	1	
Flow between Parallel Plates with problems	1	2
Laws of fluid friction	1	
Darcy's Equation, Minor Losses with problems	2	
Pipes in Series and Parallel with problems	1	2
Total Energy line and Hydraulic Gradient line with problems	2	
UNIT – 6: Flow Measurements		
Pitot tube	1	
Orifices-Fully submerged and partially submerged	2	2
Flow through Notches rectangle, triangle	2	

Trapezoidal, stepped notches with problems	1	
Total No. of Periods:	**56**	**28**

12. Seminar Topics

- Classification of Flows
- Flow measurements

UNIT –I

Objective:

The course will introduce fluid mechanics and establish its relevance in civil engineering.

- Understand nature of fluids.
- Learn fluid properties and flow characteristics.

Syllabus:

Physical Properties of Fluids - Specific gravity, Viscosity and problems, Surface tension and Capillarity with problems, Vapor pressure and their influences on fluid motion,Hydro static law, Atmospheric, Gauge and Vacuum pressures – measurement of pressure, Pressure gauges, Manometers –Differential and Inverted Manometers, Hydrostatic forces on submerged plane, Horizontal, Vertical and Inclined surfaces, Center of pressure, buoyancy-Introduction.

Learning Outcomes:

The student will be able to

1. Understand the basic concepts of Mechanics of Fluids.
2. Describe the properties of fluids and obtain the intensity of pressure distribution.
3. Recognize the various types of fluid flow problems encountered in practice.
4. Model engineering problems and solve them in a systematic manner.
5. Compute hydrostatic forces on surfaces immersed in liquid.

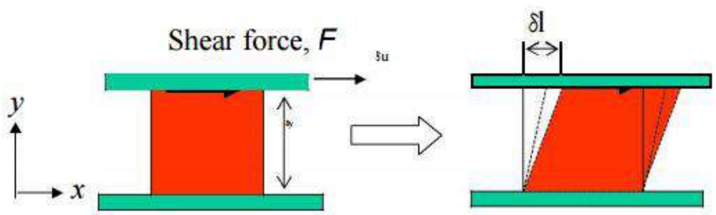
Learning Material

UNIT –1: INTRODUCTION:

Matter can be distinguished by the physical forms known as solid, liquid, and gas. The liquid and gaseous phases are usually combined and given a common name of fluid. Solids differ from fluids on account of their molecular structure (spacing of molecules and ease with which they can move). The intermolecular forces are large in a solid, smaller in a liquid and extremely small in gas.

DEFINITION OF FLUID:

A fluid is a substance which deforms continuously under the action of shearing forces, however small they may be.It follows that: If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.



Fluid deforms continuously under the action of a shear force

$\tau_{yx} = \frac{dF_x}{dA_y} = f(\text{Deformation Rate})$

Shear stress in a moving fluid:

Although there can be no shear stress in a fluid at rest, shear stresses are developed when the fluid is in motion, if the particles of the fluid move relative to each other so that they have different velocities, causing the original shape of the fluid to become distorted. If, on the other hand, the velocity of the fluid is same at every point, no shear stresses will be produced, since the fluid particles are at rest relative to each other.

Differences between solids and fluids: The differences between the behavior of solidsand fluids under an applied force are as follows:

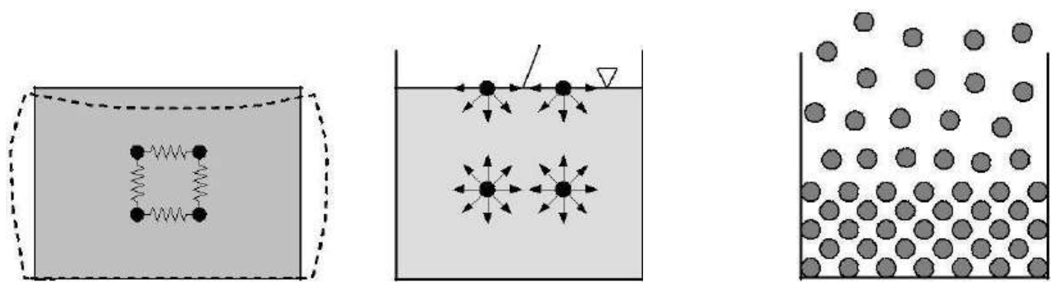
i. For a solid, the strain is a function of the applied stress, providing that the elastic limit is not exceeded. For a fluid, the rate of strain is proportional to the applied stress.
ii. The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow as long

as the force is applied and will not recover its original form when the force is removed.

Differences between liquids and gases:

Although liquids and gases both share the common characteristics of fluids, they have many distinctive characteristics of their own. A liquid is difficult to compress and, for many purposes, may be regarded as incompressible. A given mass of liquid occupies a fixed volume, irrespective of the size or shape of its container, and a free surface is formed if the volume of the container is greater than that of the liquid.

A gas is comparatively easy to compress (Fig.1). Changes of volume with pressure are large, cannot normally be neglected and are related to changes of temperature. A given mass of gas has no fixed volume and will expand continuously unless restrained by a containing vessel. It will completely fill any vessel in which it is placed and, therefore, does not form a free surface



Solid liquid gas

Definition applied to Static Fluids

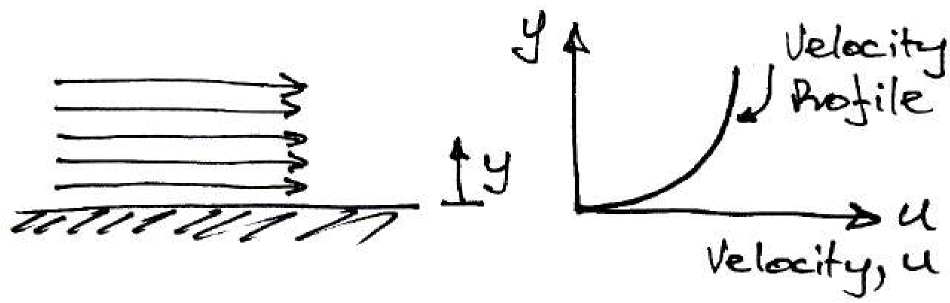
According to this definition, if we apply a shear force to a fluid it will deform and take up a state in which no shear force exists. Therefore, we can say: If a fluid is at rest there can be no shearing forces acting and therefore all forces in the fluid must be perpendicular to the planes in which they act. Note here that we specify that the fluid must be at rest. This is because it is found experimentally that fluid in motion can have slight resistance to shear force. This is the source of viscosity.

Definition applied to Fluids in Motion

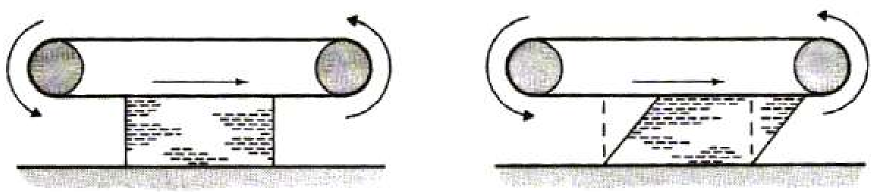
For example, consider the fluid shown flowing along a fixed surface. At the surface there will be little movement of the fluid (it will 'stick' to the surface), whilst further away from the surface the fluid flows faster (has greater velocity):

Fluid motion is mainly divided into fluid kinematics and fluid dynamics, Fluid kinematics is the section fluid motion will be described without concern with the actual forces necessary to produce the motion.

Fluid dynamics is the section fluid motion will be described with concern with the actual forces necessary to produce the motion.



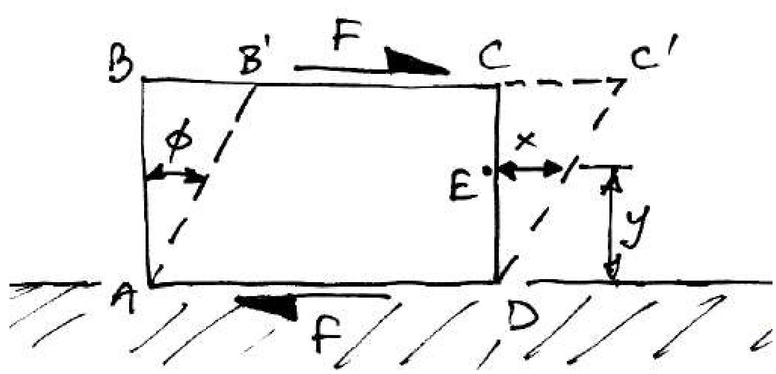
If one layer of is moving faster than another layer of fluid, there must be shear forces acting between them. For example, if we have fluid in contact with a conveyor belt that is moving we will get the behavior shown:



Ideal fluid **Real (Viscous) Fluid**

When fluid is in motion, any difference in velocity between adjacent layers has the
Same effect as the conveyor belt does.

Therefore, to represent real fluids in motion, the action of shear forces shall be considered.



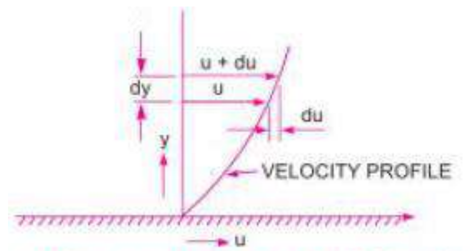
Consider the small element of fluid shown, which is subject to shear force and has a dimension *s* into the page. The force *F* acts over an area $A = BC \times s$. Hence we have a *shear stress* applied:

Stress =Force/Area

τ= F/A
Any stress causes a deformation, or strain, and a shear stress causes a *shear strain*.
This shear strain is measured by the angle φ.
Remember that a fluid *continuously* deforms when under the action of shear. This is different to a solid: a solid has a single value of φ for each value of τ. So the longer a shear stress is applied to a fluid, the more shear strain occurs. However, what is known from experiments is that the rate of shear strain (shear strain per unit time) is related to the shear stress:
Shear stress α Rate of shear strain
Shear stress= Constant x Rate of shear strain
We need to know the rate of shear strain. From the diagram, the shear strain is:
φ = x/y

Viscosity (Derivation for Newton's Law of Viscosity)

The viscosity of a fluid determines the amount of resistance to shear force. Or Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow. Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it. In case of liquids, viscosity is due to cohesive force between the molecules of adjacent layers of liquid. In case of gases, molecular activity between adjacent layers isthe cause of viscosity.



Velocity variation near a solid boundary.

When two layers of a fluid , a distance 'dy' apart, move one over the other at different velocities, say 'u' and 'u+du' shown in fig., the viscosity together with relative velocity causes shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent layer. This shear stress is proportional to the rate of change of velocity with respect to 'y'.

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

Where μ (mu) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From this we can write viscosity as $\mu = \frac{\tau}{(\frac{du}{dy})}$

Units of Viscosity

$$\mu = \frac{Shear\ stress}{\frac{change\ of\ velocity}{change\ of\ distance}} = \frac{\frac{Force}{area}}{\frac{Length}{Time} \text{X} \frac{1}{\text{Length}}}$$

$$\mu = \frac{\frac{Force}{Length^2}}{\frac{1}{Time}} = \frac{Force\ \text{X Time}}{Length^2}$$

MKS unit of viscosity = $\frac{kgf-sec}{m^2}$

CGS unit of viscosity = $\frac{dyne-sec}{cm^2}$

SI unit of viscosity = Ns/m^2 = Pa s.

Since N/m2 is also known as Pascal which is represented by Pa. Hence N/m^2 = Pa.

Newton's law of viscosity:

Let us consider a liquid between the fixed plate and the movable plate at a distance 'Y' apart, 'A' is the contact area (Wetted area) of the movable plate, 'F' is the force required to move the plate with a velocity 'U' According to Newton's law shear stress is proportional to shear strain.

$$\mu = \frac{\tau}{(\frac{du}{dy})} = \frac{shear\ stress}{rate\ of\ shear\ strain}$$

Non- Newtonian fluids

• *Plastic:* Shear stress must reach a certain minimum before flow commences.

• *Pseudo-plastic:* No minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. substances like clay, milk and cement.

• *Dilatant substances;* Viscosity increases with rate of shear, e.g. quicksand.

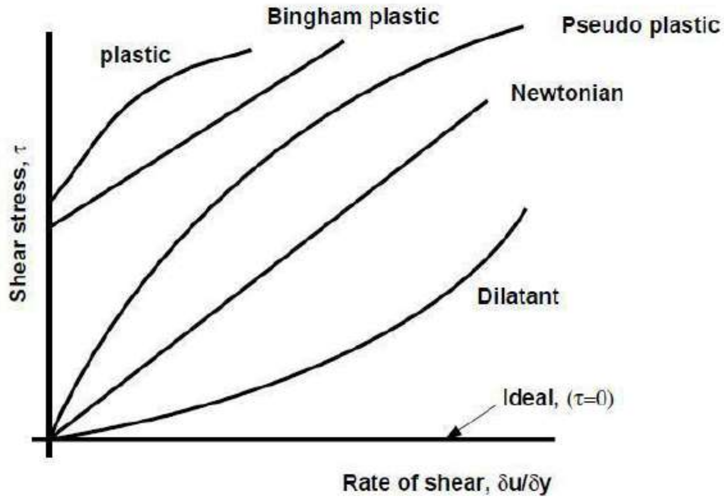
• *Viscoelastic materials:* Similar to Newtonian but if there is a sudden large change in shear they behave like plastic.

• *Solids:* Real solids do have a slight change of shear strain with time, whereasideal solids (those we idealized for our theories) do not.

Lastly, considering the *ideal fluid*, which is assumed to have noviscosity and is useful for developing theoretical solutions. It helps to achievesome practically useful solutions.

♦ *Effect of Pressure on Viscosity of fluids:*

Pressure has very little or no effect on the viscosity of fluids.



♦ *Effect of Temperature on Viscosity of fluids:*

1.Effect of temperature on viscosity of liquids: Viscosity of liquids is due to cohesive force between the molecules of adjacent layers. As the temperature increases cohesive force decreases and hence viscosity decreases.

2. Effect of temperature on viscosity of gases: Viscosity of gases is due to molecular activity between adjacent layers. As the temperature increases molecular activity increases and hence viscosity increases.

Kinematic viscosity.

Its defined as the ratio between the dynamic viscosity and density of fluid. Its denonted by the Greek symbol (υ) called 'nu'. Thus mathematically,

$$\upsilon = \frac{Viscosity}{Density} = \frac{\mu}{\rho}$$

the units of kinematic viscosity is obtained as $\upsilon = \frac{Force\ X\ Time}{Length^2\ X\ \frac{Mass}{Length^3}} = \frac{Force\ X\ Time}{\frac{Mass}{Length}}$

$$= \frac{Mass\ X\ \frac{Length}{Time^2}\ X\ Time}{\frac{Mass}{Length}} = \frac{Length^2}{Time}$$

In MKS and SI the unit of kinematic viscosity is m^2/s while in CGS its unit is cm^2/s or stroke.

1Stroke = $1cm^2/s = 10^{-4}m^2/s$

Properties of Fluids

Mass Density

The mass per unit volume of a substance usually denoted as ρ. Typical values of some substances are:

• Water: 1000 kg/m^3;

• Mercury: 13546 kg/m^3;

• Air: 1.23 kg/m^3;

• Paraffin: 800 kg/m^3.

Specific Weight

The weight of a unit volume a substance, usually denoted as γ. Essentially density times the acceleration due to gravity:

$\gamma = \rho g$

Specific Gravity

Its defined as the ratio of the specific weight/mass density of a substance to the specific weight/mass density of a standard substance.

A dimensionless measure of the density of a substance with reference to the density of some standard substance, usually water at 4°C:

Specific Gravity= density of substance/ density of water

Bulk Modulus

In analogy with solids, the bulk modulus is the modulus of elasticity for a fluid. It isthe ratio of the change in unit pressure to the corresponding volume change per unitvolume, expressed as:

$$\frac{changeinvolume}{originalvolume} = \frac{changeinpressure}{bulkmodulus}$$

$K = -V\frac{dp}{dV}$

In which the negative sign indicates that the volume reduces as the pressure increases. The bulk modulus changes with the pressure and density of the fluid, but for liquids can be considered constant for normal usage.

Typical values are:

• Water: 2.05 GN/m^3;

• Oil: 1.62 GN/m^3.

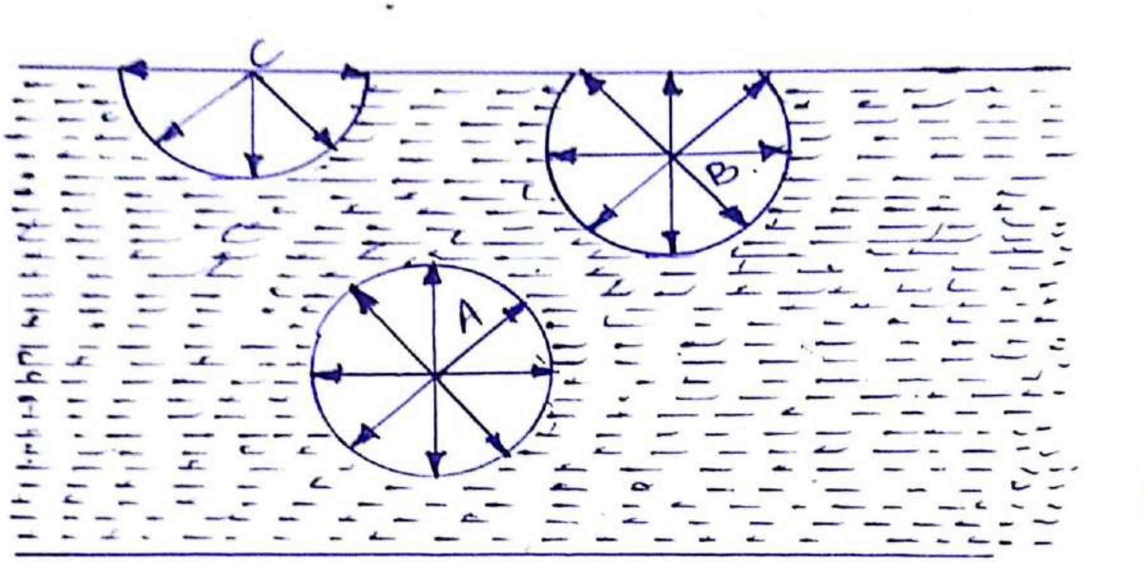
The units are the same as those of stress or pressure

Compressibility:

It's the inverse of the bulk modulus.

SURFACE TENSION

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension/ The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ.



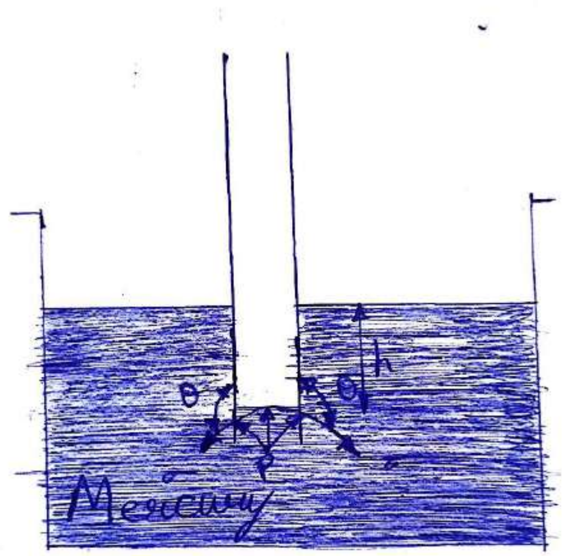
Surface tension

Vapor pressure

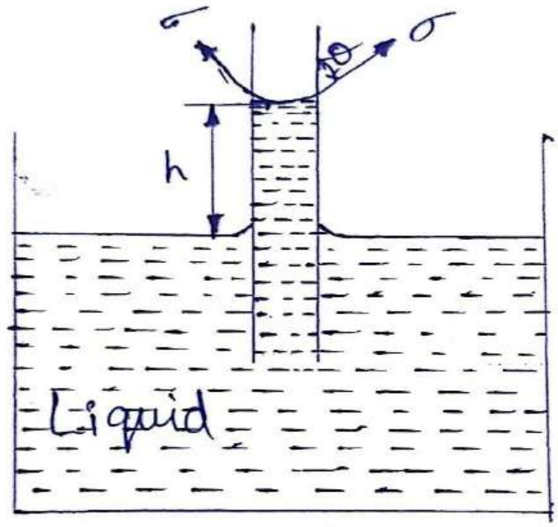
The **pressure** exerted by the gas in equilibrium with a solid or liquid in a closed container at a given temperature is called the vapor pressure.

Capillarity:

Any liquid between contact surfaces attains curved shaped surface as shown infigure. The curved surface of the liquid is called Meniscus. If adhesion is more thancohesion then the meniscus will be concave. If cohesion is greater than adhesionmeniscus will be convex.



Capillary fall



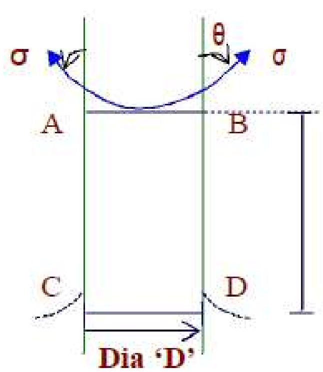
Capillary rise

Capillarity is the phenomena by which liquids will rise or fall in a tube of smalldiameter dipped in them. Capillarity is due to cohesion adhesion and surface tension ofliquids. If adhesion is more than cohesion then there will be capillary rise. If cohesion isgreater than adhesion then will be capillary fall or depression. The surface tensile forcesupports capillary rise or depression.

To derive an expression for the capillary rise of a liquid in small tube dipped in it:

Let us consider a small tube of diameter 'D' dipped in a liquid of specific weight γ. 'h' is the capillary rise. For the equilibrium,

Vertical force due to surface tension = Weight of column of liquid ABCD



$[\sigma(\Pi D)]$ cos $\theta = \gamma$ x volume

$[\sigma(\Pi D)]$ cos $\theta = \gamma$ x$(\frac{\pi D2}{4})$ x h

$h = \frac{4\,\sigma cos\theta}{\gamma D}$

It can be observed that the capillary rise is inversely proportional to the diameterof the tube.

Note:

The same equation can be used to calculate capillary depression. In such cases ‘ θ ’ willbe obtuse ‘h’ works out to be –ve.

Hydrostatics

Introduction

Pressure

In fluids we use the term pressure to mean:

The perpendicular force exerted by a fluid per unit area.

This is equivalent to stress in solids, but we shall keep the term pressure. Mathematically, because pressure may vary from place to place, we have:

$P=\lim_{\Delta\to 0}\frac{\Delta F}{\Delta A}$

As we saw, force per unit area is measured in N/m2 which is the same as a pascal (Pa). The units used in practice vary:

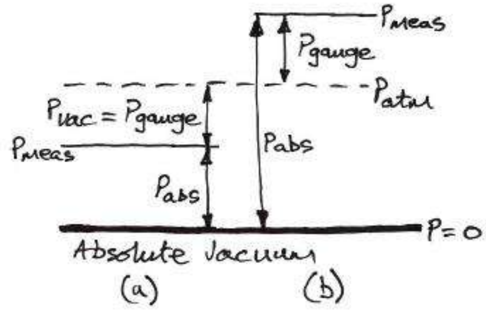
- 1 kPa = 1000 Pa = 1000 N/m^2
- 1 MPa = 1000 kPa = 1 × 10^6 N/m^2
- 1 bar = 10^5 Pa = 100 kPa = 0.1 MPa
- 1 atm = 101,325 Pa = 101.325 kPa = 1.01325 bars = 1013.25 millibars

For reference to pressures encountered on the street which are often imperial:

- 1 atm = 14.696 psi (i.e. pounds per square inch)
- 1 psi = 6894.7 Pa ≈ 6.89 kPa ≈ 0.007 MPa

Pressure Reference Levels

The pressure that exists anywhere in the universe is called the *absolute pressure*, *P abs* . This then is the amount of pressure greater than a pure vacuum. The atmosphere on earth exerts *atmospheric pressure*, *P atm*, on everything in it. Often when measuring pressures we will calibrate the instrument to read zero in the open air. Any measured pressure, *P meas*, is then a positive or negative deviation from atmospheric pressure. We call such deviations a *gauge pressure*, *P gauge*. Sometimes when a gauge pressure is negative it is termed a *vacuum pressure*, *P vac*.



The above diagram shows:

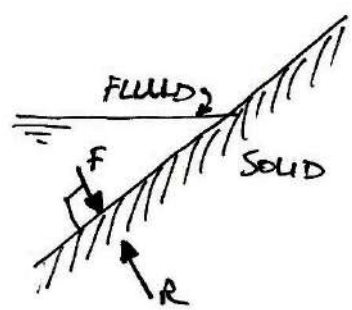
(a) the case when the measured pressure is below atmospheric pressure and so is anegative gauge pressure or a vacuum pressure;

(b) the more usual case when the measured pressure is greater than atmosphericpressure by the gauge pressure.

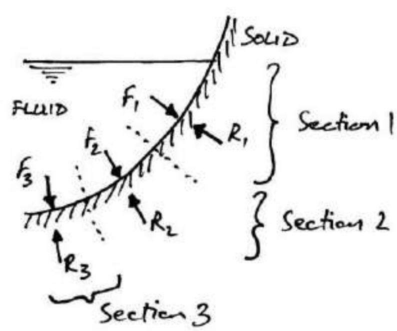
Fluid pressure at a point

We applied the definition of a fluid to the static case previously and determined thatthere must be no shear forces acting and thus only forces normal to a surface act in afluid.

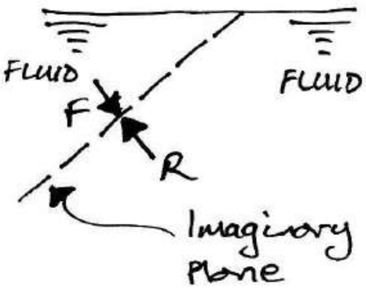
For a flat surface at arbitrary angle we have:



A curved surface can be examined in sections:



And we are not restricted to actual solid-fluid interfaces. We can consider imaginary planes through a fluid:



Pascal's Law

This law states:

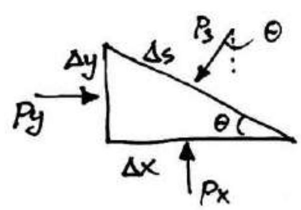
The pressure at a point in a fluid at rest is same in all directions.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in fig. below. Let the width of the element perpendicular to the plane of paper is unity and p_x, p_y and $p_z$are the pressures or intensity of pressure acting on the faces. Let θ be the angle between the two sides

Then the forces acting on the element are:

1. Pressure force normal to the surface.
2. Weight of the element in the vertical direction

As with the static objects the forces in the x and y direction should balance. Hence $\Sigma F_x = 0$



$P_y.\Delta y - P_s.\ \Delta s.\ \sin\theta = 0$

But $\sin\theta = \frac{\Delta y}{\Delta s}$, therefore

$Py.\ \Delta y - Ps.\ \Delta s.\ \frac{\Delta y}{\Delta s} = 0$

$Py.\ \Delta y = Ps.\ \Delta y$

$Py = Ps$

$\Sigma Fy = 0$

$Px.\ \Delta x - Ps.\ \Delta s.\ \cos\theta = 0$

But $\cos\theta = \frac{\Delta x}{\Delta s}$, therefore

$Px.\ \Delta x - Ps.\ \Delta s.\ \frac{\Delta x}{\Delta s} = 0$

$Px.\ \Delta x = Ps.\Delta x$

$Px = Ps$

Hence for any angle $Px = Py = Ps$

And so the pressure at a point is the same in any direction. Note that we neglected the weight of the small wedge of fluid because it is infinite small. This is why Pascal's Law is restricted to the pressure at a point.

Pressure Measurement

Pressure Head

Pressure in fluids may arise from many sources, for example pumps, gravity, momentum etc. Since $p = \rho gh$, a height of liquid column can be associated with the pressure p arising from such sources. This height, h, is known as the pressure head.

Pressure Variation with Depth(Hydrostatic law):

The pressure at any point in a fluid at rest is obtained by the Hydrostatic law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point. This is proved as

Consider a small fluid element as shown in Fig.

Let ΔA = Cross-sectional area of element

ΔZ = Height of fluid element

P = pressure on face AB

Z = distance of fluid element from free surface

The forces acting on the fluid element are:

1. Pressure force on AB = p x ΔA and acting perpendicular to the face AB in the downward direction.

2. Pressure force on CD = (p + $\frac{\partial p}{\partial Z}\Delta Z$) x ΔAand acting perpendicular to the face C Dverticallyupward direction.

3. Weight of fluid element = ρ x g x ΔA x ΔZ.

4. Pressure forces on surfaces BC and AD are equal and opposite. For equilibrium of the fluid element we have p x ΔA - (p + $\frac{\partial p}{\partial Z}\Delta Z$) x ΔA + ρ x g x ΔA x ΔZ =0

Or $\quad p \times \Delta A - p\Delta A - \frac{\partial p}{\partial Z}\Delta Z \times \Delta A + \rho \times g \times \Delta A \times \Delta Z = 0$

Or $\quad - \frac{\partial p}{\partial Z}\Delta Z \times \Delta A + \rho \times g \times \Delta A \times \Delta Z = 0$

Or $\quad \frac{\partial p}{\partial Z} = \rho \times g = wg \qquad (\rho \times g = w)$

w = weight density of fluid

The above equation states that the rate of increase of pressure in a vertical direction is equal to the weight density of the fluid at that point. This is

Hydrostatic law.

By integrating the above equation for liquids we get

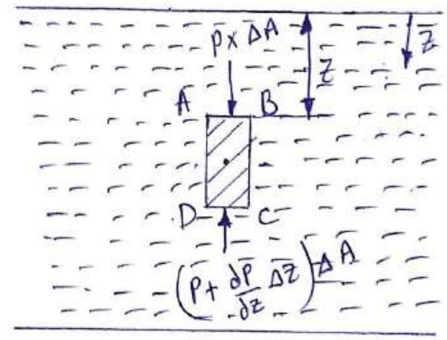
$\int dp = \int \rho g dZ$

$P = \rho g Z$

Where p is the pressure above atmospheric pressure and Z is the height of the surfaces.

From this we can write $Z = \frac{P}{\rho g}$

Here Z is called pressure head.



Forces on a fluid element

Measurement of pressure

The various devices adopted for measuring fluid pressure may be broadly classified as

1. Manometers
2. Mechanical gauges.

Manometers

A manometer (or liquid gauge) is a pressure measurement device which uses the relationship between pressure and head to give readings.

The manometers may be classified as

a. Simple manometers

i. Piezometer

ii. U-tube Manometer

iii. Single Column Manometer

b. Differential manometers

i. Two-Piezometer Manometer

ii. Inverted U-tube Manometer

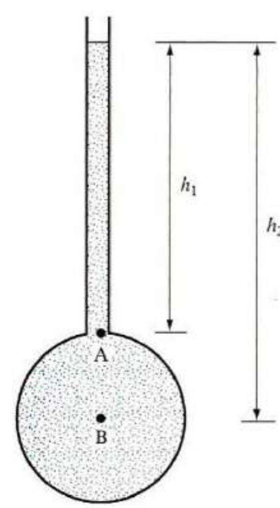
iii. U-tube Differential Manometer

iv. Micromanometer

In the following, we wish to measure the pressure of a fluid in a pipe.

Piezometer

This is the simplest gauge. A small vertical tube is connected to the pipe and its top is left open to the atmosphere, as shown.



The pressure at *A* is equal to the pressure due to the column of liquid of height h_1

$P_A = \rho g h_1$

Similarly,

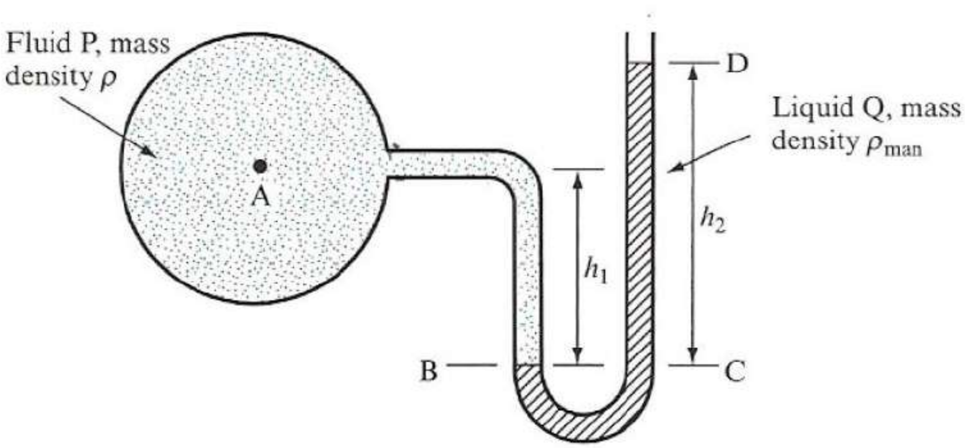
$P_B = \rho g h_2$

The problem with this type of gauge is that for usual civil engineering applications, if the pressure is large (e.g. 100 kN/m^2) the height of the column is impractical (e.g.10 m).

Such a gauge is useless for measuring gas pressures.

U-tube Manometer

To overcome the problems with the piezometer, the U-tube manometer seals the fluidby using a measuring (manometric) liquid:



Choosing the line *BC* as the interface between the measuring liquid and the fluid,

Pressure at *B*, P_B= Pressure at *C*, P_C

For the left-hand side of the U-tube:

$$p_B = p_A + \rho g h_1$$

For the right hand side:

$$p_C = \rho_{man} g h_2$$

Neglectingatmospheric pressure and dealing with gauge pressures.

$$p_B = p_C$$
$$p_A + \rho g h_1 = \rho_{man} g h_2$$

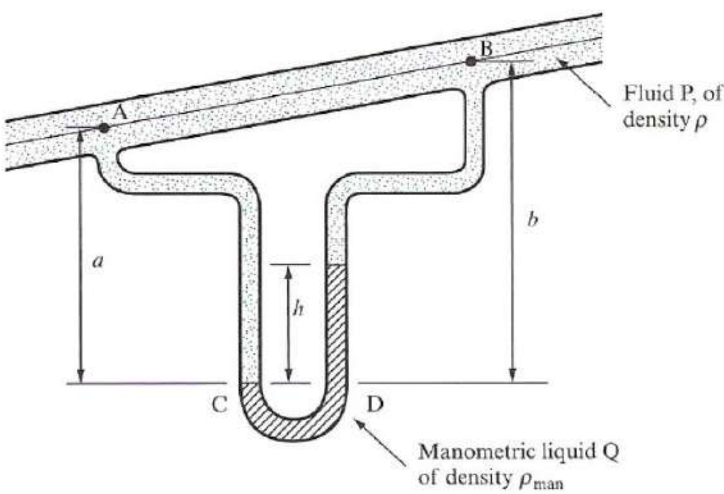
And so:

$$p_A = \rho_{man} g h_2 - \rho g h_1$$

In any continuous fluid, the pressure is the same at any horizontal level.

Differential Manometer

To measure the pressure difference between two points we use a u-tube as shown:

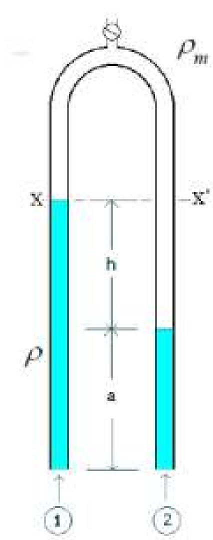


$$\text{Pressure at } C,\ p_C = \text{Pressure at } D,\ p_D$$

$$p_A + \rho g a = p_B + \rho g (b - h) + \rho_{man} g h$$

Hence the pressure difference is:

$$p_A - p_B = \rho g (b - a) + h g (\rho_{man} - \rho)$$



Inverted U- Tube manometer

It is used for measuring pressure differences in liquids. The space above the liquid in the manometer is filled with air, which can be admitted or expelled through the tap A in order to adjust the level of the liquid in the manometer.

The pressure at level x will be: $Px = P1 - \rho g\ (a+h)$

The pressure at level x' will be: $P2 - \rho m\ g\ h - \rho\ g\ a$

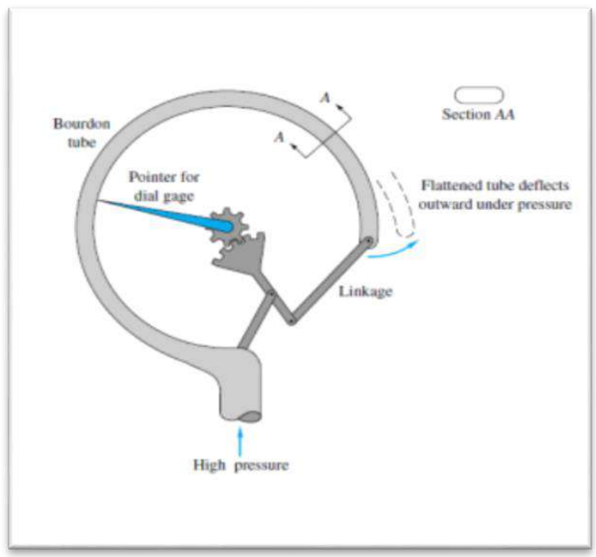
Since Px = Px' (at same level)

Then $P1 - P2 = (\rho - \rho m)\ gh$

Mechanical Gage

Whenever a very high fluid pressure is to be measured, and a very great sensitivity a mechanical gauge is best suited for these purposes. They are also designed to read vacuum pressure. A mechanical gauge is also used for measurement of pressure in boilers or other pipes, where tube manometer cannot be conveniently used.

i. The Bourdon tube pressure gauge or simply Bourdon pressure gauge



It's a type of mechanical gauge.

The pressure to be measured is applied to a curved tube, oval in cross section, and the deflection of the end of the tube is communicated through a system of levers to a recording needle. This gauge is widely used for steam and compressed gases, and frequently forms the indicating element on flow controllers.

ii. Diaphragm Pressure gauge

The pressure responsive element in this gage is an elastic steel corrugated diaphragm. The elastic deformation of the diaphragm under pressure is transmitted to a pointer by a similar arrangement as in the case of Bourdon tube pressure gage. This is used to measure relatively low pressure intensities.

Hydrostatic forces on submerged planes

Total Pressure and Center of Pressure

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and center of pressure is to be determined. The submerged surfaces may be:

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface and
4. Curved surface

Forces on Submerged Surfaces in Static Fluids

We have seen these features of static fluids

- Hydrostatic vertical pressure distribution
- Pressures at any equal depths in a continuous fluid are equal
- Pressure at a point acts equally in all directions (Pascal's law).
- Forces from a fluid on a boundary acts at right angles to that boundary.

<u>Fluid pressure on a surface</u>

Pressure is force per unit area.

Pressure p acting on a small area dA exerted force will be

$F = \rho g dA$

Since the fluid is at rest the force will act at right-angles to the surface.

Vertical Plane Surface Submerged in liquid

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. below.

Let A = total area of the surface

$\bar{h}$ = distance of C.G of the area from free surface of liquid

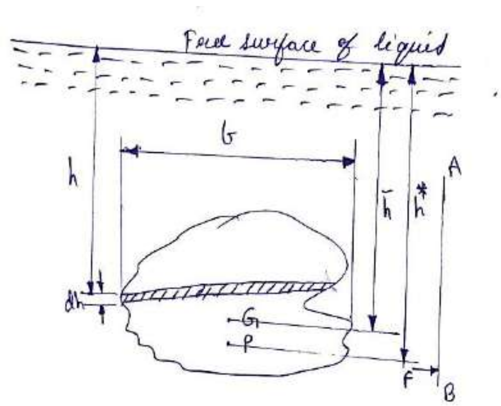
G = centre of gravity of plane surface

P = Centre of pressure

h^* = distance of centre of pressure from free surface of liquid.

Total pressure (F). the total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small stip.

Consider a strip of thickness dh and width b at the depth of h from free surface of liquid as shown in fig.



Vertical plane surface Immersed in liquid

Pressure intensity on the strip p = ρgh

Area of strip dA = b x dh

Total pressure force on the strip dF = p x dA

= ρgh xb x dh

Therefore total pressure force on the whole surface,

F = $\int dF = \int \rho g h \, x \, b \, x \, dh$ = ρg$\int b \, x \, h \, x \, dh$

But $\int b \, x \, h \, x \, dh = \int h \, x \, dA$

= Moment of surface area about the free surface of the liquid

= Area of surface x distance of C.G from the free surface

= A x $\bar{h}$

Therefore P = ρgh.A

(a) Total pressure force (F): the total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig.3.1

pressure intensity of the strip, p = ρgh

Area of the strip, dA = b x dh

Total pressure force on the strip, dF = p x Area

= ρgh xb x dh

Total pressure force on the whole surface,

F = $\int dF = \int \rho g h \, x \, b \, x \, dh$ = ρg$\int b \, x \, h \, x \, dh$

But $\int b \, x \, h \, x \, dh = \int h \, x \, dA$

= Moment of surface area about the free surface of the liquid

= Area of surface x distance of C.G from the free surface

= A x $\bar{h}$

P = ρgh.A

For water the value of ρ = 1000Kg/m^{3}and g =9.81m/s^2. The force will be in Newton.

(b) Centre of pressure (h*): Center of pressure is calculated by using the "principal of moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h* from free surface of the liquid as shown in Fig. Hence moment of the force F about free surface of the liquid = F x h*

Moment of force dF, acting on a strip about free surface of liquid

=dF x h {Since dF=ρgh x b x dh}

=ρgh x b x dh x h

Sum of moments of all such forces about free surface of liquid

$=\int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$

$=\rho g \int b h^2 dh = \rho g \int h^2 dA$

But $\int h^2 dA = \int b h^2 dh$

=Moment of inertia of the surface about free surface of liquid

$=I_o$

Sum of moments about free surface

$=\rho g I_o$

$F \times h^* = \rho g I_o$

But $F = \rho g A\bar{h}$

Therefore $\rho g A\bar{h} \times h^* = \rho g I_o$

Or $h^* = \frac{\rho g I_o}{\rho g A\bar{h}} = \frac{I_o}{A\bar{h}}$

By the theoremof parallel axis, we have

$I_o = I_G + A \times \bar{h}^2$

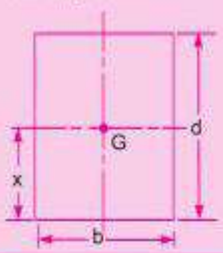
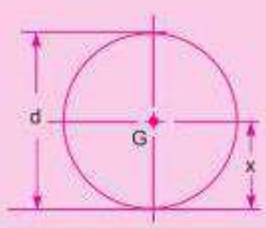
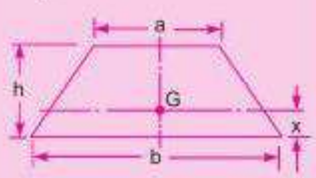
Where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

(i) Center of pressure (i.e.,h*) lies below the center of gravity of the vertical surface.

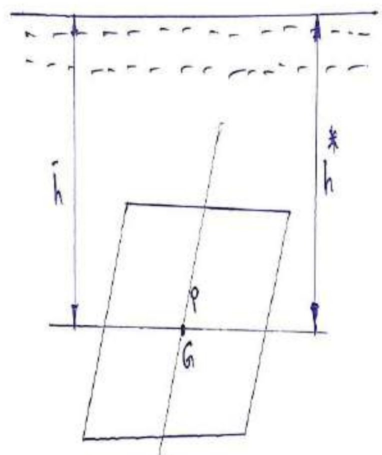
(ii) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

The moments of inertia and other geometric properties of some important plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Horizontal Plane Surface Submerged in a liquid

Consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to, $p = \rho g h$, whereh is depth of surface.



Horizontal plane surface submerged in liquid

Let A = Total area of surface

Then total force, F, on the surface

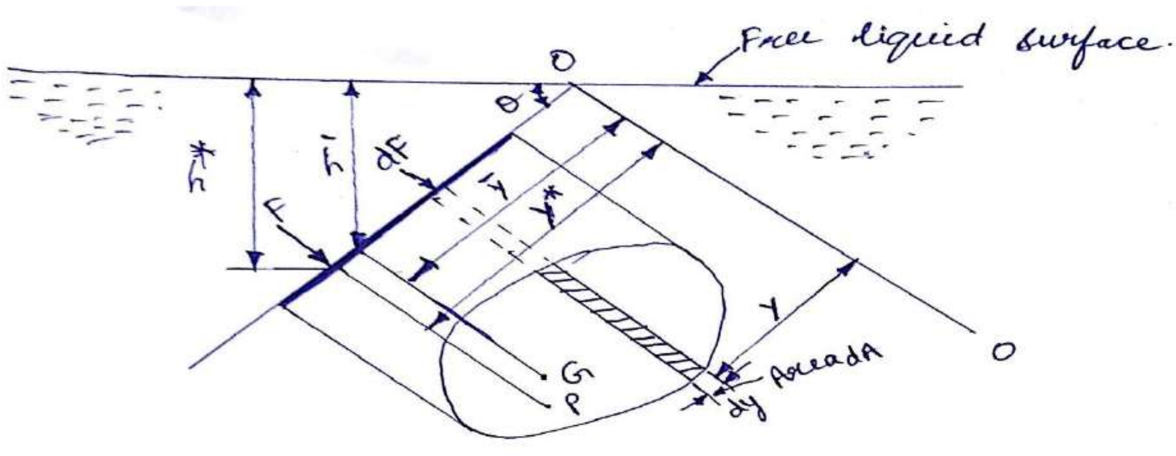
$= p$ x Area = ρg x h x A = $\rho g A\bar{h}$

Where $\bar{h}$ = Depth of C.G from free surface of liquid = h

Also h*= Depth of centre of pressure from free surface = h.

Inclined Plane Surface Submerged in a liquid

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in Fig.



Inclined immersed surface.

Let A = Total area of inclined surface

$\bar{h}$ = Depth of C.G. of inclined area from free surface

h*= Distance of centre of pressure from free surface of liquid

θ= Angle made by the plane of the surface with free liquid surface.

Let the plane surface, if produced meet the free liquid surface at *0*. Then *0-0* is the axis perpendicular to the plane of the surface.

Let $\bar{y}$ = distance of the C.G. of the inclined surface from *0-0*

y* = distance of the centre of pressure from *0-0.*

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis

0-0 as shown in Fig.3.18.

Pressure intensity on the strip, p = ρgh

∴ Pressure force, dF, on the strip, $dF = p$ x Area of strip = ρgh x dA

Total pressure force on the whole area, F = $\int dF = \int \rho g h dA$

But from Fig.3.18, $\frac{h}{y} = \frac{\bar{\bar{h}}}{y} = \frac{h*}{y*} = \sin\theta$

∴ $h = y \sin\theta$

∴ $F = \int \rho g \; x \; y \; x \sin\theta \; x \; dA = \rho g \sin\theta \int y dA$

But $\int ydA = A\bar{y}$

Where $\bar{y}$= Distance of C.G. from axis *0-0*

$F = \rho g \sin\theta \; \bar{y}$ x A

$= \rho g A\bar{h}$

Center of Pressure

Pressure force on the strip, $dF = \rho g h dA$

$= \rho g y \sin\theta \; dA$

Movement of the force, dF, about axis *0-0*

$= dF$ x y = $\rho g y \sin\theta \; dA$x y = $\rho g \sin\theta \; y^2 dA$

Some of the movements of all such forces about *0-0*

$= \int \rho g \sin\theta \; y^2 \; dA = \rho g \sin\theta \int y^2 dA$

But $\int y^2 dA$ = M.O.I. of the surface about *0-0* = I_o

∴ Sum of movements of all forces about *0-0* = $\rho g \sin\theta \; I_o$

Movement of the total force ,F, about *0-0* is also given by

= F x y*

Where y* = Distance of centre of pressure from *0-0.*

Equating the two values given by equation (3.7) and (3.8)

F x y* = $\rho g \sin\theta \; I_o$

Or $$y^* = \frac{\rho g \sin\theta\, Io}{F}$$

Now $$y^* = \frac{h*}{\sin\theta},\ F = \rho g A\bar{h}$$

And I_o by the theorem of parallel axis = $I_G + A\,\bar{y}^2$

Substituting these values in equation (3.9), we get

$$\frac{h*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}}\,[I_G + A\,\bar{y}^2]$$

∴ $$h^* = \frac{\sin^2\theta}{A\bar{h}}\,[I_G + A\,\bar{y}^2]$$

But $$\frac{\bar{h}}{\bar{y}} = \sin\theta \quad \text{or} \quad \bar{y} = \frac{\bar{h}}{\sin\theta}$$

∴ $$h^* = \frac{\sin^2\theta}{A\bar{h}}\,[I_G + A \text{ x } \frac{\bar{h}^2}{sin^2\theta}$$

Or $$h^* = \frac{IG\sin^2\theta}{A\bar{h}} + \bar{h}$$

Buoyancy:

When a body is immersed in a fluid either wholly or partially it is subjected to an upward force which tends to lift (or buoy) it up. This tendency for an immersed body to be lifted up in the fluid, due to an upward force opposite to the action of gravity, it is known as **BUOYANCY**.

The force tending to lift up the body under such condition is known as ***buoyant force*** (or force of buoyancy or up thrust)

The point of application of the force of buoyancy on the body is known as ***center of buoyancy***.

The magnitude of buoyant force can be determined by the well-known **Archimedes' principle**,

Which states that when a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force which is equal to the weight of the fluid displaced by the body. It is due to this upward force acting on a body immersed in a fluid, either wholly or partially, that there occurs an apparent loss in the weight of the body.

Assignment-Cum-Tutorial Questions

Unit 1: Introduction

Assignment-Cum-Tutorial Questions

Part- A Objective type Questions

1. The unit of dynamic viscosity of a fluid is

   (a) m^2/s (b) $N\ s/m^2$ (c) $Pa.\ s/m^2$ (d) $kg\ s^2/m^2$

2. If a liquid has greater cohesion than adhesion with the solid, then the liquid in the capillary tube will

   (a) rise with concave surface upward

   (b) rise with convex surface upward

   (c) fall with concave surface upward

   (d) fall with convex surface upward

3. If the shear stress 'τ' and shear rate (du/dy) relationship of a material is plotted with τ on Y-axis and (du/dy) on X-axis, the behavior of an ideal fluid is

   (a) a straight line passing through the origin and inclined to X-axis

   (b) the positive X-axis

   (c) the positive Y-axis

   (d) a curved line passing through the origin

4. At room temperature, the dynamic and kinematic viscosity of water

   (a) Are both greater than that of air

   (b) Are both less than of air

   (c) Are respectively greater than and less than that of air

   (d) Are respectively less than and greater than of air

5. If no resistance is encountered by displacement, such substance is known as

   (a) Fluid (b) water (c) gas (d) ideal fluid

6. Mercury is often used in barometer because

   (a) It is the best liquid

   (b) The height of barometer is less

   (c) The vapour pressure is negligibly low

   (d) Both b and c

9. The space between two parallel plates kept 3mm apart is filled with a dynamic viscosity of 0.2 Pa.s. The shear stress on lower fixed plate, if the upper plate is moved with velocity of 1.5 m/s.

(a) 500 N/m^2 (b) 100 N/m^2(c) 150 N/m^2 (d) none

Match the following

Set A	**Set B**
7. 1. density	A. N/m
2. surface tension	B. N/m^3
3. kinematic viscosity	C. kg/m^3
4. specific weight	D. m^2/s
8. 1. droplet formation	A. zero viscosity
2. weight	B. constant viscosity
3. ideal fluid	C. gravitational acceleration
4. Newtonian fluid	D. surface tension

10. The capillary rise in a 3mm tube immersed in a liquid is 15mm. If another tube of dia. 4mm is immersed in the same liquid the capillary rise would be (mm)
    (a) 11.25 (b) 20.00 (c) 8.44 (d) 26.67
11. If mercury in a barometer is replaced by water, the height of 3.75 cm of mercury will be following cm of water
    (a) 51 cm (b) 50 cm (c) 52 cm (d) 52.2 cm (e) 51.7 cm. Ans: a
12. A pressure of 25 m of head of water is equal to
    (a) 25 kN/m2 (b) 245 kN/m2 (c) 2500 kN/m2 (d) 2.5kN/m2 (e) 12.5 kN/m2. Ans: b

13. If 850 kg liquid occupies volume of one cubic meter, men 0.85 represents its
    (a) specific weight (b) specific mass (c) specific gravity (d) specific density (e) none of the above. Ans: c

14. The total pressure on the surface of a vertical sluice gate 2 m x 1 m with its top 2 m surfa ce being 0.5 m below the water level will be
    (a) 500 kg (b) 1000 kg (c) 1500 kg (d) 2000 kg (e) 4000 kg. Ans: d
15. An odd shaped body weighing 7.5 kg and occupying 0.01 m3 volume will be completely submerged in a fluid having specific gravity of
    (a) 1 (b) 1.2 (c) 0.8 (d) 0.75 (e) 1.25. Ans: d

16. A square surface 3 m x 3 m lies in a vertical line in water pipe its upper edge at water sur face. The hydrostatic force on square surface is

    (a) 9,000 kg (b) 13,500 kg (c) 18,000 kg (d) 27,000 kg (e) 30,000 kg. Ans: b

17. The capillary rise at 20°C in a clean glass tube of 1 mm bore containing water is approxi mately (a) 1 mm (b) 5 mm (c) 10 mm (d) 20 mm (e) 30 mm. Ans: e

Part-B Subjective Questions

II) Descriptive Questions

1. Name the different properties of fluid?
2. What are the units for specific gravity, specific volume, bulk density?
3. Define Viscosity? Derive an expression for Newton's law of Viscosity?
4. If $5m^3$ of certain oil weighs 4000Kg. Calculate the specific weight, mass density and specific gravity of this oil.
5. A plate 0.0254mm distant from a fixed plate, moves at 61cm/s and requires a force of $0.2Kg(f)/m^2$ to maintain this speed. Determine the dynamic viscosity of the fluid between the plates.
6. At a certain point in a castor oil the shear stress is $0.216N/m^2$ and the velocity gradient $0.216s^{-1}$. If the mass density of castor oil is $959.42Kg/m^3$, find kinematic viscosity.
7. If a certain liquid has viscosity $4.9*10^{-4}$ $Kg(f)\text{-}s/m^2$ and kinematic viscosity $3.49*10^{-2}$ stokes, what is its specific gravity?
8. What is the pressure within a droplet of water 0.05mm in diameter at 20˚C, if the pressure outside the droplet is standard atmospheric pressure of 1.03 $Kg(f)/cm^2$? Given σ=0.0075 Kg(f)/m for water at 20˚C
9. Prove that the pressure is the same in all directions at a point in a static fluid
10. Briefly explain the principle employed in the manometers used for the measurement of pressure.
11. Derive an expression for pressure intensity inside a droplet, soap bubble and liquid jet.
12. Differentiate between simple and differential type of manometers.
13. The left leg of a U – tube mercury manometer is connected to a pipe-line conveying water, the level of mercury in the leg being 0.6m below the center of pipe-line, and the right leg is open to atmosphere. The level of mercury in the right leg is 0.45m above that in the left leg and the space above mercury in the right leg contains Benzene (specific gravity 0.88) to a height of 0.3m. Find the pressure in the pipe.

14. A plate of metal 1m x 1m x 2 mm is to be lifted up with velocity of 0.1 m/s through an infinitely extending gap 20 mm wide containing an oil of sp. gr. 0.9 and viscosity of 2.15 N-S/m2. Find the force required assuming the plate to remain midway in the gap. Weight of the plate is 29.5 N.
15. A 400mm diameter shaft is rotating at 200 r.p.m in a bearing of length 120mm. If the thickness of oil film is 1.5 mm and the dynamic viscosity of oil is 0.7 N-S /$m^{2.}$
Determine i) Torque required to overcome friction in bearing
ii) Power utilized in overcoming viscous resistance.
16. Derive expression for the depth of Centre of pressure from free surface of liquid of an inclined plane surface submerged in the liquid.
17. What is buoyant force? What causes it?
18. Write the application of surface tension, Capillarity, Viscosity related to Civil Engineering field.
19. A differential manometer is connected to two pipes whose centres are at 3.2 m difference in height. Higher level pipe is carrying liquid of specific gravity of 0.85 at a pressure of 150 kN/m 2 and another pipe is carrying liquid at specific gravity of 1.4 at a pressure of 90 kN/m 2 . The centre of pipe carrying low pressure liquid is 2 m above the higher level of the mercury in the manometer. Find out the difference in mercury level in the manometer in m.
20. A pipe containing water at 172 kN/m^2 pressure is connected by a differential gauge to another pipe1.5 m lower than the first pipe and containing water at high pressure. If the difference in the heights of the two mercury columns of the gauge is equal to 75 mm, what is the pressure in the lower pipe? Specific gravity of mercury is 13.6.
21. A cylinder 0.1 m diameter rotates in an annular sleeve 0.102 m internal diameter at 100 r.p.m. The cylinder is 0.2 m long. If the dynamic viscosity of the lubricant between the two cylinders is 1.0 poise, find the torque needed to drive the cylinder against viscous resistance. Assume that Newton's Law of viscosity is applicable and the velocity profile is linear.

Part C- Gate questions/ competitive exams

1. A fluid mass occupies 2 m^3. Calculate the density, specific weight, and specific gravity if the fluid mass is

(a) 4 kg (b) 8 kg (c) 15 kg (d) 20kg

2. The SI unit of kinematic viscosity (υ) is:

(a) m^2/s (b) kg/m-s (c) m/s^2 (d) m^3/s^2

3. For a Newtonian fluid

(a) Shear stress is proportional to shear strain

(b) Rate of shear stress is proportional to shear strain

(c) Shear stress is proportional to rate of shear strain

(d) Rate of shear stress is proportional to rate of shear strain

4. Match List-I (Type of fluid) with List-II (Variation of shear stress) and select the correct answer:

List-I	**List-II**
A. Ideal fluid	1. Shear stress varies linearly with the rate of strain
B. Newtonian fluid	2. Shear stress does not vary linearly with the rate of strain
C. Non-Newtonian fluid	3. Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain
D. Bingham plastic	4. Shear stress is zero

Codes: A B C D	**A B C D**
(a) 3 1 2 4	(b) 4 2 1 3
(c) 3 2 1 4	(d) 4 1 2 3

5. What is the pressure difference between inside and outside of a droplet of water?

(a) $2\sigma / d$ (b) $4\sigma / d$ (c) $8\sigma / d$ (d) $12\sigma / d$

Where 'σ ' is the surface tension and‘d’ is the diameter of the droplet.

FLUID MECHANICS

UNIT – II

FLUID KINEMATICS

Objective:

- To understand the concept of fluid motion
- To learn the different types of fluid flow

Syllabus:

Methods of describing fluid motion, Classification of flows - Steady, Unsteady, Uniform, Non Uniform, Laminar, Turbulent, Rotational and Irrotational, Three , two and one dimensional flows, Stream line, Path line, Streak line, Equation for acceleration, Convective and Local acceleration with problems, Continuity Equation with problems, Velocity Potential and Stream Function, Vortex Flow – Free and Forced Flow.

Learning Outcomes:

Student will be able to

- Determine the different types of fluid flow
- Illustrate the characteristics of fluid flow
- Identify the fluid motion

Learning Material

Fluid Kinematics

It is a branch of science which deals with motion of fluid particles without considering the forces causing the motion.

Path lines:

A path line is the path fallowed by a fluid particle in motion. A path line shows the direction of particular particle as it moves ahead.

Stream line:

It is defined as an imaginary line with in the flow so that the target at any point on it indicates the velocity at that point.

- Stream lines cannot intersect nor cross
- Stream line spacing varies inversely as the velocity

Stream tube:

A stream tube is a fluid mass bounded by a group of stream lines.

- The contents of a stream tube are known as current filament.
- Stream tube has finite dimension.
- There is no perpendicular flow to stream lines and no flow crosses the surface.
- The shape of a stream tube changes from one instant to another because of change in position of stream lines.

Streak lines:

The streak line is a curve which gives an instantaneous picture of the location of the fluid particle.

Types of fluid flows:

The fluid flows is classified as

i. Steady and unsteady flow
ii. Uniform and non-uniform flow

iii. Laminar and turbulent flow
iv. Compressible and incompressible flow
v. Rotational and irrotational flow
vi. One, two and three dimensional flow

Steady and unsteady flow:

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time.

Mathematically, unsteady flow is that type of flow, in which the velocity, pressure or density at a point change with respect to time.

Uniform and non-uniform flows:

Uniform flow is a type of flow in which the velocity at any given time does not change with respect to space.

Non-uniform flow is a type of flow in which the velocity at any given time changes with respect to space.

Laminar and turbulent flows:

Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream line flow or viscous flow.

Turbulent is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place with are responsible for high energy loss.

Compressible and incompressible flows:

Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density is not constant for the fluids.

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible.

Rotational and irrotational flows:

Rotational is that type of flow in which the fluid particles while flowing along stream lines, also rotate about their own axis**.**

If the fluid particles while flowing along stream-lines do not rotate about their own axis then that type of flow is called irrotational flow.

One, two and three dimensional flows:

One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space coordinate only say ‘x’.

Two dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and two rectangular space coordinate only say ‘x and y’.

Three dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and three space coordinate only say ‘x, y and z’.

Rate of flow or Discharge:

The quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

Units of Q is m^3/s or litres/s for liquids

Kg(f)/s or Newton/s

$Q = AxV$

Continuity Equation:

Principle:

Principle of conservation of mass.

- For a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

Derivation:

Consider two cross sections of a pipe

Let V_1= average velocity at 1-1

ρ_1= density at 1-1

A_1= Area of pipe at 1-1

And V_2, ρ_2, A_2 are corresponding values at 2-2

Rate of flow at section 1-1= $\rho_1 A_1 V_1$

Rate of at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at 1-1 = rate of flow at 2-2

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

the above equation is called continuity equation which is applicable to both compressible and incompressible fluids.

Continuity equation in three dimensional:

Derivation:

Consider a fluid element of lengths *dx, dy*and *dz* in the direction of *x, y and z.* let*u, v and w*are the inlet velocity components in *x, y and z* directions respectively.

Mass of fluid entering the face ABCD per second = ρ X velocity in *x*- direction X area of ABCD

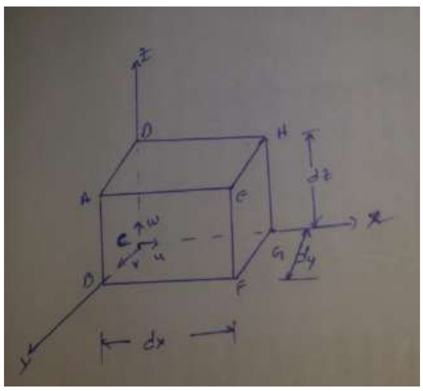
= ρ. *u .dy. Dz*

Then mass of fluid leaving the face EFGH per second = ρ*udydz* +

Gain of mass in x- direction = mass through ABCD –mass through EFGH per second

= ρ*udydz*-ρudydz-

= -



Similarly, the net gain of mass in y- direction

=-

And in z- direction = -

Net gain of masses = - [

Since the mass is neither be created nor be destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But mass of the fluid in the element is ρ. and its rate of increase with time is

- [=

+ = 0

This continuity eq is applicable to:

1. Steady and unsteady flows,
2. Uniform and non uniform flow, and
3. Compressible and incompressible fluids.

Velocity and acceleration:

Let V is the resultant velocity at any point in a fluid flow. Let *u, v and w* are its component in *x, y and z* directions. The velocity components are functions of space co-ordinates and time.

Mathematically,

$u = f_1(x, y, z, t)$

$v = f_2(x, y, z, t)$

$w = f_3(x, y, z, t)$

and resultant velocity, V = $ui+vj+wk$

let a_x, a_y ,*and* a_z *are the total acceleration in x, y and z* directions respectively.

a_x=

and

So

$a_x =$

$a_y =$

$a_z =$

Local acceleration

It is defined as rate of increase of velocity with respect to time at a given point in a flow field. In the above equations is known as local accelerations.

Convective acceleration

It is defined as the rate of change of velocity due to the change of position of fluid particle in a fluid flow. The expression other than   in the above equation are known as convective acceleration.

Velocity potential function:

It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is defined by Φ.

Mathematically, the velocity, potential is defined as $\Phi = f(x, y, z)$ for steady flow

Such that

$$u = -$$

$$v = -$$

$$w = -$$

where u, v and w are the components of velocity in *x, y and z* directions respectively.

The continuity equation for an incompressible steady flow is

Substituting the values of *u, v and w.*

Streamlines and equipotential lines:

A property of the stream function is that the difference of its values at two points represents the flow across any line joining the points. Therefore, when two points lie on the same streamline, then since there is no flow across a streamline, the difference between the stream ψ1 function ψ2 and at these two points is equal to zero, i.e.,( )=0.in other words, it means that streamline is given by ψ=constant.

Similarly φ=constant, represent a curve for which the velocity potential is same at every point, and hence it represents an equipotential line.

Consider two curves, viz.,φ =constant and ψ = constant, interesting each other at any point. The slope of this curves at the point of intersection may be determined as indicated below.

For the curves φ =constant,

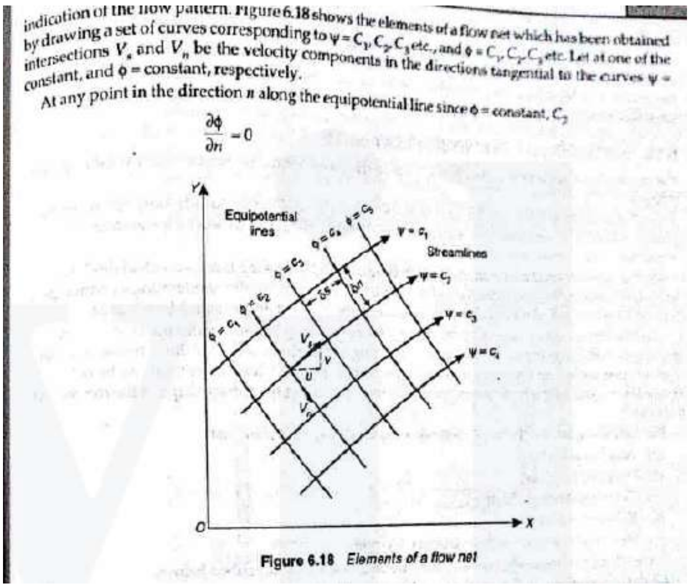
Slope= = =

Similarly for the curve ψ = constant

Slope =

From the above equations, the product of above two slopes will give the value of '-1'. This can say that stream line and equipotential lines are perpendicular to each other.

FLOW NET: Agrid obtained by drawing a series of stream lines and equipotential lines is known as flow net.


indication of the flow pattern. Figure 6.18 shows the elements of a flow net which has been obtained by drawing a set of curves corresponding to $\psi = C_1, C_2, C_3$ etc., and $\phi = C_1, C_2, C_3$ etc. Let at one of the intersections V_s and V_n be the velocity components in the directions tangential to the curves ψ = constant, and ϕ = constant, respectively.

At any point in the direction n along the equipotential line since ϕ = constant, C_2

$$\frac{\partial \phi}{\partial n} = 0$$

Figure 6.18 *Elements of a flow net*

METHODS OF DRAWING FLOW NETS: The construction of the flow net for the flows is restricted by certain conditions which are indicated below are as

1.The flow should be steady. This is so because it is only for the steady flow the s , pattern remain constant. For unsteady flow the streamline pattern will be instantaneous and it may change from instant to instant.

(ii) The flow should be irrotational, which is possible if the flowing fluid is an ideal fluid (having no viscosity) or it has negligible viscosity. However, in case of rapidly accelerating or converging flow of fluids, even if the fluids have low viscosity, the flow net analysis may be adopted.

(iii) The flow is not governed by the gravity This is so because under the action of gravity the shape of the free surface is constantly changing and with the shape of the extreme bouoday surface (free surface in this case) undergoing a change, no fixed flow net pattern can be obtained. However, in such cases the flow nets can be drawn after fixing the correct shape of the free surface boundary.

The following are the different methods used for drawing the flow nets :

(1) Analytical Method.

(2) Graphical Method.

(3) Electrical Analogy Method.

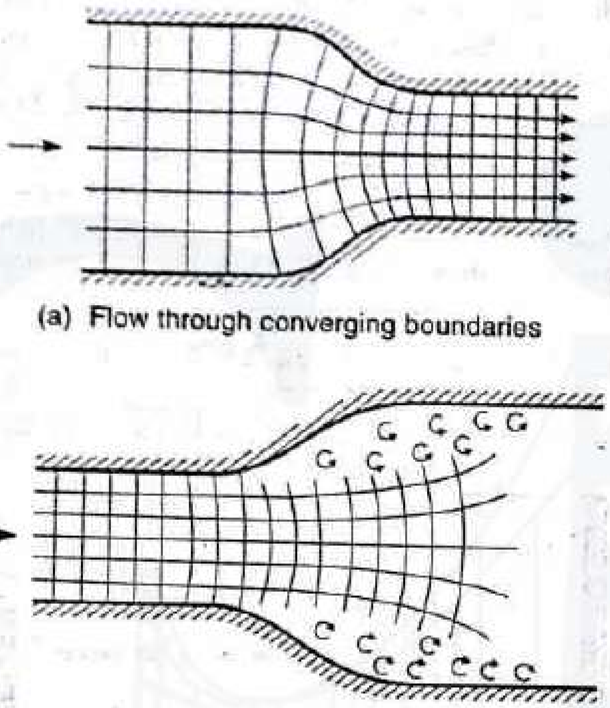
(4) Relaxation Method.

(5) Fide Shaw or Viscous flow Analogy Method.

USE OF THE FLOW NET: For a given set of boundary configuration there is only one possible pattern of the flow for an ideal fluid and a correctly drawn flow net will represent this pattern. As such after a flow net for a given boundary configuration has been obtained, it maybe used for all irrotational flows with geometrically similar boundaries. Once the flow net is drawn, the spacing between the adjacent streamlines is determined and the application and the continuity equations gives the velocity flow at any Point,if the velocity of flow at any reference point is known.

Further the flow net analysis assists in the determination of the efficient boundary shapes for which the flow does not separate from the boundary surface.

Although the flow net analysis is based on an ideal fluid flow concept, it may also be applied to the flow of a real fluid within certain limit. The ideal fluid theory neglects the effect of fluid friction or viscosity, which is however possessed by a real fluid. But the viscosity effects of a real fluid are most pronounced at or near a solid boundary and diminish rapidly with distance from the boundary.As such in the regions where viscosity effects are not predominant, the real fluid behaves more or less like an ideal fluid, and in these regions the flow net analysis may be applied to the real fluids. with sufficient accuracy.



(a) Flow through converging boundaries

(b) Flow through diverging boundaries

Figure 6.20 *Typical flow nets*

Figure 6.20 shows a few examples of the flow nets. It is observed in Fig. 6.20 (a) that since the boundaries are converging the streamlines also converge rapidly. In this case the accelerating flow is developed and the actual flow pattern approximates closely to that represented by the flow net. On the other hand as observed in Fig. 6.20 (b) the boundaries in the direction of flow are diverging and therefore the streamlines also tend to diverge. In the region where the streamlines diverge, a phenomenon known as separation of flow generally occurs. That is, in such cases the flowing fluid does not remain in contact with the boundary surface or it separates from the boundary, thereby developing regions of flow separation in which eddies are developed. Therefore, in such cases where the flow separation takes place, the flow net which is constructed with streamlines conforming to the boundaries does not describe the actual pattern of flow field.

Limitations Of Flow Net: A flow net always indicates some velocity at the boundary, but a real fluid must have zero velocity adjacent to the boundary on account of the fluid friction or viscosity. As h the flow net analysis cannot be applied in the region close to the boundary where the effects of viscosity are predominant. As stated earlier the flow net analysis can also not be applied to a sharply diverging flow, since the actual flow pattern is not represented by the flow net. Further in the case of flow of a fluid past a solid body, while the flow net gives a fairly accurate picture of the flow pattern for the upstream part of the solid body, it can give little information concerning the flow conditions at the rear because of separation and eddies. The disturbed flow in the rear of the solid body is known as wake, the formation of which is not indicated by a flow net.

Unit II: Fluid Kinematics

Assignment-Cum-Tutorial Questions

Part- A Objective type Questions

1. The rate of increase of velocity with respect to change in the position of fluid particle in a flow field is called as ____________
2. Blood circulation through arteries is _____
3. The continuity equation is the result of application of the following law to the flow field
    a) First law of thermodynamics
    b) Conservation of energy
    c) Newton's second law of motion
    d) Conservation of mass
4. A path line is the
    (a) Mean direction of a number of particles at the constant same instant of time.
    (b) Instantaneous picture of positions of all particles in the flow which passed a given point.
    (c) trace made by a single particle over a period of time
    (d) path traced by continuously injected tracer at a point
5. A streamline is defined as the line
    (a) Parallel to central axis flow (b) parallel to outer surface of pipe
    (c) of equal velocity in a flow (d) along which the pressure drop is uniform
6. The imaginary line drawn in the fluid in such a way that the tangent to any point gives the direction of motion at the point, is called as
    a) Path line
    b) Streak line
    c) Filament line
    d) Stream line
7. In the case of steady flow of a fluid, the acceleration of any fluid particle is
    (a) Constant (b) variable (c) zero (d) zero under limiting conditions (e) never zero.

8. Non uniform flow occurs when

   (a) The direction and magnitude of the velocity at all points are identical

   (b) The velocity of successive fluid particles, at any point, is the same at successive periods of time

   (c) The magnitude and the direction of the velocity do not change from point to point in the fluid

   (d) The fluid particles move in plane or parallel planes and the streamline pat-terns are identical in each plane

9. During the opening of a valve in a pipe line, the flow is

(a)steady (b) unsteady (c) uniform (d) laminar (e) free vortex type

10. The flow in which conditions do not change with time at any point, is known as

    (a) one dimensional flow (b) uniform flow (c) steady flow (d) turbulent flow (e) streamline flow.

11. A stream function
    a) Is defined only for incompressible flow
    b) Is defined for irrotational flow
    c) Is defined when the flow is continuous
    d) Is defined only for steady flow

Part-B Subjective Questions

1. Define and distinguish between steady and unsteady flow
2. Define and distinguish between uniform and non-uniform flow
3. A stream function is given by $\psi = 3x^2 - y^3$. Determine the magnitude of velocity components at the point (2,1).
4. A stream function in a two-dimensional flow is $\psi = 2xy$. Show that the flow is irrotational and determine the corresponding velocity potential φ.
5. Define and distinguish between stream line and path line and streak line.
6. What is meant by one-dimensional, two-dimensional and three dimensional flows?
7. Differentiate between laminar and turbulent flow.

8. An incompressible fluid flows steadily through two pipes of diameter 0.15m and 0.2m which combine to discharge in a pipe of 0.3m diameter. If the average velocities in the 0.15m and 0.2m diameter pipes are 2m/s and 3m/s respectively, then find the average velocity in the 0.3m diameter pipe.
9. A nozzle is so shaped that the velocity of flow along the centre line changes linearly from 1.5m/s to 15m/s in a distance of 0.375m. Determine the magnitude of the convective acceleration at the beginning and end of this distance.
10. The bucket of spillway has a radius of 6m. When the spillway is discharging $5m^3$ of water per second per meter length of crest, the average thickness of the sheet of water over the bucket is 0.4m. Compare the resulting normal acceleration (or centripetal acceleration) with the acceleration due to gravity.
11. When 2500 lit of water flows per minute through a 0.3m diameter pipe which later reduces to a 0.15 diameter pipe, calculate the velocities of flow in the two pipes.
12. Two velocity components are given in the following cases, find the third component such that they satisfy the continuity equation. $U = x^3 + y^2 + 2z^2$; $v = -x^2y - yz - xy$.
13. If for two-dimensional flow the stream function is given by $\psi = 2xy$, calculate the velocity at a point (3,6). Show that the velocity potential φ exists for this case and deduce it. Also draw the streamlines corresponding to $\psi = 100$ and $\psi = 300$ and equipotential lines correspond to $\varphi = 100$ and $\varphi = 300$.
14. Derive the equation of stream function and velocity potential for a uniform stream of velocity V in a two-dimensional field, the velocity V being included to the x-axis at a positive angle α.

Part C- Gate questions/ competitive exams

1. You are asked to evaluate assorted fluid flows for their suitability in a given laboratory application. The following three flow choices, expressed in terms of the two-dimensional velocity fields in the xy-plane, are made available. P. $u = 2y$, $v = -3x$ Q. $u = 3xy$, $v = 0$ R. $u = -2x$, $v = 2y$ which flow(s) should be recommended when the application requires the flow to be incompressible and irrotational?

(a) P and R (b) Q (c) Q and R (d) R

2. Streamlines, path lines and streak lines are virtually identical for

(a) Uniform flow (b) Flow of ideal fluids

(c) Steady flow (d) Non uniform flow

3. In a flow field, the streamlines and equipotential lines

(a) Are Parallel

(b) Are orthogonal everywhere in the flow field

(c) Cut at any angle

(d) Cut orthogonally except at the stagnation points

4. Irrotational flow occurs when:

(a) Flow takes place in a duct of uniform cross-section at constant mass flow rate.

(b) Streamlines are curved.

(c) There is no net rotation of the fluid element about its mass center.

(d) Fluid element does not undergo any change in size or shape.

5. A streamline is a line:

(a) Which is along the path of the particle

(b) Which is always parallel to the main direction of flow

(c) Along which there is no flow

(d) On which tangent drawn at any point given the direction of velocity

UNIT – III

Fluid Dynamics

Objective:

- Understand the fluid motion with respect to the force acting on it
- Learn the momentum conservation of the fluid motion.

Syllabus:

Surface and body forces Euler's Equation of motion. Bernoulli's Equation from Euler's Equation with problems, Applications -Venturimeter and orifice meter, Momentum principle, Application of Momentum equations, Force exerted on a Pipe Bend with problems

Learning Outcomes:

- To determine the motion with respect to the force acting on it
- To determine the momentum of fluid flow, energy correction factor for the fluid

Learning Material

Fluid Dynamics

The study of fluids in motion, where pressure forces are considered for the fluid in motion, that branch of science is called Fluid Dynamics.

Equations of motion

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of `$x$` is equal to mass `$m$` of the fluid element multiplied by the acceleration `$a_x$` in the `$x$` direction. Thus mathematically

$$F_x = m.a_x \qquad (1)$$

In the fluid flow the following forces are present:

I. F_g gravity force.
II. F_P the pressure force.
III. F_v the force due to viscosity.
IV. F_t the force due to turbulence.
V. F_c the force due to compressibility.

Thus in equation (1) the net force

$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$

If the force due to compressibility, F_c is negligible. The resulting net force

$$F_x = (F_g)_x + (F_P)_x + (F_v)_x + (F_t)_x$$

And the equation of motion is called **Reynolds equation of motion**

i. For flow, Where (F_t) is negligible, the resulting equation of motion are known as **Navier Stokes of Equation.**
ii. If the flow is assumed to be ideal. Viscous force (F_v) is zero and the equations of motions are known as **Euler equation of motion.**

EULER`S EQUATION OF MOTION

This equation of motion in which the force due to gravity and pressure are taken in to consideration. This is derived by considering the motion of a fluid element along a stream line as:

Consider a stream-line in which the flow is taking place in s- direction as shown in the figure. Consider a cylindrical element of cross section *dA* and length *dS*

The forces acting on cylindrical elements are

1. The pressure force *pdA* in the direction of flow.
2. The pressure force (*ds) dA* opposite to the direction of flow.
3. Weight of the element *ρgdAdS*

Let θ is the angle between the direction of flow and the line of action of the weight of the element.

The resultant force on the fluid element in the direction of *S* must be equal to the mass of the element × acceleration in the direction of *S*

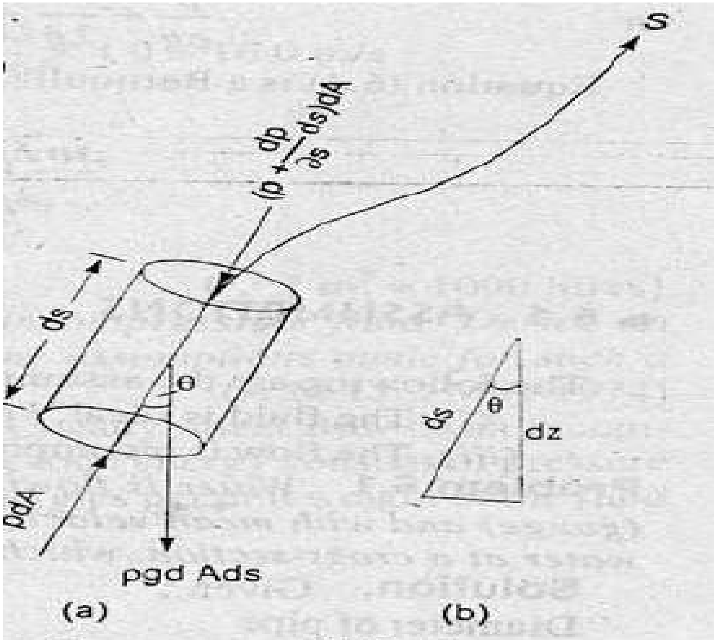
$pdA - (ds)\, dA - \rho g dA dS \cos\theta$

$= \rho g dA dS \times a_s$ (2)

Where a_s is the acceleration in the direction of *S*

Now $a_s =$, where v is a function of *S* and *t*

= + = therefore



Forces on a fluid element.

If the flow is steady, $= 0$

$a_s =$

Substituting the value of a_s in equation (2) and simplifying the equation, we get $dSdA - \rho g\, dAdS \cos\theta = \rho dAdS \times$

Dividing by $\rho dSdA - - g \cos\theta -$

Or $+ g \cos\theta + v = 0$

But from fig 1(b) , we have $\cos\theta =$

Therefore $+ g + = 0$ or $+ gdZ + vdv = 0$

$+ gdZ + vdv = 0$ (3)

Equation (3) is known as Euler`s equation of motion

BERNOULLI`S EQUATION FROM EULER `S EQUATION

Bernoulli`s equation is obtained by integrating the Euler`s equation of motion (3) as

$\int + \int gdZ + \int vdv =$ constant

If the flow is incompressible. ρ is constant and

Therefore + gz + = constant

+ z + = constant (4)

Equation (4) is a Bernoulli`s equation in which

= pressure energy per unit weight of fluid or pressure head.

= kinetic energy per unit weight or kinetic head.

Z = potential energy per unit weight or potential head

ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli`s equation :

i. The fluid is ideal that is viscosity is zero
ii. The flow is steady
iii. The flow is incompressible
iv. The flow is irrotational

Statement of Bernoulli`s theorem: It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are:

Pressure energy =

Kinetic energy =

Datum energy = z

Thus mathematically Bernoulli`s theorem is written as + z + = constant

BERNOULLI`S EQUATION FOR REAL FLUID

Bernoulli`s equation was derived on the assumption that the fluid is invicid (non-viscous) and therefore frictionless. But all the real fluids are viscous and hence offers resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli`s equation, these losses have to be taken in to consideration. Thus the Bernoulli`s equation for real fluids between points 1and 2 is given as

=

Where h_L is loss of energy between points 1 and 2.

PRACTICAL APPLICATIONS OF BERNOULLI`S EQUATION

Bernoulli`s equation is applied in all problems of in compressible fluid flow where energy considerations are involved. But we shall consider its applications to the following measuring devices:

1. Venturimeter
2. Orificemeter
3. Pitot-tube

VENTURI METER

A Venturimeter is a device which is used for measuring the rate of a flow of fluid through a pipe. The principle of the Venturimeter was first demonstrated in 1979 by Italian physicist G.B. Venturi.

Principle:

The basic principle on which a Venturi meter works is that by reducing the cross-sectional area of the flow passage, a pressure difference is created and the measurement of the pressure difference enables the determination of the discharge through the pipe.

Description:

The inlet section the Venturimeter is of the same diameter as that of the pipe which is followed by a convergent cone. The convergent cone is a short pipe which tapers from the original size of the pipe to that of the throat of the Venturimeter. The throat of the Venturimeter is a short parallel-sided tube having its cross-sectional area smaller than that of the pipe. The divergent cone of the Venturimeter is a gradually diverging pipe with its cross-sectional area increasing from that of the throat to the original size of the pipe. At the inlet section and the throat of the Venturi meter, pressure taps are provided through pressure rings.

The convergent cone of a Venturi meter has a total included angle of $21^o \pm 1^o$ and its length parallel to the axis is approximately equal to 2.7 times (D-d), where D is the diameter of the inlet section and d is the diameter of the throat. The length of the throat is equal to d. the divergent cone has a total included angle lying between 5^o to 15^0. This results in the convergent of the Venturi meter to be of smaller length than its divergent cone. This is so because from the consideration of the continuity equation it is obvious that in the convergent cone the fluid id being accelerated from the inlet section 1 to the throat section 2, but in the divergent cone the fluid is retarted from the throat section 2, to the end section 3, of the Venturi meter. The acceleration of the flowing fluid may be allowed to take place rapidly in a relatively small length, without resulting in appreciable loss of energy. However, if the retardation of flow is allowed to take place rapidly in small length, then the flowing fluid will not remain in contact with the boundary of the diverging flow passage or in other words the flow separates from the walls, and eddies are formed which in turn result in excessive energy loss. Therefore, in order to avoid the possibility of flow separation and the consequent energy loss, the divergent cone of the Venturi meter is made longer with a gradual divergence. Since the separation of flow may occur in the divergent cone of the Venturi meter, this portion is not used for the discharge measurement.

Since the cross-sectional area of the throat is smaller than the cross-sectional area of the inlet section, the velocity of flow at the throat will become greater than that at the inlet section, according to the continuity equation. The increase in the velocity of flow at the throat results in the decrease in the pressure. As such a pressure difference is developed between the inlet section and the throat of the Venturi meter. The pressure difference between these sections can be determined either by connecting a differentional manometer between the pressure taps provided at these sections or by connecting a separate gage at each pressure taps.

The measurements of the pressure difference between these sections enable the rate of flow of fluid to be calculated. For a greater accuracy in the measurement of the pressure difference the cross-sectional area of the throat should be reduced considerably, so that the pressure at the throat is very much reduced. But if the cross-sectional area of the throat of Venturi meter is reduced so much that the pressure at this section drops below the vapour pressure of the flowing liquid, then the flowing liquid may vapourise and vapour pockets or bubbles may be formed in the liquid at this section. Further liquids ordinarily contain some dissolved air which is released as the pressure is reduced and it too may from air pockets in the liquid. The formation the vapour and air pockets in the liquid ultimately result in a phenomenon called Cavitation. Therefore, in order to avoid the phenomenon of Cavitation to occur, the diameter of the throat can be reduced only up to certain limited value which is restricted on account of the above noted factors. In general, the diameter of the throat may vary from of the pipe diameter and more commonly the diameter of the throat is kept equal to of the pipe diameter.

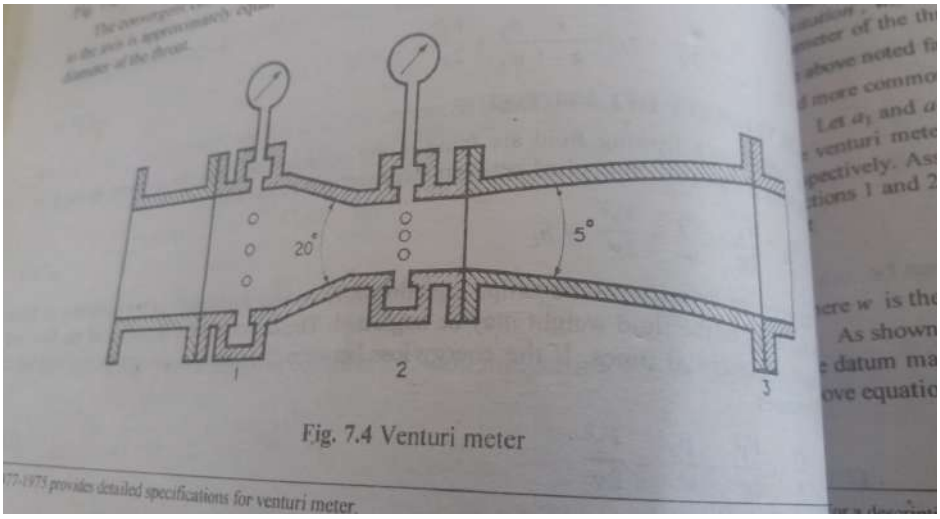


FIG 1: Venturi Meter

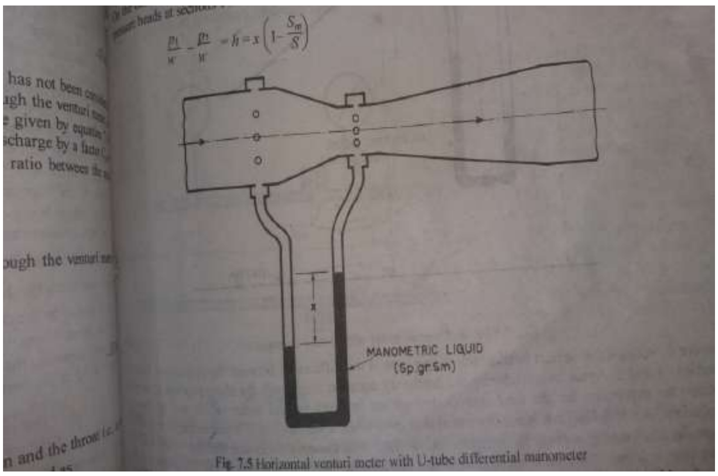


FIG 2: horizontal venturi meter with U- tube differential manometer

Derivation:

Let and be the cross-sectional areas at the inlet section and the throat (i.e., sections 1 and 2) of the venturi meter respectively, at which let the pressures and and the velocities and respectively. Assuming that the flowing fluid is incompressible and there is no loss of energy between the sections 1 and 2 of the venturi meter, then applying Bernoulli's equation between the sections 1 and 2, we get

...…………………(

Where is the specific weight of the flowing fluid

As shown in fig 2 if the venturi meter is connected in a horizontal pipe then (Or in this case the datum may be assumed to be passing through the axis of the venturi meter so that. Then equation (is reduced to

……………………(

In the above expression is the difference between the pressure heads at the sections 1 and 2 which is known as venturi head and is denoted by . That is

……………………(

Further if represents the discharge through the pipe, then by continuity equation

and ……………………(

By substituting the values of and from equation (in (, then we get

Equation 5 gives only the theoretical discharge because the loss of energy has not been considered. But in actual practice there is always some loss of energy as the fluid flows through the venturi meter, on account of which the actual discharge will be less than the theoretical given by equation 5. The actual discharge may be obtained by multiplying the theoreticaldischarge by a factor called coefficient of discharge of the venturi meter which is defined as the ratio between the actual discharge and theoreticaldischarge of the venturi meter. That is

Where represents the actual discharge. Therefore the actual discharge through venturi meter given by

Since for a given venturi meter the cross-sectional areas of the inlet sections and the throat are fixed and therefore we may introduce constant of the Venturimeter C, expressed as

……………………..

Introducing equation in

The coefficient of discharge of the venturi meter also accounts for the effects of non-uniformity of velocity distribution at section 1 and 2. The coefficient of discharge of the venturi meter varies somewhat with the rate of flow, the viscosity of the fluid and the surface roughness, but in general for the fluids of low viscosity a value about 0.98 is usually adopted for of the venturi meter.

As shown in Fig.2 if a U-tube manometer is used for measuring the difference between the pressure heads at sections 1 and 2, then for a difference in the levels of the manometric liquid in the two limbs equal to, we have

and are the specific weights of the manometric liquid and the liquid flowing in the venturi meter respectively. If and are respectively the specific gravities of the manometric liquid and the liquid flowing in the venturi meter, then the expression for the venturi head becomes

On the other hand if an inverted U-tube manometer is used for measuring the difference between the pressure heads at sections 1 and 2, since , we have

Venturi meter can also be used for measuring the discharge through a pipe which is laid either in an inclined or in vertical position. Consider a venturi meter connected in an inclined as shown in Fig 3. Applying Bernoulli’s equation between sections 1 and 2 for no loss of energy, we get

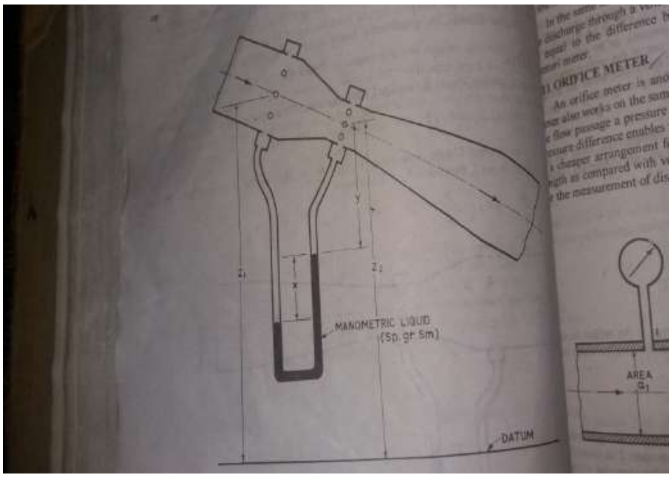


FIG 3. Inclined venturi meter with U-tube manometer

Where h is again the venturi head which in this case is the difference between the piezometric heads at sections 1 and 2. Again by considering the continuity equation along with the above expression, we may obtain an expression for the discharge Q through an inclined venturi meter which will be same as equation *vi* or *viii*. Thus it may stated that even when a venturi meter is connected in a inclined pipe the discharge Q is given by equation *vi* or *viii* with only the difference that the venturi head in this case is equal to difference between the piezometric heads at sections 1 and 2. If a U-tube manometer is connected between the pressure tabs at sections 1 and 2, then for a difference in the levels of the manometric liquid in the two limbs equal to, the following manometric equation may be formed

Therefore it may be stated that even in the case of venturi meter laid in an inclined position the venturi head may be determined simply by noting the difference in the levels of the manometric liquid in the limbs of the manometer.

ORIFICE METER

AREA a_1

1

2

HEAZ

Z

AREA a_0

FLUID JET

Fig: 4 ORIFICE METER

Use:

An orifice meter is another simple device used for measuring the discharge through pipes.

Principle:

Orifice meter also works on the same principle as that of venturi meter. However, an orifice meter is a cheaper arrangement for discharge measurement through pipes and its installation requires a smaller length as compared with venturi meter. As such where space is limited, the orifice meter may be used for the measurement of discharge through pipes.

Description:

An orifice meter consists of a flat circular plate with a circular hole called orifice, which is concentric with the pipe axis. The thickness of the plate "t" is less than or equal to 0.05 times the diameter of the pipe. From the upstream face of the plate edge of the orifice is made flat for a thickness t_1 less than or equal to 0.02 times the diameter of the pipe and for the remaining thickness of the plate it is bevelled with the bevel angle lying between 30^0 to 45^0.

Derivation:

Let be the pressures and velocities at sections 1 and 2 respectively. Then for an incompressible fluid, applying Bernoulli's equation between the sections 1 and 2 neglecting the losses, we have

…………………

………….

where is the difference between the piezometric heads at sections 1 and 2. However, if the orifice meter is connected in horizontal pipe then .

from equation (ii) above we obtain

Since in deriving the above expression the losses have not been considered this expression this expression gives the theoretical velocity of flow at section 2. In order to obtain the actual velocity at section 2 it must be multiplied by a factor called coefficient of velocity which is defined as the ratio between the actual velocity and the theoretical velocity. Thus the actual velocity of flow at section 2 is obtained as

Furtherif are the cross-sectional area of the pipe at section 1 and that of the jet at section 2 respectively and Q represents the actual discharge through the pipe, then by continuity equation

Coefficient of contraction which is defined as the ratio between the area of the jet at vena contracta and the area of the orifice. Thus introducing the value of in equation (v), we get

By substituting the value of equation (iv) we get

Solving, we get

Now

And

Where is the coefficient of the discharge of the orifice. It is usual to simply the above expression for the discharge through the orifice meter by using a coefficient expressed as

The above equation gives the discharge through an orifice meter. The coefficient C introduced in equation 6 may be considered as the coefficient of discharge of an orifice meter.

Orifice meter reference Diagram

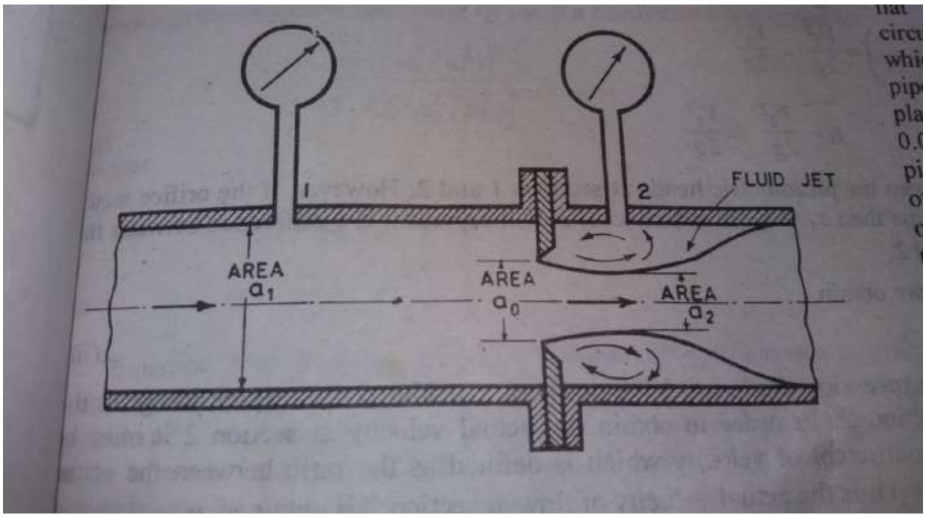


FIG. 4 ORIFICE METER

The momentum principle:

The net force acting on a fluid mass is equal to the change in momentum of flow per unit time that direction.

The force acting on a mass '*m*' is given by the Newton's second law of motion,

F = m. a

Where *a* is the acceleration acting in the same direction as force *F.*

a =

F = m.

F =

The above equation is known as a momentum principle

The momentum equation:

It is based on the law of conservation of momentum or on the momentum principle.

From the above equation it can be written as

$$F.\,dt = d(mv)$$

Which is known as impulse momentum equation which states that the impulse of a force acting on a fluid of mass in a short interval of time is equal to the change of momentum in the direction of force

Forces exerted by a flowing fluid on a pipe bend:

The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections 1 and 2 as shown in fig.

Let v_1 = velocity of flow at section 1

p_1 = pressure intensity at 1

A_1 = *area of cross section of pipe at 1*

Similarly consider at section 2

Let F_x and F_y be the forces exerted by the flowing fluid on the bend in *x*- and *y*-directions respectively. Then the force exerted by the bend on the fluid in the direction of *x* and *y* will be equal to F_x and F_y but in the opposite directions. Hence component of the force exerted by the bend on the fluid in the x-direction = - F_x and in the direction of y = - F_y.

The other external forces acting on the fluid are p_1A_1 and p_2A_2 on the sections 1 and 2 respectively. Then momentum equation in x – direction is given by

Net force acting on fluid in the direction of x = Rate of change of momentum in x direction

$p_1A_1 - p_2A_2\cos\theta - F_x$ = *mass per sec. change in velocity*

$$= \rho Q(V_2 \cos\theta - V_1)$$

$$F_x = \rho Q(-V_2 \cos\theta + V_1) + p_1A_1 - p_2A_2\cos\theta$$

Similarly in y direction,

$$F_y = \rho Q(-V_2 \sin\theta) - p_2A_2\sin\theta$$

Resultant force acting on the bend =

Angle made by the resultant

$$\tan\theta =$$

UNIT-IV
DIMENSIONAL ANALYSIS

SYLLABUS: Dimensional analysis-Rayleigh's method and Buckingham's pi theorem-Study of hydraulic models-Hydraulic similitude-Dimensionless numbers; Model laws

2.1 Fundamental Dimensions
All physical quantities are measured by comparison which is made with respect to a fixed value.Length, Mass and Time are three fixed dimensions which are of importance in fluid mechanics and fluid machinery. In compressible flow problems, temperature is also considered as a fundamental dimension.

2.2 Secondary Quantities or Derived Quantities
Secondary quantities are derived quantities or quantities which can be expressed in terms of two or more fundamental quantities.

2.3 Dimensional Homogeneity
In an equation if each and every term or unit has same dimensions, then it is said to have dimensional Homogeneity.

$$V = u + at$$
$$m/s \quad m/s \quad m/s$$
$$LT^{-1} = (LT^{-1}) + (LT^{-2})\ (T)$$

2.4 Uses of Dimensional Analysis
1. It is used to test the dimensional homogeneity of any derived equation.
2. It is used to derive equation.
3. Dimensional analysis helps in planning model tests.

2.5. Dimensions of quantities

1. Length LM^oT^o
2. Mass L^oMT^o
3. Time L^oM^oT
4. Area $L^2M^oT^o$
5. Volume $L^3M^oT^o$
6. Velocity LM^oT^{-1}
7. Acceleration LM^oT^{-2}
8. Momentum LMT^{-1}
9. Force LMT^{-2}
10. Moment or Torque L^2MT^{-2}
11. Weight LMT^{-2}
12. Mass density $L^{-3}MT^o$
13. Weight density $L^{-2}MT^{-2}$
14. Specific gravity $L^oM^oT^o$
15. Specific volume $L^3M^{-1}T^o$
16. Volume flow rate $L^3M^oT^{-1}$
17. Mass flow rate L^oMT^{-1}
18. Weight flow rate LMT^{-3}
19. Work done L^2MT^{-2}
20. Energy L^2MT^{-2}
21. Power L^2MT^{-3}
22. Surface tension L^oMT^{-2}
23. Dynamic viscosity $L^{-1}M^{+1}T^{-1}$
24. Kinematic viscosity $L^2M^oT^{-1}$
25. Frequency $L^oM^oT^{-1}$
26. Pressure $L^{-1}MT^{-2}$

27. Stress $L^{-1}MT^{-2}$
28. Compressibility $LM^{-1}T^{2}$
30. Efficiency $L^{o}M^{o}T^{o}$
31. Angular velocity $L^{o}M^{o}T^{-1}$
32. Thrust LMT^{-2}
33. Energy head (Energy/unit mass) $L^{2}M^{o}T^{-2}$
34. Energy head (Energy/unit weight) $LM^{o}T^{o}$

2.6 Methods of Dimensional Analysis

There are two methods of dimensional analysis.

1. Rayleigh's method
2. Buckingham's (π– theorem) method

2.6.1. Rayleigh's method

Rayleigh's method of analysis is adopted when number of parameters or variables is less (3 or 4 or 5).

Methodology

X_1 is a function of

$X_2, X_3, X_4, \ldots\ldots, X_n$ then it can be written as

$X_1 = f(X_2, X_3, X_4, \ldots\ldots, X_n)$

$X_1 = K\,(X_{2a}, X_3^{b}, \ldots\ldots)$

Taking dimensions for all the quantities

$$[X_1] = [X_2]_a[X_3]_b\,[X_4]_c \ldots\ldots$$

Dimensions for quantities on left hand side as well as on the right hand side are written and using the concept of Dimensional Homogeneity a, b, c …. can be determined.

Then,

$$X_1 = K \cdot X_{2a} \cdot X_{3b} \cdot X_{4c} \cdot \ldots\ldots$$

2.6.2. Buckingham's Π Method

This method of analysis is used when numbers of variables are more.

Buckingham's Π Theorem

If there are n – variables in a physical phenomenon and those n-variables contain 'm' dimensions, then the variables can be arranged into (n-m) dimensionless groups called Π terms.

Explanation:

If $f(X_1, X_2, X_3, \ldots\ldots\ldots X_n) = 0$ and variables can be expressed using m dimensions then.

$$f(\Pi_1, \Pi_2, \Pi_3, \ldots\ldots\ldots \Pi_{n-m}) = 0$$

Where, $\Pi_1, \Pi_2, \Pi_3, \ldots\ldots\ldots$ are dimensionless groups.

Each Π term contains (m + 1) variables out of which m are of repeating type and one is of non-repeating type.

Each Π term being dimensionless, the dimensional homogeneity can be used to get each Π term.

Selecting Repeating Variables

1. Avoid taking the quantity required as the repeating variable.
2. Repeating variables put together should not form dimensionless group.
3. No two repeating variables should have same dimensions.
4. Repeating variables can be selected from each of the following properties.
   a. Geometric property → Length, height, width, area
   b. Flow property → Velocity, Acceleration, Discharge
   c. Fluid property → Mass density, Viscosity, Surface tension

Before constructing or manufacturing hydraulics structures or hydraulics machines tests are performed on their models to obtain desired information about their performance. Models are small scale replica of actual structure or machine. The actual structure is called prototype.

2.7. Geometrical Similitude / Similarity

It is defined as the similarity between the prototype and its model.

Types of Similarity

There are three types of similarity.

- Geometric similarity
- Kinematic similarity
- Dynamic similarity

Geometrical Similarity

Geometric similarity is said to exist between the model and prototype if the ratio of corresponding linear dimensions between model and prototype are equal.

i.e. $\frac{L_p}{L_m} = \frac{h_p}{h_m} = \frac{H_p}{H_m} \text{...........} = L_r$

$L_r \rightarrow$ scale ratio / linear ratio

$$\frac{A_p}{A_m} = (L_r)^2 \quad \frac{V_p}{V_m} = (L_r)^3$$

Kinematic Similarity

Kinematic similarity exists between prototype and model if quantities such at velocity and acceleration at corresponding points on model and prototype are same.

(V)   (V)   (V)

$$1\ p = \quad 2\ p = \quad 3\ p\ \ldots\ldots\ldots\ldots = V$$

$(V_1)_m \quad (V_2)_m \quad (V_3)_m \qquad\qquad r$

$V_r \rightarrow$ Velocity ratio

Dynamic Similarity

Dynamic similarity is said to exist between model and prototype if ratio of forces at corresponding points of model and prototype is constant.

(F) (F
) (F)

$$1 \ p = \ 2 \ p = \ 3 \ p \ .... = F$$

$(F_1)_m \quad (F_2)_m \quad (F_3)_m \qquad\qquad R$

$F_R \rightarrow$ Force ratio

Following dimensionless numbers are used in fluid mechanics.

1. Reynold's number
2. Froude's number
3. Euler's number
4. Weber's number
5. Mach number

2.8. Dimensionless numbers

There are five dimensionless numbers to analyze various

1. Reynolds number

It is defined as the ratio of inertia force of the fluid to viscous force.

$$\therefore N_{Re} = \frac{F_i}{F_v}$$

Expression for N_{Re}

F_i = Mass x Acceleration

$F_i = \rho$ x Volume x Acceleration

Change in velocity

$F_i = \rho$ x Volume x Time

$F_i = \rho$ x Q x V

$F_i = \rho A V_2$

F_V → Viscous force

$F_V = \tau$ x A

V

$$F_V = \mu \; {}_{y} \; A$$

$$F_V = \mu \frac{V}{L} A$$

$$N_{Re} = \frac{\rho A V_2}{V}$$

μ_L^A

$$N_{Re} = \frac{\rho V L}{\mu}$$

2. Froude's Number (F_r)

It is defined as the ratio of square root of inertia force to gravity force.

$F_r =$

$F_i = m \times a$

$F_i = \rho \times \text{Volume} \times \text{Acceleration}$

$F_i = \rho A V_2$

$F_g = m \times g$

$F_g = \rho \times \text{Volume} \times g$

$F_g = \rho \times A \times L \times g$

$F_r =$

$F_r =$

$F_r =$

3. Euler's Number (E_u)

It is defined as the square root of ratio of inertia force to pressure force.

$E_u =$

$F_i = \text{Mass} \times \text{Acceleration}$

Velocity

$F_i = \rho \times \text{Volume} \times$ Time

$F_i = \rho \times Q \times V$

$F_i = \rho A V^2$

$F_p = p \times A$

$E_u =$

$E_u =$

$E_u =$

$E_u =$

$E_u =$

4. **Weber's Number (W_b)**

It is defined as the square root of ratio of inertia force to surface tensile force.

$W_b =$

$F_i = \rho A V^2$

$F_\sigma = \sigma \times L$

$W_b =$

$W_b =$

$W_b =$

$W_b =$

$W_b =$

5. Mach Number (M)

It is defined as the square root of ratio of inertia force to elastic force.

$M =$

$F_i = \rho A V^2$

$F_e = K \times A$

$K \rightarrow$ Bulk modulus of elasticity

$A \rightarrow$ Area

$M=$

$M=$

$M=$

2.9. Model Laws

1. **Reynold's Model Law**

For the flows where in addition to inertia force, similarity of flow in model and predominant force, similarity of flow in model and prototype can be established if Re is same for both the system.

This is known as Reynold's Model Law.

Re for model = Re for prototype

$$(Re)_m = (Re)_p$$

Applications:

i) In flow of incompressible fluids in closed pipes.

ii) Motion of submarine completely under water.

iii) Motion of air-planes.

2. Froude's Model Law

When the force of gravity is predominant in addition to inertia force then similarity can be established by Froude's number. This is known as Froude's model law.

$$(Fr)_m = (Fr)_p$$

Applications:

i) Flow over spillways.

ii) Channels, rivers (free surface flows).

iii) Waves on surface.

iv) Flow of different density fluids one above the other.

The other dimensionless numbers also can be used for model investigation just like Reynold's model law and Fraud's model law

Unit IV Hydraulic Similitude

Assignment-Cum-Tutorial Questions

Part- A Objective type Questions

1. Expression for Reynold's number is ………….
2. Expression for Froude number is ……………
3. Formula to get Euler number is …………….
4. Mach number is a ratio of …………….
5. Forces involved in Weber number are ………………
6. The scale- effect is the discrepancy caused if complete similitude doesn't exist between the model and it's ………….
7. The model of completely submerged objects such as aeroplanes are governed by ………… model laws
8. A small scale replica of the actual structure or machine is called ………..
9. The expression for a derived quantity in terms of primary quantities is called the …………….. of the physical quantity.
10. Reynold's number is a ratio of ……………….

Part- B Subjective Questions

1. Explain the terms distorted models and undistorted models. What is the use of distorted models
2. Define the terms: model, prototype, model analysis, and hydraulic similitude.
3. What are the methods of dimensional analysis? Describe the Rayleigh's method for dimensional analysis.
4. What is meant by geometric, kinematic and dynamic similarities?
5. What do you mean by dimensionless number? Name any three dimensionless number
6. Explain the different types of hydraulic similarities that must exist between a prototype and its model.
7. State Buckingham's Π-theorem. Why this theorem is considered superior over the

Rayleigh's method for dimensional analysis.

8. What do you mean by repeating variables? How are the repeating variables selected for dimensional analysis?
9. Determine the dimensions of the quantities given below i) angular velocity ii) angular acceleration iii) discharge iv) Kinematic viscosity v) Force vi) Specific weight
10. Time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.
11. Derive the expression for the power P, developed by a pump when P depends upon the head H, the discharge Q and specific weight w of the fluid.
12. The efficiency η of a fan depends on the density ρ, the dynamic viscosity μ of the fluid, the angular velocity ω, diameter D of the rotor and the discharge Q. Express η in terms of dimensionless parameters.
13. Explain Reynolds's number, Froude's number and Mach number. Derive expressions for any above two numbers.
14. A model of submarine of scale 1/40 is tested in a wind tunnel. Find the speed of air in wind tunnel if the speed of submarine in sea is 15m/s. Also find the ratio of resistance between the model and its prototype. Take values of kinematic viscosities for sea water and air as 0.12 stokes and 0.016 stokes respectively. The density of sea water and of air is given as 1030kg/m^3 and 1.24 kg/m^3 respectively.

Part C- Gate questions/ competitive exams

1. Water is flowing through a pipe of diameter 30cm at a velocity of 4m/s. Find the velocity of oil flowing in another pipe of diameter 10 cm, if the condition of dynamic similarity is satisfied between the two pipes. The viscosity of water and oil is given as 0.01 poise and .025 poise. The specific gravity of oil=0.8.

   a) 37.5m/s b) 42.5m/s c) 45m/s d) 50m/s

2. A pipe of diameter 1.5m is required to transport an oil of specific gravity of 0.90 and viscosity 3 X10^{-2} poise and at the rate of 3000litre/s. Tests were conducted on a 15cm diameter pipe using water at 20^{o}c. The rate of flow in the model when Viscosity of water at 20^{o}c=0.01 poise would be

   a) 5.1 m^3/s b) 5.5m^3/s c) 6m3/s d) 9m^3/s

3. A 1:15 model of a flying boat is towed through water. The prototype is moving in sea water of density of 1024kg/m^3 at a velocity of 20m/s. The corresponding speed of the model would be

   a) 5.165m/s b)3.215m/s c) 6.3m/s d)8m/s

4. In a model experiment with weir, if the dimensions of the model are reduced by a factor ‘k’, the flow rate through the model weir is the following fraction of the flow rate through the prototype.

a) $k^{2.5}$ b) k^2 c) k^{-1} d) k^{-2}

5. The number of π parameters needed to express the equation with 6 variables is

a) 3 b) 2 c) 4 d) 5

6. A 1:30 ogee spillway model crest records and acceleration of 1.3m/sec^2 at certain location. The homologous value of the acceleration in prototype of m/sec^2 is

a) 0.043 b) 0.237 c) 1.30 d) 7.2

Unit - 5

Closed conduit flow

Objective:

- To determine the characteristics of different flows
- To determine the flow between the parallel plates and long tubes

Syllabus:

Reynold's experiment, Characteristics of Laminar & Turbulent flows, Flow through circular pipe, Flow through Parallel Plates with problems, Laws of fluid friction, Darcy's Equation, Minor Losses with problems, Pipes in Series and Parallel with problems, Total Energy line and Hydraulic Gradient line with problems.

Learning Outcomes:

- Ability to identify the energy grade lines
- Discuss the Characteristics of different flows
- Discuss the flow between parallel plates.
- Illustrate the different types of energy grade lines

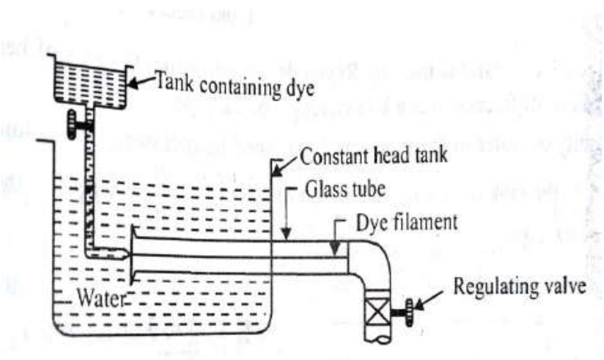
Learning material

Laminar flow is the flow of fluid particles move along straight parallel path in layers. Laminar flow is possible only at low velocities and when the fluid is highly viscous. But when the velocity increased the fluid particles doesn't move in straight paths. The fluid particles move in random manner resulting in general mixing of the particles. This type of flow is called turbulent flow.

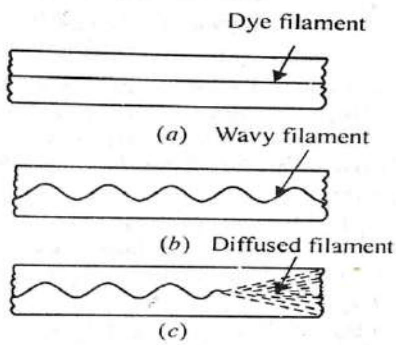
Reynolds experiment

The type of flow is determined from Reynolds number i.e. . This was

de
m
on
str
at
ed
by



Reynolds apparatus.



Appearance of dye filament in (a) laminar flow, (b) transition, and (c) turbulent flow.

O. Reynold in 1883.

This apparatus consist of

i) A tank containing water at constant head.

ii) A small tank containing some dye.

iii) A glass tube having a bell mouthed entrance at one end and a regulating valve at outer ends.

The water the tank was allowed to flow through the glass tube. The velocity of flow was varied by the regulating valve. A liquid dye having same specific as water was introduced into the glass tube.

The following observations were made:

i) When velocity of flow was low, the dye filament in the glass tube was in the form of a straight line. The straight line of dye filament was parallel to the glass tube, which was in case of laminar flow

ii) With the increase of velocity of flow, the dye filament was no longer a straight line but it becomes a wavy, this shows that flow is no longer laminar.

iii) With increase of velocity of flow, the wavy dye filament broke and finally diffused in water. This means that the fluid particles of the dye at this higher velocity are moving in random fashion, which shows a case of turbulent flow.

Characteristics of Laminar and Turbulent flows.

	Laminar	**Turbulent**
Visible characteristics	Flow in layers parallel to boundary	Chaotic velocity fluctuations
Reynolds number	Low less than 2000	Greater than 4000
Mixing	Small molecular diffusion	Large scale eddies
Shear stress	Lower	Higher
Velocity profile	Parabolic	Logarithmic

Flow of fluid through circular pipe:

The flow through the circular pipe will be viscous or laminar, if the Reynolds number is less than 2000.

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe. Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius (r+ dr).

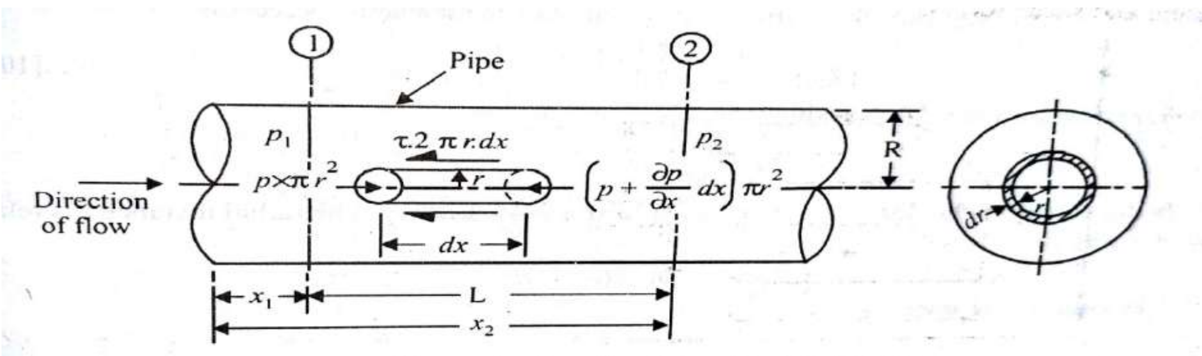
let the length of fluid element be Δx. if p is the intensity of pressure on the face AB,

forces acting on the fluid element are:

1. The pressure force on face AB = p.
2. The pressure force on face CD =
3. The shear force on surface of fluid element =τ.2

As there is no acceleration, hence the summation of all forces in the direction of flow must be zero.

- τ.2 =
0



Viscous/laminar flow through a circular pipe.

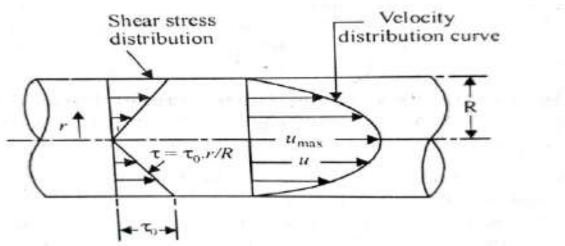
By simplification we get τ = -

Velocity distribution:

To obtain the velocity distribution across a section, the value of shear stress is substituted in above equation

‘y’ is measured from the pipe wall

Y= R-r and dy = -dr



Shear stress and velocity distribution across a section.

τ= μ = - μ

by substituting the equation of τ and integrating with limiting values from R to 0 we get

Ratio of maximum velocity to average velocity:

The velocity is maximum, when r=0 in above equation. We get,

The average velocity is obtained by dividing discharge of fluid across the section by the area of the pipe

By integrating we get

Average velocity

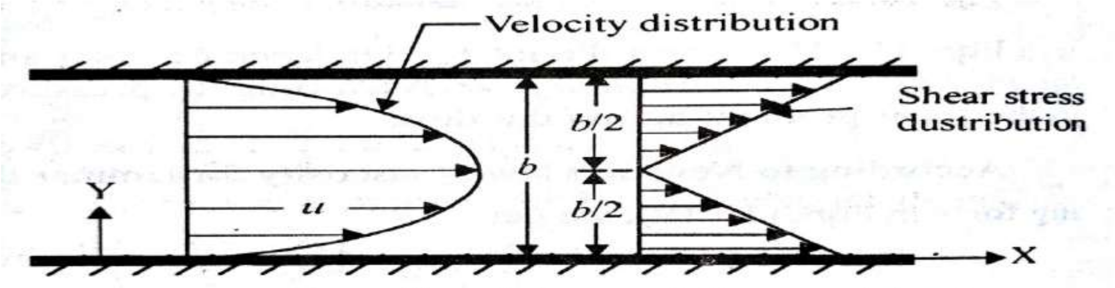
By dividing the two equations of max velocity and average velocity

Flow between parallel plates:

Consider two parallel fixed plates keep at a distance 't' apart. A viscous fluid is flowing between these two plates from left to right. Consider a fluid element of length and thickness at a distance y from the lower fixed plate. If p is the intensity of pressure on the face AB.

If the width of the element in the direction perpendicular to the paper is unity then the forces acting on the fluid element are:

1. The pressure force on face AB = p.dy.1
2. The pressure force on face CD =)
3. The shear force on face BC = τ.
4. The shear force on face AD =(τ+



Flow between stationary plates.

For steady and uniform flow, there is no acceleration and hence the resultant force in the direction of flow is zero.

By solving, we get

i. **Velocity distribution:**

To obtain the velocity distribution across a section, the value of shear stress from newtons law of viscosity.

By integrating and applying limitations we get,

ii. **Ratio of maximum velocity to average velocity:**

The velocity is maximum, when y=t/2. By substituting the value in the above equation we get

The average velocity is obtained by dividing the discharge across the section by the area of section(tx1)

By integrating the above equation we get the discharge Q

We know that Q= A.V

Average velocity =

From the above equations we get,

iii. **Drop of pressure head for a given length:**

=

Integrating this equation w.r.t. x,

If is the drop of pressure head, then

iv. **Shear stress distribution:**

It is obtained by substituting the value of u from the above equations in shear stress equation

Hence τ varies linearly with y, and remaining parameters in the equation is constants. Shear stress is maximum when y=0 or at wall and zero at y=t/2 that at the centre line between the two plates.

The maximum shear stress is given by

Laws of fluid friction

Different laws are obeyed by the frictional resistance in the laminar and turbulent flow. On the basis of the experimental observations the laws of fluid friction for the two types flows may be narrated as follows.

(a) Laws of fluid friction for laminar flow. The frictional resistance in the laminar flow is

i. Proportional to the velocity of flow
ii. Independent of the pressure
iii. Proportional to the area of surface in contact
iv. Independent of the nature of the surface in contact
v. Greatly affected by the variation of the temperature of the flowing fluid.

(b) Laws of fluid friction forturbulent flow.The frictional resistance in the turbulent flow is

i. Proportional to (velocity)^n where the index n varies from 1.72 to 2.0
ii. Independent of the pressure

iii. Proportional to the density of the flowing fluid
iv. Slightly effected by the variation of the temperature of the flowing fluid
v. Proportional to the area of the surface in contact
vi. Dependent of the nature of the surface in contact.

Losses in pipes:

Losses in pipes are mainly classified into two types. They are major losses and minor losses

Major loss:

Major loss is formed due to friction in pipes

Equation for head loss in pipes due to friction – Darcy –Weisbach equation

Consider a horizontal pipe of cross sectionalarea A carrying a fluid with a mean velocity V. Let 1 and 2 be the two sections of the pipe L distance apart, where let the intensities of pressure beP_1 and P_2 respectively.

By applying Bernoulli's equation between the sections 1 and 2, we obtain

Since $V_1 = V_2 = V$ and $Z_1 = Z_2$

Loss of head = h_f =

i.e; the pressure intensity will be reduced by the frictional resistance in the direction of flow and the difference of heads between any two sections is equal to the loss of head due to friction between these sections.

Further let f^1 be the frictional resistance per unit area at unit velocity,

then frictional resistance = f^{11}*area*V^n

= f^1*PL*V^n where P is the wetted perimeter of the pipe

The pressure forces at the sections 1 and 2 are P_1A and P_2A respectively. Thus resolving all the forces horizontally, we have $P_1A = P_2A$ + frictional resistance

Or $(P_1 – P_2)A = f^1$*PL*V^n

Or $(P_1 - P_2) = (f^1 * PL * V^n)/A$

Dividing both sides by specific weight W of the flowing fluid

But , then

The ratio of the cross sectional area of the flow (wetted area) to the perimeter in contact with the fluid (wetted perimeter) i.e; (is called hydraulic mean depth (H.M.D.) or hydraulic radius and its represented by 'm or R'.

Then

For pipes running full

m = = =

Substituting this in the equation for h_f and assuming n = 2

Putting

Where f is known as friction factor, which is a dimensionless quantity.

The above equation is known as Darcy-Weisbach equation which is commonly used for computing the loss of head due to friction in pipes. It may be noted that the head loss due to friction is also expressed in terms of the velocity head ($V^2/2g$) corresponding to the mean velocity. Further the observations show that the coefficient *f* is not a constant but its value depends on the roughness condition of the pipe surface and the Reynolds number of the flow. As such in order to determine the loss of head due to friction correctly, its essential to estimate the value of the friction *f* correctly.

Minor losses

Local or minor losses are caused by certain features or disturbances and these are also called as SECONDARY LOSSES. These losses occur whenever there is a sudden change in the area of flow and or the direction of flow. These would affect the velocity of distribution and may result in an eddy formation. For example, losses that occur in pipelines because of bends, elbows, joints etc., are called MINOR LOSSESS.

The loss of head or energy due to friction in a pipe is known as major losses, while the loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy. In almost all the cases, the minor loss is determined by experiment. However, one important exception is the head loss due to a sudden expansion in a pipeline.

Minor losses are the following:

a. loss of head due to Sudden Enlargement (Carnot or Borda equation)

$$h_e = (1-a_1/a_2)^2 V_1^2/2g$$

$$=k\ V_1^2/2g$$

$$k=(1-a_1/a_2)^2$$

b. Loss of Head Due to Sudden Contraction

$$h_c$$

c. Loss of head at Entrance in Pipe

d. Loss of Head at the Exit of a Pipe

e. Loss of Head at the Bends

f. Loss of Head Due to Fittings

Flow Through parallel Pipes

When two or more pipes are connected as shown in fig such that they branch out from a single point (at M) and after equal or unequal lengths join at a single point (at N), then the pipes are said to be parallel. A parallel pipe is added to an existing pipe to increase the discharge.

The following two principles are used in the solution of any types of problem connected with parallel pipes.

1. Continuity principle: $Q = Q_1 + Q_2 + \ldots\ldots\ldots$
2. The difference in piezometric heads between junctions (M & N) is equal to the head loss in each of the parallel pipes between junctions. That is,

sum of minor loss coefficients, i=1, 2, or 3

if only loss due to firction is considered

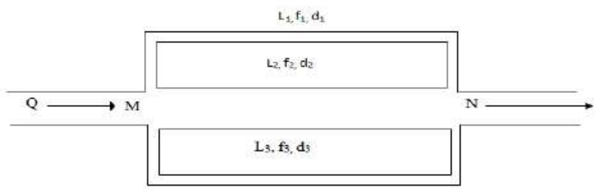


Fig: Pipes in Parallel

Hydraulic Gradient and Total Energy Line

Hydraulic Gradient Line (H.G.L): It is defined as the line which gives the sum of pressure head and datum head of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L.

Total Energy Line (T.E.L): It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates, showing the pressure head and kinetic head from the centre of the pipe.

Flow through pipes in the series or flow through compound pipes

Pipes in series or compounds pipes are defined as the pipes of different lengths and different diameters connected end to end (in series) to from a pipe line as shown in figure

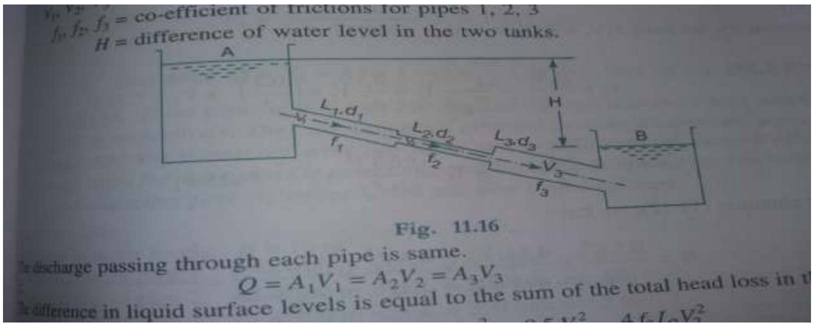
Let L_1, L_2, L_3= length of pipes 1, 2 and 3

d_1, d_2, d_3= diameter of pipes 1, 2 and 3

V_1, V_2, V_3= velocity of flow through pipes 1, 2 and 3

f_1, f_2, f_3 = coefficient of frictions for pipes 1, 2 and 3

H= difference of water level in the two tanks



The discharge passing through each pipe is same.

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

If the minor losses are neglected, then above equation becomes as

If the coefficient of friction is same for all pipes

, then above equation becomes as

Hydraulic gradient and total energy line:

Hydraulic gradient line:

It is defined as the line which gives the sum of pressure head and datum head of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head of flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L.

Total energy line:

It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L.

Variation of friction factor with respect of Reynolds number:

The friction factor varies with respect to the Reynolds number because flow behaviour varies with the Reynolds number

Coefficient of friction f= .... for laminar flow

= for turbulent flow in smooth pipes

= 0.0008+ for but

= 2 for rough pipes

Unit V Closed Conduit Flow

Assignment-Cum-Tutorial Questions

Part- A Objective type Questions

1. Reynolds number () = ...
2. Energy per unit weight of water is called ....
3. Loss of head due to friction = ...

4. Major loss in pipes is formed due to …………..
5. Equation for the loss of energy due to sudden enlargement is ……………
6. Equation for the loss of energy due to sudden contraction is …………….
7. Equation for loss of energy at the entrance of a pipe is ……………….
8. Expression for loss of energy at the exit from a pipe is ……………..
9. Expression for loss of energy in bends is …………………
10. Expression for loss of energy in various pipe fittings is …………………..

Part-B Subjective Questions

1. What are the characteristics of laminar flow?
2. What are the characteristics of turbulent flow?
3. Derive Darcy's equation.
4. What are the laws of fluid friction?
5. What is energy line?
6. What are the minor losses in pipes?
7. Water is flowing through a horizontal pipe line 1500m long and 200 mm in diameter. Pressures at the two ends of the pipe line are 12 kpa and 2 kpa. If f = 0.015, determine the discharge through the pipe in lit/min. Consider only friction loss.
8. Two pipes each 300m long are available for connecting to a reservoir from which a flow of $0.085m^3/s$ is required. If the diameters of the two pipes are 0.30m and 0.15m respectively, determine the ratio of the head lost when the pipes are connected in series to the head lost when they are connected in parallel. Neglect minor losses.
9. Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400 mm, 200 mm and 300 mm respectively. They are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks where difference of water levels is 16 m. assume the friction factor as 0.005. Determine the discharge through the compound pipe neglecting minor losses?
10. Water flows from a reservoir through a pipe of 0.15m diameter and 180m long to a point13.5m below the open surface of the reservoir. Here it branches into two pipes, each of 0.1m diameter, one of which is 48m long discharging to atmosphere at a point 18m below reservoir level, and the other 60m long discharging to atmosphere 24m below the reservoir level. Assuming a constant friction coefficient of 0.032, calculate

the discharge from each pipe. Neglect any losses at the junction.

11. A pipe line ABC 180m long, is laid on an upward slope of 1 in 60. The length of the portion AB is 90m and its diameter is 0.15m.At B the pipe section suddenly enlarges to 0.30m diameter and remains so for the remainder of its length BC, 90m. A flow of 50 liters per second is pumped into the pipe at its lower end A and is discharged at the upper end C into a closed tank. The pressure at the supply end A is 137.34 kN/m^2. Sketch a) the total energy line, b) the hydraulic grade line and also find the pressure at the discharge end C. Take f = 0.02.
12. The population of a city is 8,00,000 and its to be supplied with water from a reservoir 6.4 km away. Water is to be supplied at the rate of 140 liters per head per day and half the supply is to be delivered in 8 hours. The full supply level of the reservoir is R.L 180.00 and its lowest water level is R.L. 105.00. The delivery end of the main is at R.L. 22.50 and the head required there is 12m. Find the diameter of the pipe. Take f = 0.04.
13. A pipe line having a length of 6 km and diameter 0.70m connects two reservoirs A and B, the difference between their water levels is 30m. Halfway along the pipe there is a branch through which water can be supplied to a third reservoir C. Taking f = 0.024 determine the rate of flow of reservoir B when a) no water is discharged to the reservoir C; b) the quantity of water discharge to reservoir C is 0.15 m^3/s. Neglect minor losses.

Part C- Gate questions/ competitive exams

1. For steady incompressible flow through a closed-conduit of uniform cross-section, the direction of flow will always be:

   a) from higher to lower elevation b)from higher to lower pressure

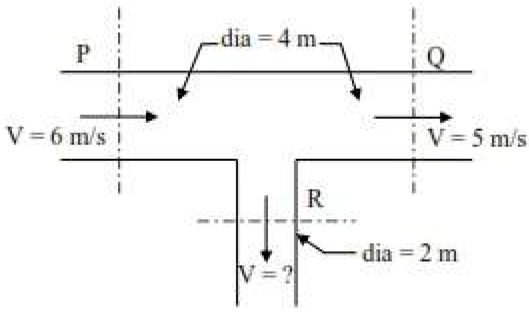
   c)from higher to lower velocity d) from higher to lower piezometric head

2. An incompressible homogeneous fluid is flowing steadily in a variable diameter pipe having the large and small diameters as 15 cm and 5 cm, respectively. If the velocity at a section at the 15 cm diameter portion of the pipe is 2.5 m/s, the velocity of the fluid (in m/s) at a section falling in 5 cm portion of the pipe is ___________
3. A 2 km long pipe of 0.2 m diameter connects two reservoirs. The difference between water levels in the reservoirs is 8 m. The Darcy-Weisbach friction factor of the pipe is 0.04. Accounting for frictional, entry and exit losses, the velocity in the pipe (in m/s)

is:

a) 0.63 b)0.35 c)2.52 d)1.25

4. The circular water pipes shown in the sketch are flowing full. The velocity of flow (in m/s) in the branch pipe "R" is



Unit - 6

Measurement of flow

Objective:

- To determine the rate of flow by using different devices
- To determine the rate of flow in open channel flow

Micro Syllabus:

Pitot tube, Orifices-fully submerged and partially submerged orifices;, Flow through Notches rectangle, triangle, Trapezoidal, stepped notches with problems.

Learning Outcomes:

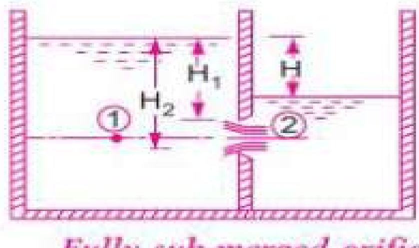
Student will be able to:

- Calculate the rate of flow by using different devices
- Calculate the rate of flow in open channel flow

Learning material

DISCHARGE THROUGH FULLY SUB-MERGED ORIFICE

Fully sub-merged orifice is one which has its whole of the outlet side sub-merged under liquid so that it discharges a jet of liquid into the liquid of the same kind. It is also called totally drowned orifice. Fig shows the fully sub-merged orifice. Consider two points (1) and (2), point 1 being in the reservoir on the upstream side of the orifice and point 2 being at the vena-contracta as shown in Fig.



Fully sub-merged orifice.

Fig. Fully submerged Orifice

Let H_1 = Height of water above the top of the orifice on the upstream side.

H_2 = Height of water above the bottom of the orifice

H = Difference in water levels,

b=Width of orifice,

Cd = Co-efficient of discharge.

Height of water above the centre of orifice on upstream side

$$= H_1 + \;=$$

Height of water above the centre of orifice on downstream side

$$= \; - H$$

Applying Bernoulli's equation at (1) and (2), we get

$$+ \quad = \quad +$$

Now $=$ $=$ - H and V_1 is negligible.

$$+ 0 = \; - H +$$

$$= H$$

$$V_2 =$$

Area of orifice = b x (H_2-H_1)

Discharge through orifice = C_d x Area x Velocity

Q = C_d x b x (H_2 - H_1) x V_2 =

Discharge through a partially sub-merged orifice:

Partially sub-merged orifice is one which has its outlet side-partially sub-merged under liquid as shown in Fig. . It is also known as partially drowned orifice. Thus the partially sub-merged orifice has two portions. The upper portion behaves an orifice discharging free while the lower portion behaves as a sub-merged orifice. Only a large orifice can behave as a partially sub-merged orifice. The total discharge Q through partially sub-merged orifice is equal to the discharges through free and the sub-merged portions.

Discharge through the sub-merged portion is given by equation

$$Q_1 = C_d \text{ x } b \text{ x } (H_2 - H) \text{ X}$$

Discharge through the free portion is given by equation as

$$Q_2 = C_d \times b \times [H_2^{3/2} - H_1^{3/2}]$$

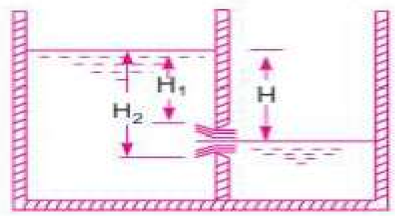


Fig. Partially sub-merged orifice.

Fig. Partially submerge orifice

Total discharge $Q = Q_1 + Q_2$

$$= [C_d \times b \times (H_2 - H) X] + [C_d \times b \times [H_2^{3/2} - H_1^{3/2}]]$$

Pitot tube

A pitot tube is a pressure measurement instrument used to measure fluid flow velocity. The Pitot tube was invented by the French engineer Henri Pitot in the early 18th century and was modified to its modern form in the mid-19th century by French scientist Henry Darcy. It is widely used to determine the airspeed of an aircraft, water speed of a boat, and to measure liquid, air and gas velocities in industrial applications. The Pitot tube is used to measure the local velocity at a given point in the flow stream and not the average velocity in the pipe or conduit.

Working of Pitot tube

The basic Pitot tube consists of a tube pointing directly into the fluid flow. As this tube contains fluid, a pressure can be measured; the moving fluid is brought to rest (stagnates) as there is no outlet to allow flow to continue. This pressure is the stagnation pressure of the fluid, also known as the total pressure or (particularly in aviation) the pitot pressure.

The liquid flows up the tube and when equilibrium is attained, the liquid reaches a height above the free surface of the water stream.

Since the static pressure, under this situation, is equal to the hydrostatic pressure due to its depth below the free surface, the difference in level between the liquid in the glass tube and the free surface becomes the measure of dynamic pressure. Therefore, we can write, neglecting friction,

$$p_0 - p = h\rho g$$

where p_0, p and V are the stagnation pressure, static pressure and velocity respectively at point A.

$$v = (2gh)^{1/2}$$

For an open stream of liquid with a free surface, this single tube is sufficient to determine the velocity. But for a fluid flowing through a closed duct, the Pitot tube measures only the stagnation pressure and so the static pressure must be measured separately.

Measurement of static pressure in this case is made at the boundary of the wall. The axis of the tube measuring the static pressure must be perpendicular to the boundary and free from burrs, so that the boundary is smooth and hence the streamlines adjacent to it are not curved. This is done to sense the static pressure only without any part of the dynamic pressure.

A Pitot tube is also inserted as shown (Fig.-5) to sense the stagnation pressure. The ends of the Pitot tube, measuring the stagnation pressure, and the piezo metric tube, measuring the static pressure, may be connected to a suitable differential manometer for the determination of flow velocity and hence the flow rate.

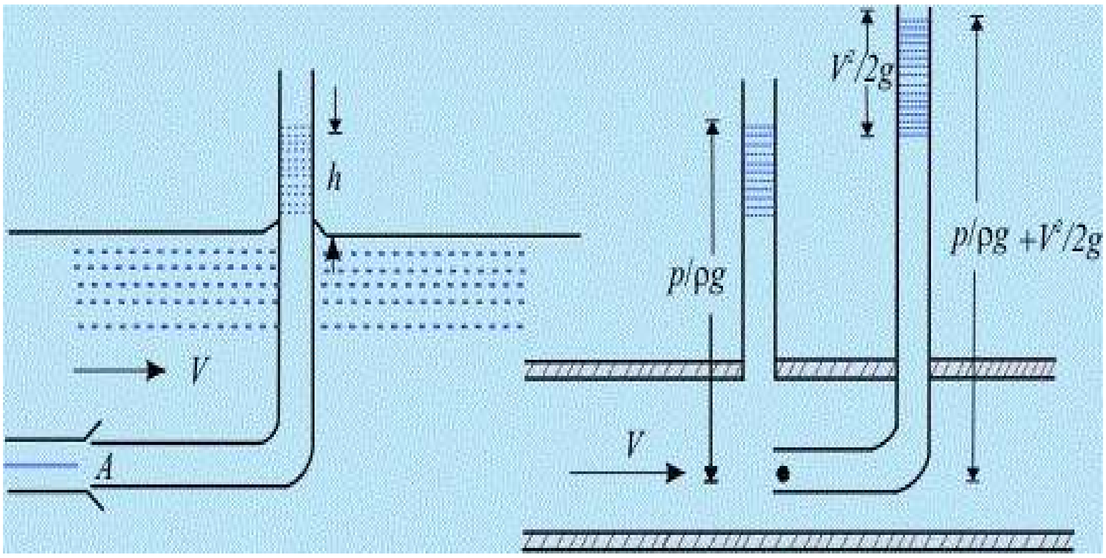


Fig. 5 Pitot tube used for measuring velocities in a stream and in a pipe

A pitot tube is simply a small cylinder that faces a fluid so that the fluid can enter it. Because the cylinder is open on one side and enclosed on the other, fluid entering it cannot flow any further and comes to a rest inside of the device. A diaphragm inside of the pitot tube separates the incoming pressure (static pressure) from the stagnation pressure (total pressure) of a system. The difference between these two measurements determines the fluid's rate of flow.

In industry, the velocities being measured are often those flowing in ducts and tubing where measurements by an anemometer would be difficult to obtain. In these kinds of measurements, the most practical instrument to use is the Pitot tube. The Pitot tube can be inserted through a small hole in the duct with the pitot connected to a U-tube water gauge or some other differential pressure gauge for determining the velocity inside the ducted wind tunnel. One use of this technique is to determine the volume of air that is being delivered to a conditioned space.

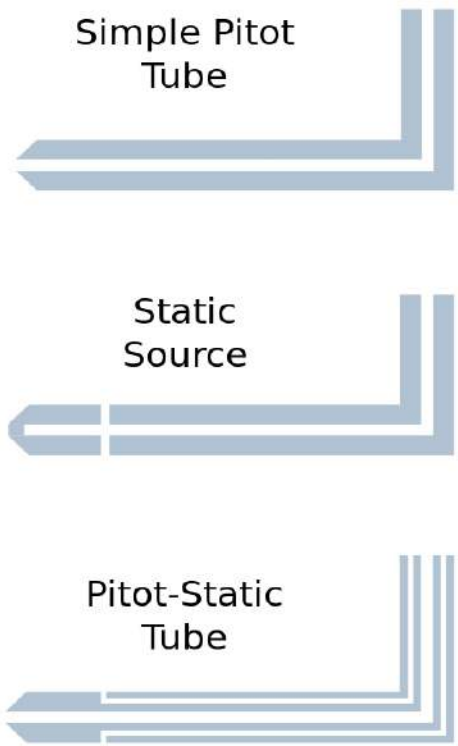


Fig. 6 Types of Pitot tube

Advantages:
Pitot tubes measure pressure levels in a fluid. They do not contain any moving parts and routine use does not easily damage them. Also, pitot tubes are small and can be used in tight spaces that other devices cannot fit into.

Disadvantages:
Foreign material in a fluid can easily clog pitot tubes and disrupt normal readings as a result. This is a major problem that has already caused several aircraft to crash and many more to make emergency landings.

Notch:

Notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that liquid surface in the tank or channel is below the top edge of the opening.

Nappe or vein:

The sheet of water flowing through a notch or over a weir

Crest or still:

The bottom edge of a notch or a top of a weir over which the water flows

Classification of notches:

1. According to the shape of the opening:
   (a) Rectangular notch,
   (b) Triangular notch,
   (c) Trapezoidal notch and
   (d) Stepped notch.
2. According to the effect of the sides on the nappe:
   (a) Notch with end contraction.
   (b) Notch without end contraction or suppressed notch.

Discharge over a rectangular notch or a weir:

Consider a rectangular notch or a weir provided in a channel carrying water .

Let H = Head of water over the crest

L = length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water.

The area of the strip = L. dh

Theoritical velocity of strip =

The discharge dQ =

The total discharge Q =

By solving above equation

Q =

Discharge over a triangular notch or a weir:

Let H = Head of water over the crest

θ = angle of notch

Consider a horizontal strip of water thickness *dh* at a depth of h from the free surface of water.

Width of the strip = 2.(H-h)tan(θ/2)

Area of the strip = 2.(H-h)tan(θ/2).dh

Theoretical velocity =

dQ =

Q =

By solving above equation we get,

Q =

Discharge over a trapezoidal notch or a weir:

A trapezoidal notch or weir is a combination of rectangular and triangular notch. Thus the total discharge will be equal to the sum of discharge through a rectangular and triangular.

Let H = Head of water over the crest

L = length of the notch or weir

Q = =

Discharge over a stepped notch or a weir:

A stepped notch is a combination of rectangular notches. The discharge through stepped notch is equal to the sum of the discharges through the different rectangular notches. Let H_1 = Head of water over the crest1

L_1 = length of the notch or weir1

H_2, L_2 and H_3, L_3 corresponding values of notches 2 and 3 respectively.

Q =

Q = +

Unit VI Measurement of Flow

Assignment-Cum-Tutorial Questions

Part- A Objective type Questions

1. Velocity through the pipe system by using Pitot tube is …………
2. The principle of Pitot tube is ………..
3. Equation for the discharge of water flowing over a rectangular weir or notch…………
4. Equation for the discharge of water flowing over a rectangular weir or notch when the velocity of approach is taken into account is …………..
5. Expression for discharge of water flowing over a triangular weir or notch is ……….
6. Expression for discharge of water flowing over broad crested weir is………

Part- B Subjective Questions

1. A pitot static tube placed in the centre of a 200mm pipe line, has one orifice pointing upstream and the other perpendicular to it. If the pressure difference between the two orifices is 40mm of water when the discharge through the pipe is 1,365 litres per minute, calculate the coefficient of the Pitot tube. Take the mean velocity in the pipe to be 0.83 of the central velocity.
2. Find the expression for discharge over a broad crested weir.

3. Determine the height of a rectangular weir of length 6m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 108m and discharge is 2000 lit/sec. take Cd = 0.6 and neglect end contractions
4. Water flows through a triangular right angled weir first and then over a rectangular weir of 1m width. The discharge coefficients of the triangular and rectangular weirs are 0.6 and 0.7 respectively. If the depth of water over the triangular weir is 360mm. find the depth of water over the rectangular weir.
5. A rectangular orifice of 2m width and 1.2m deep is fitted in one side of a large tank. The water level on one side of orifice is 3m above the top edge of orifice, while on the other side of the orifice, the water level is 0.5m below its top edge. Calculate the discharge through the orifice if cd = 0.64.
6. Water is supplied from a tank into a canal through a rectangular sluice 1m wide and 1.75m high. The water level in the tank is 2m above the top edge of the opening and the canal water level is 0.3m below the top edge. If the coefficient of discharge is 0.62 for both the free and submerged portions, calculate the discharge.
7. What are the different types of coefficients for an orifice?
8. What do you understand by the terms wholly submerged orifice and partially submerged orifice?
9. Differentiate between a large and small orifice. Obtain the expression for discharge through a large rectangular orifice.
10. What is a notch? How are the notches classified?

Part C- Gate questions/ competitive exams

1. A fluid jet is discharging from a 100 mm nozzle and the vena contracta formed has a diameter of 90 mm. If the coefficient of velocity is 0.95, then the coefficient discharge for the nozzle is:

   a) 0.855 b)0.81 c) 0.9025 d) 0.7695

2. A standard 90° V-notch weir is used to measure discharge. The discharge is Q1 for heights H1 above the sill and Q2 is the discharge for a height H2. If H2/ H1 is 4, then Q2/ Q1 is:

   a) 32 b)16 c) 16 d)8

3. A triangular notch is more accurate measuring device than the rectangular notch for measuring which one of the following?

   a) low flow rates b)Medium flow rates c)High flow rates d) All flow rates