# GUDLAVALLERU ENGINEERING COLLEGE <br> (An Autonomous Institute with Permanent Affiliation to JNTUK, Kakinada) <br> Seshadri Rao Knowledge Village, Gudlavalleru - 521356. 

## DEPARIMESNT OF CIVIL EJNGINEERISNG



## HANDOUT

on

## Vision

To provide quality education embedded with knowledge, ethics and advanced skills and preparing students globally competitive to enrich the civil engineering research and practice.

## Mission

$>$ To aims at imparting integrated knowledge in basic and applied areas of civil engineering to cater the needs of industry, profession and the society at large.
> To develop faculty and infrastructure making the department a centre of excellence providing knowledge base with ethical values and transforming innovative and extension services to the community and nation.
$>$ To make the department a collaborative hub with leading industries and organizations, promote research and development and combat the challenging problems in civil engineering which leads for sustenance of its excellence.

## Program Educational Objectives

PEO1: Exhibit their competence in solving civil engineering problems in practice, be employed in industries and undergo higher studies.

PEO2: Adapt to changing technologies with societal relevance for sustainable development in the field of their profession.

PEO 3: Develop multidisciplinary team work with ethical attitude \&social responsibility and engage in life - long learning to promote research and development in the profession.

## GEOTECHNICAL ENGINEERING-II

| Class\& Sem. | $:$ III B.Tech - II Semester | Year $: 2016-17$ |
| :--- | :--- | :--- |
| Branch | $:$ CIVIL ENGINEERING | Credits $: 3$ |

1. Brief History and Scope of the Subject

In the Autonomous curriculum, the subject "Geotechnical Engineering-II" is offered at III B.Tech II-Semester level. Soil engineering is an applied science dealing with the applications of principles of soil mechanics to practical problems. It has a much wider scope than soil mechanics, as it deals with all engineering problems related with soils. It includes site investigations, design and construction of foundations, earth-retaining structures and earth structures.

Geotechnical engineering is a broader term which includes soil engineering, rock mechanics and geology.

The field of Soil Mechanics is very vast. The Civil Engineering has many diversified and important encounters with soil. Apart from testing and classifying the various types of soils in orlder to know its physical properties, the knowledge of soil mechanics is particularly helpful in the design of foundations, rigid and flexible pavements, underground and earth retaining structures, embankments and earth dams.

## 2. Pre-Requisites

- Geotechnical Engineering -I
- Engineering Geology and GIS applications

3. Course Objectives:

- To understand the slope stability.
- To impart the knowledge on lateral earth pressure and stability of retaining walls.
- To understand load carrying capacity of shallow and deep foundations.


## 4. Course Outcomes:

Student will be able to
CO1: Carry out soil investigation and prepare the bore log data.
CO2: Analyse stability of slopes of an embankment.
CO3: Determine lateral earth pressure and check the stability of retaining walls.
CO4: Determine bearing capacity of soil.
CO5: Analyse the load carrying capacity of deep foundation.
5. Program Outcomes:

Graduates of the Civil Engineering Program will have
a. An ability to apply knowledge of mathematics, science and engineering principles to civil engineering problems.
b. An ability to analyze design and conduct experiments and interpret the resulting data.
c. An ability to design a system, component or process to meet desired goals in civil engineering applications.
d. An ability to function on multi disciplinary teams.
e. An ability to identify, formulate and solve challenging engineering problems.
f. An understanding of professional and ethical responsibility.
g. An ability to communicate effectively through verbal, written and drawing presentations.
h. An ability to understand the impact of engineering solutions in a global, economical and social context with a commitment on environmental and safety issues.
i. An ability to recognize the need of engaging in lifelong learning and acquiring further knowledge in specialized areas.
j. Ability to excel in competitive examinations, advanced studies and become a successful engineer in construction industry.
k. An ability to use the techniques, skills and modern engineering tools and software for engineering design and practices.

1. The understanding of basic finance \& management techniques and construction practices including work procurement and legal issues.
2. Mapping of Course Outcomes with Program Outcomes:

|  | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{c}$ | $\mathbf{d}$ | $\mathbf{e}$ | $\mathbf{f}$ | $\mathbf{g}$ | $\mathbf{h}$ | $\mathbf{i}$ | $\mathbf{j}$ | $\mathbf{k}$ | $\mathbf{l}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CO 1 | L | H |  |  |  |  | M |  |  |  |  |  |
| CO 2 |  | H |  |  | M |  |  |  |  | H | M |  |
| CO 3 | L | M |  |  | M |  |  |  | L | H |  |  |
| CO 4 |  |  |  |  | H |  |  |  | M | L | M |  |
| CO 5 |  | L |  |  |  |  |  |  |  | H | M |  |

7. Prescribed Text Books
a) Soil Mechanics and Foundation Engg.By K.R. Arora, Standard Publishersand Distributors, Delhi.
b) Geotechnical Engineering by C. Venkataramiah, New age International Pvt.Ltd,(2002).
c) Geotechnical Engineering Principles and practices of soil mechanics andfoundation engineering by VNS Murthy - CRC Press.
8. Reference Text Books
a) Basic and Applied Soil Mechanics by GopalRanjan\& ASR Rao, New ageInternational Pvt. Ltd, New Delhi
b) Soil Mechanics - T.W. Lambe and Whitman, Mc-Graw Hill Publishing Company, Newyork.
c) Geotechnical Engineering by Manoj Dutta \& Gulati S.K - Tata Mc.Grawhill Publishers New Delhi.
d) Foundation Analysis and Design by Joseph E. Bowles - McGraw-Hill
9. URLs for E-Learning Resources
a. http://nptel.ac.in/courses/105106142/
b. http://nptel.ac.in/courses/105104137/
c. http://nptel.ac.in/courses/105101083/

## 10. Digital Learning Materials:

- http://nptel.ac.in/courses/105105039/
- http://nptel.ac.in/courses/105101084/50
- http://nptel.ac.in/courses/105101084/55
- http://nptel.ac.in/courses/105107120/

11. Lecture Schedule / Lesson Plan

| Topic | No. of Periods |  |
| :---: | :---: | :---: |
|  | Theory | Tutorial |
| UNIT -1: Soil Exploration |  |  |
| Need of soil exploration | 2 | 1 |
| Methods of soil exploration,Boring | 2 |  |
| Methods of sampling | 2 |  |
| Penetration tests, Pressure meter | 1 | 1 |
| Plate load test | 1 |  |
| Planning of programme and preparation of soil investigation report | 1 |  |
| UNIT - 2: Earth Slope Stability |  |  |
| Types of failures, Factor of safety of infinite slopes | 1 | 1 |
| Stability analysis by Swedish arc method | 2 |  |
| Infinite and finite earth slopes | 2 |  |
| Bishops simplified method | 2 | 1 |
| Taylors stability number | 1 |  |
| Problems on earth slop stability | 1 |  |
| UNIT - 3: Earth Pressure Theories |  |  |
| Rankines theory of earth pressure | 2 | 1 |
| Coulombs theory of earth pressure | 2 |  |
| Rehbanns graphical method | 2 |  |
| Culmans graphical method | 1 | 1 |
| Types of retaining walls, Design approach of gravity Retaining wall | 1 |  |
| Cantilever Retaining wall | 1 | 1 |
| Problems on earth pressure theories | 1 |  |
| UNIT - 4: Shallow Foundations and Settlement Criteria |  |  |
| Bearing capacity,criteria for determination of bearing capacity,factors influencing bearing capacity | 1 | 1 |
| Determine bearing capacity byTerzaghi method | 2 |  |
| Determine bearing capacity by Meyerhof and IS code methods | 1 |  |
| Problems on bearing capacity | 1 |  |
| Safe bearing pressure based on N-value | 1 |  |
| Settlement from plate load test | 1 | 1 |
| Settlement analysis | 1 |  |
| Problems on Settlement | 1 |  |
| UNIT - 5: Pile Foundation |  |  |
| Classification of piles | 1 | 1 |
| Load carrying capacity of piles based on Static pile formulae | 2 |  |
| Load carrying of piles base on Dynamic pile formulae | 2 | 1 |
| Pile load test | 1 |  |
| Load carrying capacity of pile group in sand and clays | 2 | 1 |
| Problems on plie foundation | 2 |  |
| UNIT - 6: Well Foundation |  |  |
| Different shapes of well foundation | 1 | 1 |
| Forces acting on well foundation, Design of individual components | 2 |  |
| Construction and Sinking of well foundation | 2 |  |
| Tilts and Shifts | 2 | 1 |
| Problems on Well foundation | 2 |  |
| Total No. of Periods: | 56 | 14 |

## 12. Seminar Topics

- Soil Exploration
- Earth Pressure Theories
- Pile foundations


## Objective:

To understand the subsoil exploration and write the investigation report.

## Syllabus: SOIL EXPLORATION

Need of soil exploration, Type of Boring, Types of sampling, Standard Penetration test, Pressure meter, Plate load test, planning of programme and preparation of soil investigation report

## Learning Outcomes:

After completion of this unit the student will be able to

- List various types of soil exploration.
- Explain different types of borings to be used depending upon type of soil.
- Choose the type of sampler depending upon type of soil.
- Plan the suitable field test depending upon the subsoil.
- Outline the subsoil report.


## Learning Material

## 1. NEED OF SOIL EXPLORATION

Soil exploration is essential for design any foundation of a structure in a project. The extent of the exploration depends upon the magnitude and importance of the project. Projects such as buildings, power plants, fertilizer plants, bridges etc., are of national importance. The area occupied by such projects may vary from a few square meters to many square kilometers. Transmission lines, railway lines, roads and other similar projects extend along a narrow path. The length of such projects may be several kilometers. Each project has to be treated as per its requirements. The principle of soil exploration remains the same for all the projects but the program and methodology may vary from project to project.

The elements of soil exploration depend mostly on the importance and magnitude of the project, but generally should provide the following:

1. Information to determine the type of foundation required such as a shallow or deep foundation.
2. Necessary information with regards to the strength and compressibility characteristics of the subsoil to allow the design consultant to make recommendations on the safe bearing pressure or pile load capacity.

Soil exploration involves broadly the following:

1. Planning of a program for soil exploration.
2. Collection of disturbed and undisturbed soil or rock samples from the holes drilled in the field. The number and depths of holes depend upon the project.
3. Conducting all the necessary in-situ tests for obtaining the strength and compressibility characteristics of the soil or rock directly or indirectly.
4. Study of ground-water conditions and collection of water samples for chemical analysis.
5. Geophysical exploration, if required.
6. Conducting all the necessary tests on the samples of soil /rock and water collected.
7. Preparation of drawings, charts, etc.
8. Analysis of the data collected.
9. Preparation of report.

## 2. STAGES IN SUB-SURFACE EXPLORATION

It is generally carried out in 3 stages
a) Reconnaissance
b) Preliminary exploration
c) Detailed exploration

### 2.1. Reconnaissance

Site reconnaissance is the first step in a sub-surface exploration programme. It includes a visit to the site and to study the maps and others relevance records. It helps in deciding future programme of site investigations, scope of work, methods of exploration to be adopted, types of samples to be taken and the laboratory testing and in-situ testing.

### 2.2.Preliminary Exploration

The aim of a preliminary exploration is to be determine the depth, thickness, extent and composition of each soil stratum at the site. The depth of the bed rock and the ground water table is also determine.

The preliminary explorations are generally in the form of a few borings or test pits. Tests are conducted with cone penetrometers and sounding rods to obtain information about the strength and compressibility of soils.

Geophysical methods are also used in preliminary explorations for locating the boundaries of different strata.

### 2.3.Detailed Exploration

The purpose of the detailed explorations is to be determine the engineering properties of the soils in different strata. It includes an extensive boring programme, sampling and testing of the samples in a laboratory

Field tests, such as vane shear tests, plate load tests and permeability tests, are conducted to determine the properties of the soils in natural state. The tests for determination of dynamic properties are also carried out, if required.

For complex project involving heavy structures, such as bridges, dams, multi-storey buildings, it is essential to have detailed explorations. However, for small projects, especially at sites where the strata are uniform, detailed investigations may not be required. The design of such projects is generally based on the data collected during reconnaissance and preliminary explorations.

## 3. DEPTH OF EXPLORATION

The depth of exploration required at a particular site depends upon the degree of variation of the sub-surface data in the horizontal and vertical directions.

The depth of exploration should be about 1.5 times the width of square footing as shown in Fig1 and about 3 times the width of strip footing. However, if the footing are closely spaced, the whole of the loaded area acts as a raft foundation. In that case, the depth of boring should be at least 1.5 times the width of the entire loaded area as shown in Fig.2.In case of pile foundation, the depth of exploration below the tip of bearing piles is kept at least 1.5 timesthe width of the pile group. However, in the case of frication piles, the depth of exploration is taken 1.5 times the lower thired point as shown in Fig.3.


Fig. 1 Depth of exploration


Fig. 2 Depth of exploration for closely spaced footing


Fig. 3 Depth of exploration of friction pile
In case of multi-storied buildings, the depth of exploration can be taken from the following formula.

$$
\mathrm{D}=\mathrm{C}(\mathrm{~S})^{0.7}
$$

where $\mathrm{D}=$ depth of exploration (m),
$\mathrm{C}=$ constant, 3 for light steel buildings, it is equal to 6 for heavy steel buildings,
$\mathrm{S}=$ number of storeys.

## 4. LATERAL EXTENT OF EXPLORATIONS

The lateral extent of exploration and the spacing of bore holes depend mainly on the variation of the strata in the horizontal direction. For small and less important buildings even one bore hole or a trial pit in the centre may suffice but for compact buildings, covering an area of about 0.4 hectares, there should be at least 5 bore holes, one at the centre and four near the corners as shown in Fig. 4.


Fig. 4
For large, multi-storeyed buildings, the bore holes should be drilled at all the corners and also at important locations. The spacing between the bore holes is generally kept between 10 to 30 m , depending upon the variation in the subsurface conditions and loading (Fig. 5).

For highway, subsurface explorations are usually carried out along the proposed centre line or along the propose ditch line. The spacing of bore hole usually carried out along the proposed centre line or along the proposed ditch line. The spacing of bore holes usually varies between 150 and 300 m . If the sub-strata is erratic, the spacing may be reduced to even 30 m .

In case of concrete dams, the spacing of bore holes generally varies between 40 and 80 m .


Fig. 5

## 5. METHODS OF SOIL EXPLOARTION

The subsoil exploration should enable the engineer to draw the soil profile indicating the sequence of the strata and the properties of the soils involved.

In general, the methods available for soil exploration may be classified as follows:

1. Direct methods : Test pits, trial pits or trenches
2. Semi-direct methods : Borings
3. Indirect methods : Soundings or penetration tests and geophysical methods

In an exploratory programme, one or more of these methods may be used to yield the desired information.

### 5.1.TRIAL PITS:

Applicable to all types of soils Provide for visual examination in their natural condition

Disturbed and undisturbed soil samples can be conveniently obtained at different depths Depth of investigation is limited to 3 to 3.5 m .

### 5.1.1. Advantages

i. Cost effective.
ii. Provide detailed information of stratigraphy.
iii. Large quantities of disturbed soils are available for testing.
iv. Large blocks of undisturbed samples can be carved out from the pits.
v. Field tests can be conducted at the bottom of the pits.

### 5.1.2. Disadvantages

i. Depth limited to about 6 m .
ii. Deep pits uneconomical.
iii. Excavation below groundwater and into rock difficult and costly.
iv. Too many pits may scar site and require backfill soils.

### 5.1.3. Limitations

Undisturbed sampling is difficult.
Collapse in granular soils or below ground water table.

### 5.2. BORING:

Making or drilling bore holes into the ground with a view to obtaining soil or rock samples from specified or known depths is called 'boring'.
The common methods of advancing bore holes are:
i. Auger boring
ii. Wash boring
iii. Rotary drilling
iv. Percussion drilling
v. Core drilling

### 5.2.1. AUGER BORING

### 5.2.1.1.Hand Operated Augers

Auger boring is the simplest of the methods. Hand operated or power driven augers may be used. Two types of hand operated augers are in use as shown in Fig. 6

The depths of the holes are normally limited to a maximum of 10 m by this method. These augers are generally suitable for all types of soil above the water table but suitable only in clayey soil below the water table (except for the limitations given below). A string of drill rods is used for advancing the boring. The diameters of the holes normally vary from 10 to 20 cm . Hand operated augers are not suitable in very stiff to hard clay nor in granular soils below the water table. Hand augering is not practicable in dense sand nor in sand mixed with gravel even if the strata lies above the water table.


Fig. 6 Hand operated augers

### 5.2.1.2.Power Driven Augers

In many countries the use of power driven continuous flight augers is the most popular method of soil exploration for boring holes. The flights act as a screw conveyor to bring the soil to the surface. This method may be used in all types of soil including sandy soils below the water table but is not suitable if the soil is mixed with gravel, cobbles etc. The central stem of the auger flight may be hollow or solid. A hollow stem is sometimes preferred since standard penetration tests or sampling may be done through the stem without lifting the auger from its position in the hole. Besides, the flight of augers serves the purpose of casing the hole. The hollow stem can be plugged while advancing the bore and the plug can be removed while taking samples or conducting standard penetration tests (to be described) as shown in Fig. 7. The drilling rig can be mounted on a truck or a tractor. Holes may be drilled by this method rapidly to depths of 60 m or more.


Fig. 7 Hollow-stem auger
(a) Plugged while advancing the auger, and
(b) Plug removed and sampler inserted to sample soil below auger

### 5.2.2. WASH BORING

Wash boring is commonly used for boring holes. Soil exploration below the ground water table is usually very difficult to perform by means of pits or auger-holes. Wash boring in such cases is a very convenient method provided the soil is either sand, silt or clay. The method is not suitable if the soil is mixed with gravel or boulders.

Figure 8 shows the assembly for a wash boring. To start with, the hole is advanced a short depth by auger and then a casing pipe is pushed to prevent the sides from caving in. The hole is then continued by the use of a chopping bit fixed at the end of a string of hollow drill rods. A stream of water under pressure is forced through the rod and the bit into the hole, which loosens the soil as the water flows up around the pipe. The loosened soil in suspension in water is discharged into a tub. The soil in suspension settles down in the tub and the clean water flows into a sump which is reused for circulation. The motive power for a wash boring is either mechanical or man power. The bit which is hollow is screwed to a string of hollow drill rods supported on a tripod by a rope or steel cable passing over a pulley and operated by a winch fixed on one of the legs of the tripod.

The purpose of wash boring is to drill holes only and not to make use of the disturbed washed materials for analysis. Whenever an undisturbed sample is required at a particular depth, the boring is stopped, and the chopping bit is replaced by a sampler. The sampler is pushed into the soil at the bottom of the hole and the sample is withdrawn.


Fig. 8 Shows the assembly for a wash boring

### 5.2.3. ROTARY DRILLING

In the rotary drilling method a cutter bit or a core barrel with a coring bit attached to the end of a string of drill rods is rotated by a power rig. The rotation of the cutting bit shears or chips the material penetrated and the material is washed out of the hole by a stream of water just as in the case of a wash boring. Rotary drilling is used primarily for penetrating the overburden between the levels of which samples are required. Coring bits, on the other hand, cut an annular hole around an intact core which enters the barrel and is retrieved. Thus the core barrel is used primarily in rocky strata to get rock samples.

As the rods with the attached bit or barrel are rotated, a downward pressure is applied to the drill string to obtain penetration, and drilling fluid under pressure is introduced into the bottom of the hole through the hollow drill rods and the passages in the bit or barrel. The drilling fluid serves the dual function of cooling the bit as it enters the hole and removing the cuttings from the bottom of the hole as it returns to the surface in the annular space between the drill rods and the walls of the hole. In an uncased hole, the drilling fluid also serves to support the walls of the hole. When boring in soil, the drilling bit is removed and replaced by a sampler when sampling is required, but in rocky strata the coring bit is used to obtain continuous rock samples. The rotary drilling rig of the type given in Fig. 9 can also be used for wash boring and auger boring.


Fig. 9 Rotary drilling

### 5.2.4. PERCUSSION DRILLING

A heavy drill bit called 'churn bit' is suspended from a drill rod or a cable and is driven by repeated blows. Water is added to facilitate the breaking of stiff soil or rock. The slurry of the pulverised material is bailed out at intervals. The method cannot be used in loose sand and is slow in plastic clay. The formation gets badly disturbed by impact.

### 5.2.5. CORE DRILLING

The core drilling method is used for drilling holes and for obtaining rock cores. In this method, a core barrel fitted with a drilling bit is fixed to a hollow drilling rod. As the drilling rod is rotated, the bit advances and cuts an annular hole around an intact core. The core is then removed from its bottom and is retained by a core lifter and brought to the ground surface. Water is pumped continuously into the drilling rod to keep the drilling bit cool and to carry the disintegrated material to the groung surface.

The core drilling may be done using either a diamond studded bit or a cutting edge consisting of chilled shot. The diamond drilling is superior to the other types of drilling, but is costlier. The core barrel may consist of a single tube or a double tube. A double-tube barrel gives a good quality of the rock.

## 6. TYPES OF SOIL SAMPLERS

Soil sample are obtained during sub-surface exploration to determine the engineering properties of the soils and rocks. Soil samples are generally classified in to two categories.

## 1. Disturbed samples:

These are the samples in which the natural structures of the soil gets disturbed during sampling. However, these samples represent the composition and the mineral content of the soil. Disturbed samples can be used to determine the index properties of the soil, such as grain size, plasticity characteristics, specific gravity.

## 2. Undisturbed samples:

These are the samples in which the natural structure of the soil and the water content are retained. However, it may be mentioned that it is impossible to get truly undisturbed sample. Some disturbance is inevitable during sampling, even when the utmost care is taken. Even the removal of the sample from the ground produces a change in the stresses and causes disturbances.

Undisturbed samples are used for determining the engineering properties of the soil, such as compressibility, shear strength and permeability. Some index properties such as shrinkage limit can also be determine. The smaller the disturbance, the greater would be the reliability of the results.

### 6.1.DESIGN FEATURES AFFECTING THE SAMPLE DISTURBANCE

The disturbance of the soil depends mainly upon the following design features:

1. Area ratio: The area ratio is defined as

$$
A_{r}=\frac{D_{2}^{2}-D_{1}^{2}}{D_{1}^{2}} \times 100
$$

Where $\mathrm{D}_{1}=$ inner diameter of the cutting edge,
$\mathrm{D}_{2}=$ outer diameter of the cutting edge.
For obtain good quality undisturbed samples, the area ratio should be 10 percent or less.


Fig. 10 Soil sampler
2. Inside clearance: The inside clearance is defined as

$$
C_{i}=\frac{D_{3}-D_{1}}{D_{1}} \times 100
$$

Where $\mathrm{D}_{3}=$ inner diameter of the sampling tube.
The inside clearance allows elastic expansion of the sample when it enters the tube. It helps is reducing the frictional drag on sample. For an undisturbed sample, the inside clearance should be between 0.5 and 3 percent.
3. Outside clearance: The outside clearance is defined as

$$
C_{o}=\frac{D_{2}-D_{4}}{D_{4}} \times 100
$$

Where $\mathrm{D}_{4}=$ outer diameter of the sampling tube.
For reducing the driving force, the outside clearance should be as small as possible.
Normally, it lies between zero and 2 percent.
4. Inside wall friction: The friction on the inside wall causes disturbance of the sample. The inside surface of the sampler should be smooth. It is usually smeared with oil before use to reduce friction.
5. Design of non-return valve: The non-return valve provided on the sampler should be of proper design. It should have an orifice of large area to allow air, water or slurry to escape quickly when the sampler is driven. It should immediately close when the sampler is withdrawn.
6. Method of applying force: The degree of disturbance depends upon the method of applying force during sampling and upon the rate of penetration of the sampler. For obtaining undisturbed samples, the sampler should be pushed and not driven.

## 7. SAMPLING METHODS

### 7.1. Split-spoon sampler

The split spoon sampler is basically a thick-walled steel tube, split length wise. The sampler as standardized by the I.S.I. (IS: 2131-1986-Standard Penetration Test for soils) is shown in Fig. 11. A drive shoe attached to the lower end serves as the cutting edge. A sample head may be screwed at the upper end of split spoon. The standard size of the spoon sampler is of 35 mm internal and 50.8 mm external diameter. The sampler is lowered to the bottom of the bore hole by attaching it to the drill rod. The sampler is then driven by forcing it into the soil by blows from a hammer. The assembly of the sampler is then extracted from the hole and the cutting edge and coupling at the top are unscrewed. The two halves of the barrel are separated and the sample is thus exposed. The sample may be placed in a glass jar and sealed, after visual examination.

If samples need not be examined in the field, a liner is inserted inside the split spoon. After separating the two halves, the liner with the sample is sealed with wax.


Fig. 11 Split spoon sampler

### 7.2. Scraper bucket sampler

If a sandy deposit contains pebbles, it is not possible to obtain samples by standard split-spoon sampler. For such deposits, a scraper bucket sampler can be used. A scraper bucket sampler consists of a driving point which is attached to its bottom end as shown in Fig. 12. There is a vertical slit in the upper portion of the sampler. As the sampler is rotated, the scrapings of the soil enter the sampler through the slit. When the sampler is filled with the scrapings, it is lifted. Although the sample is quite disturbed, it is still representative.

A scraper bucket sampler can also be used for obtaining the sample of cohesionless soil below the water table.


Fig. 12 Scraper bucket sampler

### 7.3. Shelby tubes and thin-walled samplers

Thin-walled sampler, as standardized by the ISI (I.S.: 2132-1986 Code of Practice for Thin-walled Tube Sampling of Soils), is shown in Fig. 13.

The sampling tube shall be made of steel, brass, or aluminum. The lower end is levelled to form a cutting edge and is tapered to reduce wall friction.

After having extracted the sample in the same manner as in the case of split spoon type, the tube is sealed with wax on both ends and transported to the laboratory.


Fig. 13 Thin-walled sampler

### 7.4. Piston sampler

To improve the quality of samples and to increase the recovery of soft or slightly cohesive soils, a piston sampler is normally used. Such a sampler consists of a thin walled tube fitted with a piston that closes the end of the sampling tube until the apparatus is lowered to the bottom of the bore hole (Fig. 14(a)). The sampling tube is pushed into the soil hydraulically by keeping the piston stationary (Fig. 14(b)). The presence of the piston prevents the soft soils from squeezing rapidly into the tube and thus eliminates most of the distortion of the sample. The piston also helps to increase the length of sample that can be recovered by creating a slight vacuum that tends to retain the sample if the top of the column of soil begins to separate from the piston. During the withdrawal of the sampler, the piston also prevents water pressure from acting on the top of the sample and thus increases the chances of recovery. The design of piston samplers has been refined to the extent that it is sometimes possible to take undisturbed samples of sand from below the water table. However, piston sampling is relatively a costly procedure and may be adopted only where its use is justified.


Fig. 14 Piston Sampler (a) Sampler is set in drilled hole, (b) Sample tube is pushed hydraulically into the soil,
(c) Pressure is released through hole in piston rod.

### 7.5. Denison sampler

The Denison sampler is a double-welled sampler. The outer barrel rotates and cuts into the soil. The sample is obtained in the inner barrel. The inner barrel is provided with a liner. It may also be provided with a basket-type core retainer.

The sampler is lowered to the bottom of the drilled hole. A downward force is applied on the top of the sampler. A fluid under pressure is introduced through the inner barrel to cool the coring bit when the outer barrel rotates the fluid returns through the annular space between the two barrels. The rotation of the outer barrel is continued till the required length of the sample is obtained.

The Denison sampler gives a sample 140 mm in diameter and 508 mm long. Care is needed in adjusting the speed of rotation, the pressure on drilling bit and the velocity of wash water when drilling in soils and very friable rocks.

### 7.6. Hand-carved sampler

Hand-carved sample (Fig. 15) can be obtained if the soil is exposed, as in a test pit, shaft or tunnel. Hand-curved samples are also known as chunk samples.

The soil at the bottom of the pit is trimmed as a chunk to the required shape and size approximately. A cylindrical container open at both ends is placed carefully over this chunk after covering the top with paraffin wax. The bottom is scooped with a steel spatula and trimmed after reversing the box along with the sample. Paraffin wax is again used to seal the face and any gaps in the sides, before transporting it to the laboratory.


Fig. 15 Hand-carved sample

## 8. PENETRATION TESTS

Standard penetration test: It is most commonly used for cohesion-less soils. It is useful for determining the relative density and the angle of shearing resistance of cohesion less soils. It can also used to determine the unconfined compressive strength of cohesion less soils.

The standard penetration test is conducted in a bore hole using a standard split-spoon sampler. When the bore hole has been drilled to the desired depth, the drilling tools are removed
and the sampler is lowered to the bottom of the hole. The sampler is driven into the soil by a drop hammer of 63.5 kg mass falling through a height of 750 mm at the rate of 30 blows per minute (IS: 2131-1963). The number of hammer blows required to drive 150 mm of the sample is counted. The sample is further driven by 150 mm and the number of blows recorded. Likewise, the sampler is once again further driven by 150 mm and the number of blows recorded for the first 150 mm is disregarded. The numbers of blows recorded for the last two 150 mm intervals are added to give the standard penetration number ( N ). In other words, the standard penetration number is equal to the number of blows required for 300 mm of penetration beyond a seating drive of 150 mm (Fig. 16). If the number of blows for 150 mm driven exceeds 50, it is taken as refusal and the test is discontinued.

The standard penetration number is corrected for dilatancy correction and overburden correction as explained below.


Fig. 16 Standard penetration test

## Dilatancy correction

Silty fine sands and fine sands below the water table develop pore pressure which is not easily dissipated. The pore pressure increase the resistance of the soil and hence the penetration number(N).

Terzaghi and Peck (1967) recommend the following correction in case of silty fine sands when the observed value of N exceeds 15 .

The correction penetration number, $N_{c}=15+\frac{1}{2}\left(N_{R}-15\right)$
Where $N_{R}$ is the recorded value, and $N_{c}$ is the corrected value.
If $N_{R} \leq 15, \quad N_{c}=N_{R}$

## Overburden pressure correction

In granular soils, the overburden pressure affects the penetration resistance. If the two soils having same relative density but different confining pressures are tested, the one with a higher confining pressure gives a higher penetration number. As the confining pressure in cohesinless soils increases with the depth, the penetration number for soils at shallow depths is underestimated and that at greater depths is overestimated. For uniformity, the N -values obtained from field tests under different effective overburden pressures are corrected to a standard effective overburden pressure.

Gibbs and Holtz (1957) recommend the use of the following equation for dry or moist clean sand.

$$
N_{c}=N_{R} \times \frac{350}{\bar{\sigma}_{0}+70}
$$

Where $N_{R}=$ observed N -value, $N_{c}=$ corrected N -value,
$\bar{\sigma}_{0}=$ effective overburden pressure $\left(\mathrm{kN} / \mathrm{m}^{2}\right)$
This Eq. is applicable for $\bar{\sigma}_{0} \leq 280 \mathrm{kN} / \mathrm{m}^{2}$
The ratio $\left(N_{c} / N_{R}\right)$ should lie between 0.45 and 2.0. If $\left(N_{c} / N_{R}\right)$ ratio is greater than 2.0, $N_{c}$ should be divided by 2.0 to obtain the design value used in finding the bearing capacity of the soil.

## 9. PLATE LOAD TEST

The allowable bearing pressure can be determined by conducting a plate load test at the site. The conduct a plate load test, a pit of the size $5 B_{p} \times 5 B_{p}$, where $B_{p}$ is the size of the plate, is excavated to a depth equal to the depth of foundation $\left(D_{f}\right)$. the size of the plate is usually 0.3 m square. It is made of steel and is 25 mm thick. Occasionally, circular plates are also used. Sometimes, large size plates of 0.6 m square are used.

A central hole of size $B_{p} \times B_{p}$ is excavated in the pit. The depth of the central hole ( $D_{p}$ ) is obtained from the following realation.

$$
D_{p} / B_{p}=D_{f} / B_{f}
$$

Where $B_{p}$ is the size of the plate, and $\mathrm{B}_{\mathrm{f}}$ is the width of the pit.
For conducting the plate load test, the plate is placed in the central hole and the load is applied by means of a hydraulic jack (Fig. 17). The reaction to the jack is provided by means of a reaction beam. Sometimes, trusses are used instead of a reaction beam to take up the reaction. Alternatively, a loaded platform (kentledge) can be used to provide reaction. A seating load of $7 \mathrm{kN} / \mathrm{m}^{2}$ is first applied, which is released after some time. The load is then applied in increments of about $20 \%$ of the estimated safe load or one-tenth of the ultimate load. The settlement is recorded after $1,5,10,20,40,60$ minutes, and further after an interval of one hour. These hourly observations are continued for clayey soils until the rate of settlement is
less than 0.2 mm per hour. The test is conducted until failure or at least the settlement of about 25 mm has occurred (IS: 1888).

The ultimate load for the plate $\mathrm{q}_{\mathrm{u}}(\mathrm{p})$ is indicted by a break on the log-log plot between the load intensity q and the settlement. If the break is not well-defined, the ultimate load is taken as that corresponding to a settlement of one-fifth of the plate width $\left(\mathrm{B}_{\mathrm{p}}\right)$. on the natural plot (Fig. 18), the ultimate load is obtained from the intersection of the tangents drawn as shown.


Fig. 17 Plate load test


Fig. 18 load Vs settlement
The ultimate bearing capacity of the proposed foundation $\mathrm{q}_{\mathrm{u}}(\mathrm{f})$ can be obtained from the following relations.
a. For clayey soils, $\quad q_{u}(f)=q_{u}(p)$
b. Foe sandy soils, $\quad \mathrm{q}_{\mathrm{u}}(\mathrm{f})=\mathrm{qu}_{\mathrm{u}}(\mathrm{p}) \times \frac{B_{f}}{B_{p}}$

Where $\mathrm{B}_{\mathrm{f}}=$ foundation width

The plate load test can also be determine the settlement for a given intensity of loading $\left(\mathrm{q}_{\mathrm{o}}\right)$. The relation between the settlement of the plate $\left(\mathrm{S}_{\mathrm{p}}\right)$ and that of the foundation $\left(\mathrm{S}_{\mathrm{f}}\right)$ for the some load intensity are given below.
a. For clayey soils,

$$
S_{f}=S_{p} \times \frac{B_{f}}{B_{p}}
$$

Where $\mathrm{S}_{\mathrm{p}}$ is obtained from the load intensity-settlement curve for $\mathrm{q}_{\mathrm{o}}$.
b. For sandy soils,

$$
S_{f}=S_{p}\left[\frac{B_{f}\left(B_{p}+0.3\right)}{B_{p}\left(B_{f}+0.3\right)}\right]^{2}
$$

Where $\mathrm{B}_{\mathrm{f}}$ is the width of foundation in meters and $\mathrm{B}_{\mathrm{p}}$ is the width of the width of the plate in meters.

### 9.1 Limitations of plate load test:

i. Size effect
ii. Scale effect
iii. Time effect
iv. Interpretation of failure load
v. Reaction load
vi. Water table

## 10. PRESSURE METER

Menard developed a pressure meter, or sub-soil deformeter which can be used for determining the stress deformation characteristics of the soils in natural conditions. The volumetric deformations of the bore hole are obtained y noting the fall in water level in the water reservoir (Fig. 19). The probe is inserted into the bore hole and it is inflated by applying water pressure. The probe presses against the unlined walls of the bore hole. As the pressure is increased, the soil deforms. The volumetric deformation of the bore hole is obtained by noting the fall in water reservoir.

Fig. 20 shows a typical pressure-volumetric strain curve. The soil is initially in elastic phase but it enters the plastic phase at high pressure. After the plastic stage, there is no change in the volume with further increase in pressure. The pressure-deformation data obtained from the test may be used to determine modulus of deformation, undrained shear strength, angle of shearing resistance and other engineering properties of the soil. IS 1892-1979 describes the use of pressure meter.


Fig. 19 Pressure meter


Fig. 20 typical pressure-volumetric strain curve

## 11. SUB-SOIL INVESTIGATION REPORT

A sub soil investigation report should contain the data obtained from bore holes, site observations and laboratory results. It should also give the recommendations about the suitable type of foundation, allowable soil pressure and expected settlements.

The data obtained from a series of bore holes is presented in the form of a sub-surface profile (Fig. 21). A subsurface profile is a vertical section through the ground along the line of exploration. It indicates the boundaries of different strata, along with their classification. It is important to remember that conditions between bore holes are estimated by interpolation, which may not be correct. Obviously, the large the number of holes, the more accurate is the sub-surface profile.

The site investigation report should contain the discussion of the results. The discussion should be clear and concise the recommendations about the type and depth of foundation,
allowable soil pressure and expected settlement should be specific. The main finding of the report are given in conclusions.

A soil exploration report generally consists the following
i. Introduction, which gives scope of investigation
ii. Description of the proposed structure, the location and the geological conditions at the site.
iii. Details of field exploration programme, indicating the number of borings. Their location and depths
iv. Details the methods of exploration.
v. General description of the sub-soil conditions as obtained from in-situ tests, such as standard penetration test, cone test.
vi. Details of the laboratory test conducted on the soil samples obtained and the results obtained.
vii. Depth of the ground water table and the changes in water levels.
viii. Discussion of the results.
ix. Recommendation about the allowable bearing pressure, the type of foundation or structure.
x. Conclusions. The main findings of investigations should be clearly stated. It should be brief but should mention the salient points.

Limitations of the investigations should also be briefly stated.

## DEPTH (m)



Fig. 21 Sub-surface profile

## RECORD OPBORNG [IS: 1892.1979]

Name of boring organization:

Bored for
Ground level.
Type ofboring.
Diameter of boring.
Indination: Vertical
Bring:

Location site
Boring No
Soil sampler used
Date stated
Date completed
Recoried
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$


Fig. 22 Typical Boring log

## Assignment-Cum-Tutorial Questions

## A. Questions testing the remembering / understanding level of students

## I) Objective Questions

1. For undisturbed sampling, the area ratio for a thin-wall sampler should not normally exceeded
a) $10 \%$
b) $25 \%$
c) $30 \%$
d) $35 \%$
2. From borings $\qquad$ type of samples are obtained.
3. The depth of exploration depends upon $\qquad$
4. For what type of soils auger boring is used?
5. For isolated footings, rafts the depth of exploration should be
6. In the plate load test for determining the bearing capacity of soil, the size of square bearing plate should be
a) Less than 300 mm
b) Between 300 mm and 750 mm
c) Between 750 mm and 1 m
d) Greater than 1 m
7. The standard penetration test is useful to measures
a) Shear strength of soft clays
b) Shear strength of sands
c) Consistency of clays
d) None of the above
8. Select the incorrect statement:

For a good quality soil sample,
a) The area ratio should be low
b) The cutting edge should be thick
c) The inside clearance should be small
d) The outside clearance should be small

## II) Descriptive Questions

1) State the objectives of soil exploration?
2) Explain the importance of the depth of exploration for different structures.
3) Explain in detail various methods of boring and comment on the suitability of the method to different soils.
4) Explain the procedure of pressure meter test conducted in the field and draw the typical pressure volume curve?
5) How do you decide the bore holes depth and number of bore holes in soil investigation for a site?
6) Compute the area ratio of the sampling tube given the outside diameter $=10 \mathrm{~mm}$ and inside diameter $=9.4 \mathrm{~mm}$. In what type of soil can this tube be used for sampling?
7) What is reconnaissance? What type of information is obtained in reconnaissance? What is its use?
8) What do you understand about disturbed and undisturbed samples?
9) How do you obtain a hand-curved sample?
10) Explain the limitation of plate load test?

## B. Question testing the ability of students in applying the concepts.

## I) Multiple Choice Questions:

1. Standard penetration test was conducted at a site; the recorded values of below count for every 15 cm penetration at a depth of 1 m are 5,9 and 10 respectively. The value of SPT below count N that should be used is
2. If the actual observed value of standard penetration resistance $(\mathrm{N})$, is greater than 15 in a fine sand layer below water table, then the equivalent penetration resistance will be $\qquad$
3. In case of concrete dam, the spacing of bore hole generally various between $\qquad$
4. For an open excavation, the size of the pit recommended at bottom as per IS : 44531967 $\qquad$
5. Weight of drop hammer in standard penetration test $\qquad$
6. Depth of exploration in case of multi-storeyed building $\qquad$
7. In Standard penetration test the overburden pressure correction is done by using $\qquad$
8. An understand sample is also known as $\qquad$

## II) Descriptive Questions

1) Draw the neat sketch of $\log$ of a bore hole with complete details of exploration showing levels in cross section.
2) Explain Rotary, Percussion and Core drilling methods and its suitability depending on the site conditions.
3) How would you conduct a plate load test? Explain with a sketch.
4) Explain with neat sketch Thin-walled sampler. What are its advantages?
5) What is standard penetration number? Explain the procedure for determining it in the field. Mention the corrections to be applied to the observed values of standard penetration number.
6) Describe the salient features of a good subsoil investigation report.
7) Describe split-spoon sampler. What is its use?
8) Explain Auger and Wash boring methods and its suitability depending on the site conditions.
9) Explain with neat sketch scraper bucket and piston samplers. What are its uses?
10) Explain the design features affecting the sample disturbance.

## Academic Calendar

(Current Semester)

| Commencement of class work | $: 20-06-2016$ |
| :--- | :--- |
| I Unit of Instructions | $: 20-06-2016$ to $13-08-2016$ |
| I Mid Examinations | $: 16-08-2016$ to $28-08-2016$ |
| II Unit of Instructions | $: 17-10-2016$ to $22-10-2016$ to $15-10-2016$ |
| II Mid Examinations | $: 24-10-2016$ to $29-10-2016$ |
| Preparation and Practicals | $: 31-10-2016$ to $12-11-2016$ |

# GEOTECHNICAL ENGINEERING -II 

## UNIT -II

## Objective:

To understand the concept of slope stability.

## Syllabus: EARTH SLOPE STABILITY

Types of slopes - types of slope failures - factor of safety of infinite slopes - stability analysis by Swedish arc method - Stability of an infinite slope of cohesionless and cohesive soils - Bishop's Simplified method - Taylor's Stability Number.

## Learning Outcomes:

After completion of this unit the student will be able to

- Explain types of slope failures.
- Analyze stability of infinite slope.
- Solve the slope stability using Swedish arc method.
- Analyze the slope stability by using bishop's simplified method and Taylor's stability number.


## Learning Material

### 2.1. TYPES OF SLOPES

Slopes of earth are of two types

1. Natural slopes
2. Man-made slopes

Natural slopes are those that exist in nature and are formed by natural causes. Such slopes exist in hilly areas. The sides of cuttings, the slopes of embankments constructed for roads, railway lines, canals etc. and the slopes of earth dams constructed for storing water are examples of man-made slopes. The slopes whether natural or artificial may be

1. Infinite slopes
2. Finite slopes

The term infinite slope is used to designate a constant slope of infinite extent. The long slope of the face of a mountain is an example of this type, whereas finite slopes are limited in extent. The slopes of embankments and earth dams are examples of finite slopes. The slope length depends on the height of the dam or embankment.

### 2.2. TYPES OF SLOPE FAILURES

A slope may have any one of the following types of failures.

1. Rotational failure: This type of failure occurs by rotation along a slip surface by downward and outward movement of the soil mass (Fig.2.1). The slip surface is generally circular for homogeneous soil conditions and non-circular in case of nonhomogeneous conditions. Rotational slips are further divided into 3 types.
a. Toe failure, in which the failure occurs along the surface that passes through the toe (Fig.2.1(a)).
b. Slope failure, in which the failure occurs along a surface that intersects the slope above the toe (Fig.2.1(b)).
c. Base failure, in which the failure surface passes below the toe (Fig. 2.1(c)).

The slope failure occurs when a weak plane exists above the toe. The base failure occurs when a weak stratum lies beneath the toe. If a strong stratum exists below the toe, the slip surface of the base failure is tangential to that stratum. In all other cases, the failures are generally toe failures. Toe failures are most common.


Fig. 2.1 Rotational failure
2. Translational failure: A constant slop of unlimited extent and having uniform soil properties at the same depth below the free surface is known as an infinite slope. In practice, the slopes which are of considerable extent and in which the condition on all verticals are adequately represented by average conditions are designated as infinite slopes.
Translational failure occurs in an infinite slope along failure surface parallel to the slope (Fig. 2.2). The shape of the failure surface is influenced by the presence of any hard stratum at a shallow depth below the slope surface. Translational failures may also occur along slopes of layered materials.


Fig. 2.2 Translational failure
3. Compound failure: a compound failure is a combination of the rotational slip and the translational slip (Fig. 2.3). A compound failure surface is curved at the two ends and plane in the middle portion. A compound failure generally occurs when a hard stratum exists at considerable depth below the toe.


Fig. 2.3 Compound failure
4. Wedge failure: A failure along an inclined plain is known as plane failure or wedge failure or block failure (Fig. 2.4). It occurs when distinct blocks and wedges of the soil mass become separated.

A plane failure is similar to translational failure is many respects. However unlike translational failure which occurs in an infinite slope, a plane failure may occur even in a finite slope consisting of two different materials or in a homogeneous slope having cracks, fissures, joints or any other specific plane of weakness.


Fig. 2.4 Wedge failure
5. Miscellaneous failures: In addition to above four types of failures, some complex types of failures in the form of spreads and floes may also occurs.

### 2.3. FACTOR OF SAFETY

In stability analysis, two types of factors of safety are normally used. They are

1. Factor of safety with respect to shearing strength.
2. Factor of safety with respect to cohesion. This is termed the factor of safety with respect to height.

Let,
$F_{S}=$ factor of safety with respect to strength
$\mathrm{F}_{\mathrm{c}}=$ factor of safety with respect to cohesion
$F_{H}=$ factor of safety with respect to height
$\mathrm{F}_{\varnothing}=$ factor of safety with respect to friction
$c^{\prime}{ }_{m}=$ mobilized cohesion $m$
$\emptyset_{\mathrm{m}}^{\prime}=$ mobilized angle of friction
$\tau=$ average value of mobilized shearing strength
$s=$ maximum shearing strength.

The factor of safety with respect to shearing strength, $\mathrm{F}_{\mathrm{s}}$, may be written as

$$
F_{s}=\frac{s}{\tau}=\frac{c^{\prime}+\sigma^{\prime} \tan \emptyset^{\prime}}{\tau}
$$

The shearing strength mobilized at each point on a failure surface may be written as

$$
\begin{gathered}
\tau=\frac{c^{\prime}}{F_{s}}+\sigma^{\prime} \frac{\tan \emptyset^{\prime}}{F_{s}} \\
\text { or } \tau=c_{m}^{\prime}+\sigma^{\prime} \tan \emptyset_{m}^{\prime} \\
\text { Where } c_{m}^{\prime}=\frac{c^{\prime}}{F_{s}}, \tan \emptyset_{m}^{\prime}=\frac{\tan \emptyset^{\prime}}{F_{s}}
\end{gathered}
$$

Actually the shearing resistance (mobilized value of shearing strength) does not develop to a like degree at all points on an incipient failure surface. The shearing strains vary considerably and the shearing stress may be far from constant. However the above expression is correct on the basis of average conditions.

If the factors of safety with respect to cohesion and friction are different, we may write the equation of the mobilized shearing resistance as

$$
\tau=\frac{c^{\prime}}{F_{c}}+\sigma^{\prime} \frac{\tan \emptyset^{\prime}}{F_{\emptyset}}
$$

It will be shown later on that $F_{c}$ depends on the height of the slope. From this it may be concluded that the factor of safety with respect to cohesion may be designated as the factor of safety with respect to height. This factor is denoted by $\mathrm{F}_{\mathrm{H}}$ and it is the ratio between the critical height and the actual height, the critical height being the maximum height at which it is possible for a slope to be stable.

$$
\tau=\frac{c^{\prime}}{F_{H}}+\sigma^{\prime} \tan \emptyset^{\prime}
$$

Where $F ø$ is arbitrarily taken equal to unity.

### 2.4. SWEDISH CIRCLE METHOD

The actual shape of a slip surface in the case of finite slopes is curvilinear. For convenience, it is approximated as circular. The assumption of a circular slip surface and its application for stability analysis of slop was developed in Sweden. The method is known as the Swedish circle method or the method of slices.

Fig. 2.5 (a) shows a slope. Let AB be a circular surface with radius r and center O . The trial failure wedge above the slip surface is divided into vertical slices by drawing vertical lines, as shown. The slices are usually of equal width, but not necessarily so. In case of nonhomogeneous slopes where the slip surface passes through more than one type of material, a vertical line is always located at the point where the slip surface passes from one material to the other.

Let us consider the equilibrium of one slice. The slice is in equilibrium under the following forces.


Fig. 2.4 Swedish circle method

1. Weight (W) acting vertically through its center of gravity.
2. Cohesive force (C) acting along the curved surface in the direction opposite to the direction of probable movement of the wedge.
3. Reaction (R) at the base inclined at angle $\emptyset$ to the normal, assuming the slippage is imminent.
4. Reaction on the two vertical sides of the slice due to adjacent slices. However, in the Swedish circle method, it is assumed that the reaction on the two sides are equal and opposite and are, therefore, in equilibrium and do not affect the stability of the slice. Accordingly, only the first three forces are considered for the analysis.

The weight W is resolved into its normal component $(\mathrm{N})$ and tangential component (T). Let us take the moments about the center of rotation O of all the 3 forces.
Actuating or overturning moments,

$$
\begin{equation*}
\mathrm{M}_{\mathrm{o}}=\mathrm{Txr} \tag{a}
\end{equation*}
$$

The moment due to N - components is zero, as those components always pass through O .
Resisting moments,

$$
\begin{equation*}
M_{R}=(C \Delta L) \times r+R(r \sin \emptyset) . \tag{b}
\end{equation*}
$$

Where $\Delta L$ is the length of the curved surface of the slice.
Resolving the forces is radial direction,

$$
\begin{gather*}
N=R \cos \emptyset \quad \text { or } \quad R=N / \cos \emptyset \ldots \ldots \ldots \ldots \ldots  \tag{c}\\
\text { or } R \sin \emptyset=N \tan \emptyset \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\end{gather*}
$$

From Eqs. (b) and (d),

$$
\begin{equation*}
M_{R}=(C \Delta L) \times r+N r \tan \emptyset \tag{e}
\end{equation*}
$$

The factor of safety for the slice is equal to the ratio of the resisting moment $\left(\mathrm{M}_{\mathrm{R}}\right)$ and the overturning moment $\left(\mathrm{M}_{\mathrm{o}}\right)$. Thus

$$
F_{s}=\frac{r[c \Delta L+N \tan \emptyset]}{T r}=\frac{c \Delta L+N \tan \emptyset}{T}
$$

The factor of safety of the entire wedge is given by

$$
F_{s}=\frac{\sum c \Delta L+\sum N \tan \emptyset}{\sum T}
$$

$$
\text { If c and } \varnothing \text { are constant, } \quad F_{S}=\frac{c L_{a}+\sum N \tan \emptyset}{\sum T}
$$

Where $\mathrm{L}_{\mathrm{a}}=$ length of the entire slip surface $=\sum \Delta L$
The components N and T are determined by drawing force triangles as shown in Fig. 2.4 (b). If the angle $\alpha$ which the normal makes with the vertical is measured, the components can be computes as under.

$$
N=W \cos \alpha, \quad \text { and } \quad T=W \sin \alpha
$$

The length $\Delta L$ of the arc is given by $\mathrm{b} \sec \alpha$, where b is the width of the slice.
Therefore, Eq. can be written as

$$
F_{s}=\frac{\sum c b \sec \alpha+\sum(W \cos \alpha) \tan \emptyset}{\sum W \sin \alpha}
$$

It may be noted that the tangential component T may be negative i.e., in the direction opposite to that of movement for some of the slices near the toe.

### 2.5. STABILITY OF AN INFINITE SLOPE OF COHESIONLESS SOILS

The stability criteria of an infinite slope of cohesionless soils will depend whether the soil is dry, or submerged or has steady seepage.
a. Dry soil: Fig. 2.5 (a) shows a section of infinite slop having a slope angle of i. Let us consider the prism ABCD of the soil, with the inclined length AB equal to b . the horizontal length of the prism is 'b cosi'. The height of the prism is H (Fig. 2.5 (b)).


Fig. 2.5 Infinite slope in Dry sand

Volume of prism per unit length $=\mathrm{Hb}$ cosi
Wight of the prism per unknit length, $\quad \mathrm{W}=\gamma(\mathrm{Hb} \operatorname{cosi})$
The weight of the prism can be resolved into the normal component N and Tangential components T to plane CD .

Thus

$$
\begin{array}{r}
N=W \cos i=\gamma H b \cos ^{2} i \\
T=W \sin i=\gamma H b \cos i \sin i
\end{array}
$$

The normal and shear stresses are given by

$$
\begin{aligned}
\sigma & =\frac{N}{b}=\frac{\gamma H b \cos ^{2} i}{b}=\gamma H \cos ^{2} i \\
\tau=\frac{T}{b} & =\frac{\gamma H b \cos i \sin i}{b}=\gamma H \cos i \sin i
\end{aligned}
$$

The shear stresses $\tau$ tend to ccause the shear failure along CD. This tendency is opposed by the shearing resistance developed along the plane CD. As the soil is dry, there is no pore water pressure.

Therefore,
or

$$
\begin{gathered}
s=\bar{\sigma} \tan \emptyset^{\prime}=\sigma \tan \emptyset^{\prime} \\
s=\left(\gamma H \cos ^{2} i\right) \tan \emptyset^{\prime}
\end{gathered}
$$

The factory of safety against shear failure is given by

$$
\begin{gathered}
F_{S}=\frac{s}{\tau}=\frac{\left(\gamma H \cos ^{2} i\right) \tan \phi^{\prime}}{\gamma H \cos i \sin i} \\
F_{S}=\frac{\tan \emptyset^{\prime}}{\tan i}
\end{gathered}
$$

Above Eq. indicates that the slope is just stable when $\emptyset^{\prime}=i$. The factor of safety is greater than unity when $i$ is less than $\emptyset^{\prime}$. For the slop angle $i$ greater than $\emptyset^{\prime}$, the slop is not stable.

It is worth nothing that the factor of safety of an infinite slope of a cohesionless soil is independent of the height H of the assumed failure prism.

The angle $\varnothing^{\prime}$ in the Eq. should correspond to the actual relative density of the soil. As the soil in the surface layers is in a relatively loose state, the angle $\emptyset^{\prime}$ corresponding to the loose state is generally taken.

Eq. can be represented graphically (Fig. 2.5(c)). The ordinate PQ is equal to $\sigma \tan \emptyset^{\prime}$ and represents the shear strength. The ordinate $\mathrm{P}_{1} \mathrm{Q}$ represents the shear stress $\tau$ equal to $\sigma \tan i$. obviously, the factor of safety is given by

$$
F_{s}=\frac{s}{\tau}=\frac{P Q}{P_{1} Q}=\frac{\tan \varnothing^{\prime}}{\tan i}
$$

b. Submerged slope: If the slope is submerge under water, the normal effective stress and the shear stress are calculated using the submerged unit weight and not the bulk weight as was used for dry soil.

$$
\begin{array}{r}
\bar{\sigma}=\gamma^{\prime} H \cos ^{2} i \\
\tau=\gamma^{\prime} H \cos i \sin i
\end{array}
$$

Where $\gamma^{\prime}$ is the submerged unit weight.
Therefore, the factor of safety is given by
or

$$
\begin{gathered}
F_{s}=\frac{s}{\tau}=\frac{\left(\gamma^{\prime} H \cos ^{2} i\right) \tan \phi^{\prime}}{\gamma^{\prime} H \cos i \sin i} \\
F_{s}=\frac{\tan \phi^{\prime}}{\tan i}
\end{gathered}
$$

Comparing Eqs in above two conditions, it is observed that the factor of safety of a submerged slope is the same as that in dry condition.
c. Steady seepage along the slope: Fig. 2.6 (a) shows an infinite slope with steady seepage parallel to the surface. Fig. 2.6 (b) shows a free-body diagram of the prism ABCD of the soil. The forces acting on the vertical sides of the prism due to water and soil are equal
and opposite and, therefore, cancel. The weight of the prism W is taken corresponding to the saturated conditions.

Therefore,

$$
\begin{gathered}
W=\gamma_{s a t} H b \cos i \\
N=W \cos i=\gamma_{s a t} H b \cos ^{2} i \\
T=W \sin i=\gamma_{s a t} H b \sin i \cos i
\end{gathered}
$$

At the base of the prism, there is an upward force due to water pressure (u), given by

$$
u=\gamma_{w} H \cos ^{2} i
$$

Uplift force,

$$
U=\left(\gamma_{w} H \cos ^{2} i\right) b
$$

Thus, the net normal force $\bar{N}$ is given by

$$
\begin{aligned}
& \bar{N}=N-U=\gamma_{s a t} H b \cos ^{2} i-\left(\gamma_{w} H \cos ^{2} i\right) b \\
& \bar{N}=\gamma^{\prime} b H \cos ^{2} i, \text { where } \gamma^{\prime} \text { is submerged unit weight. }
\end{aligned}
$$

or


Fig. 2.6 Steady seepage along the slope
The effective stress is given by

Shear strength,

$$
\begin{gathered}
s=\bar{\sigma} \tan \emptyset^{\prime}=\gamma^{\prime} H \cos ^{2} i \tan \emptyset^{\prime} \\
\tau=\frac{T}{b}=\gamma_{s a t} H \sin i \cos i
\end{gathered}
$$

The shear stress is given by

Therefore, the factor of safety is given by

$$
F_{s}=\frac{s}{\tau}=\frac{\gamma^{\prime} H \cos ^{2} i \tan \emptyset^{\prime}}{\gamma_{s a t} H \sin i \cos i}
$$

$$
F_{s}=\frac{\gamma^{\prime} \tan \emptyset^{\prime}}{\gamma_{s a t} \tan i}
$$

As the submerged unit weight $\gamma^{\prime}$ is about one-half of the saturated unit weight, the factor of safety of the slope is reduced to about one-half of that corresponding to the condition when there is no seepage. The angle $\emptyset^{\prime}$ 'in the wet condition of a cohesionless soil is approximately the same as in dry condition.

### 2.6. STABILITY OF AN INFINITE SLOPE OF COHESIVE SOILS

The stability analysis of an infinite slope of cohesive soils is similar to that in the case of cohesionless soils, with one basic difference that the shear strength of a cohesive(c- $\emptyset$ soil) is given by

$$
s=c^{\prime}+\bar{\sigma} \tan \emptyset^{\prime}
$$

a. Dry soil: taking values of $\bar{\sigma}$ and $\tau$

$$
\begin{gathered}
s=c^{\prime}+\left(\gamma H \cos ^{2} i\right) \tan \emptyset^{\prime} \\
\tau=\gamma H \sin i \cos i
\end{gathered}
$$

Therefore, the factor of safety $\mathrm{F}_{\mathrm{s}}$ is given by

$$
F_{s}=\frac{c^{\prime}+\left(\gamma H \cos ^{2} i\right) \tan \phi^{\prime}}{\gamma H \cos i \sin i}
$$

Thus, the factor of safety of an infinite slope in cohesive soils depends not only on $\emptyset^{\prime}$ and $i$ but also on $\gamma, \mathrm{H}$ and $c^{\prime}$.


Fig. 2.7 Infinite slope in cohesive soil
Fig. 2.7 represents a graphical method for the determination of the factor of safety. The line $\mathrm{RP}_{2}$ represents the failure envelope. When the slop angle $\underline{i}$ is less than $\emptyset^{\prime}$, the slope is always safe as shown by line $\mathrm{OP}_{1}$. When the slope angle $i_{2}$ is greater than $\emptyset^{\prime}$, the slope line cuts the failure envelope. At point P , the slope is just stable. For normal stress greater than that indicated by point P , the shear stress is greater than the shear strength and the slope is not stable.

As the normal stress $\sigma$ depends upon the height H of the slope, an expression for the height can be found when the slope is just stable. Therefore, equating the shear stress and the shear strength corresponding to point P ,

$$
\gamma H \sin i \cos i=c^{\prime}+\gamma H \cos ^{2} i \tan \emptyset^{\prime}
$$

or

$$
\gamma H \cos ^{2} i\left(\frac{\sin i}{\cos i}-\tan \phi^{\prime}\right)=c^{\prime}
$$

or

$$
H=\frac{c^{\prime}}{\gamma \cos ^{2} i\left(\tan i-\tan \phi^{\prime}\right)}
$$

The height at which the slope is just stable is known as the critical height $\left(\mathrm{H}_{\mathrm{c}}\right)$. Thus

$$
H_{c}=\frac{c^{\prime}}{\gamma \cos ^{2} i\left(\tan i-\tan \emptyset^{\prime}\right)}
$$

For heights less than the critical height, the factor of safety is given by

$$
F_{s}=\frac{c^{\prime}+\left(\gamma H \cos ^{2} i\right) \tan \emptyset^{\prime}}{\gamma H \cos i \sin i}
$$

b. Submerged slope: As in the case of cohesionless soils, the normal and tangential components of the weight are taken for submerged unit weights and not for bulk unit weights. Thus, from above Eq.

$$
F_{s}=\frac{c^{\prime}+\gamma^{\prime} H \cos ^{2} i \tan \emptyset^{\prime}}{\gamma^{\prime} H \cos i \sin i}
$$

The value of $\emptyset^{\prime}$ in above Eq. should be taken corresponding to the submerged conditions, which may be quite different from that in dry condition in case of cohesive soils.
c. Steady seepage along the slope: The case is similar to that for a cohesionless soil. In this case, the factor of safety is given by

$$
F_{s}=\frac{c^{\prime}+\gamma^{\prime} H \cos ^{2} i \tan \emptyset^{\prime}}{\gamma_{s a t} H \sin i \cos i}
$$

The critical height is obtained corresponding to a factor of safety of unity. Thus
or

$$
c^{\prime}+\gamma^{\prime} H_{c} \cos ^{2} i \tan \emptyset^{\prime}=\gamma_{s a t} H_{c} \sin i \cos i
$$

$$
c^{\prime}=H_{c} \cos ^{2} i\left(\gamma_{s a t} \tan i-\gamma^{\prime} \tan \emptyset^{\prime}\right)
$$

or

$$
H_{c}=\frac{c^{\prime}}{\cos ^{2} i\left(\gamma_{\operatorname{sat}} \tan i-\gamma^{\prime} \tan \phi^{\prime}\right)}
$$

Sometimes, above Eq. is written as

$$
H_{c}=\frac{c^{\prime}}{\gamma_{\text {sat }}\left[\tan i-\left(\frac{\gamma^{\prime}}{\gamma_{\text {sat }}}\right) \tan \emptyset^{\prime}\right] \cos ^{2} i}
$$

### 2.7. BISHOP'S SIMPLIFIED METHOD

The conventional Swedish circle method satisfies only the overall moment equation of equilibrium. It neglects the moment equilibrium of the individual slices. It also disregards the effect of the forces acting on the sides of the individual slices and, therefore, only approximates the force equilibrium of each slice. Method of analysis which satisfy all the equilibrium equations are complicated and not convenient to use.

Bishop (1955) gave a simplified method of analysis which considers the forces on the sides of each slice. The requirement of equilibrium are applied to the slices. The factor of
safety is defined as the ratio of the maximum shear strength(s) possessed by soil on the trial surface to the shearing resistance mobilized $\left(\tau_{\mathrm{m}}\right)$.

Thus

$$
F_{s}=\frac{s}{\tau_{m}}
$$

The forces acting on the slices are given below (Fig. 2.8 (a)).

1. Weight of slice, $\mathrm{W}=\gamma h b$, where h is the average height.
2. Normal force on the base, $N^{\prime}=N-u l$, where $u$ is the pore pressure, and $l$ is the length of the base $(=\mathrm{b} \sec \alpha)$.
3. Shear force on the base, $T=\tau_{m} l$.
4. Normal forces on the sides $E_{1}$ and $E_{2}$.
5. Shear forces on the sides $X_{1}$ and $X_{2}$.
6. Any external force acting on the slice.

The problem is statically indeterminate. To solve the problem, a number of simplifying assumptions are made regarding the inter slice forces $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{E}_{1}$ and $\mathrm{E}_{2}$.

Fig. 2.8 (b) shows a slope. Taking moments about O .

$$
\sum T r=\sum W r \sin \alpha
$$


(a)

(b)

Fig. 2.8 Bishop simplified method
But

$$
T=\tau_{m} l=\frac{s}{F_{s}} \times l
$$

Therefore,

$$
\sum \frac{s}{F_{s}} \times l \times r=\sum W \times r \times \sin \alpha
$$

$$
F_{s}=\frac{\sum s \times l}{\sum W \sin \alpha}=\frac{\sum\left(c^{\prime}+\bar{\sigma} \tan \phi^{\prime}\right) l}{\sum W \sin \alpha}
$$

$$
F_{s}=\frac{\sum c^{\prime} l+\tan \phi^{\prime} \sum N^{\prime}}{\sum W \sin \alpha} \quad \text { where } N^{\prime}=\bar{\sigma} l
$$

Above Eq. is exact. However, approximations are introduced in the determination of the normal component $N^{\prime}$. In the conventional Swedish circle method, the resultant of inter-slice forces is taken as zero.

Therefore,

$$
N^{\prime}=W \cos \alpha-u l
$$

$$
\begin{gathered}
F_{s}=\frac{c^{\prime} L_{a}+\tan \emptyset^{\prime} \sum(W \cos \alpha-u l)}{\sum W \sin \alpha} \\
F_{s}=\frac{c^{\prime} L_{a}+\tan \emptyset^{\prime} \sum(N-U)}{\sum T}
\end{gathered}
$$

In Bishop's simplified method, it is assumed that the resultant forces on the sides of the slice are horizontal, i.e. $\mathrm{X}_{1}-\mathrm{X}_{2}=0$.

Resolving the forces in the vertical direction,

$$
W=N^{\prime} \cos \alpha+u l \cos \alpha+T \sin \alpha+X_{1}+X_{2}
$$

Substituting

$$
T=\frac{s}{F_{s}} \times l \quad \text { and } \quad X_{1}-X_{2}=0, \text { we have }
$$

$$
\begin{gathered}
W=N^{\prime} \cos \alpha+u l \cos \alpha+\frac{\left(c^{\prime} l+N^{\prime} \tan \emptyset^{\prime}\right)}{F_{s}} \sin \alpha \\
W=N^{\prime} \cos \alpha+u l \cos \alpha+\frac{c^{\prime} l}{F_{s}} \sin \alpha+\frac{N^{\prime}}{F_{s}} \tan \emptyset^{\prime} \sin \alpha \\
N^{\prime}\left(\cos \alpha+\frac{\tan \emptyset^{\prime}}{F_{s}} \sin \alpha\right)=W-u l \cos \alpha-\frac{c^{\prime} l \sin \alpha}{F_{s}} \\
N^{\prime}=\frac{W-u l \cos \alpha-\frac{c^{\prime} l \sin \alpha}{F_{s}}}{\cos \alpha+\frac{\tan \phi^{\prime}}{F_{s}} \sin \alpha}
\end{gathered}
$$

Substituting the above value of $N^{\prime}$

$$
\begin{aligned}
& F_{S}=\frac{\sum c^{\prime} l+\tan \emptyset^{\prime} \sum\left[\frac{W-u l \cos \alpha-\frac{c^{\prime} l \sin \alpha}{F_{S}}}{\cos \alpha+\frac{\tan \phi^{\prime}}{F_{S}} \sin \alpha}\right]}{\sum W \sin \alpha} \\
& =\frac{1}{\sum W \sin \alpha} \sum\left[c^{\prime} b \sec \alpha+\frac{\tan \phi^{\prime}\left(W \sec \alpha-u l-\frac{c^{\prime} l \tan \alpha}{F_{s}}\right)}{1+\frac{\tan \phi^{\prime} \tan \alpha}{F_{s}}}\right] \\
& =\frac{1}{\sum W \sin \alpha} \sum\left[\left\{c^{\prime} b \sec \alpha-\frac{c^{\prime} l \tan \alpha \tan \emptyset^{\prime} / F_{s}}{1+\frac{\tan \emptyset^{\prime} \tan \alpha}{F_{s}}}\right\}+\frac{\tan \emptyset^{\prime}(W \sec \alpha-u l)}{1+\frac{\tan \emptyset^{\prime} \tan \alpha}{F_{s}}}\right] \\
& =\frac{1}{\sum W \sin \alpha} \times \sum\left[\frac{\left\{c^{\prime} b \sec \alpha\left(1+\tan \alpha \tan \phi^{\prime} / F_{s}\right)-c^{\prime} b \sec \alpha \tan \alpha \frac{\tan \phi^{\prime}}{F_{s}}\right\}}{1+\frac{\tan \phi^{\prime} \tan \alpha}{F_{s}}}+\frac{\tan \phi^{\prime}(W \sec \alpha-u b \sec \alpha)}{1+\frac{\tan \phi^{\prime} \tan \alpha}{F_{s}}}\right] \\
& =\frac{1}{\sum W \sin \alpha} \sum\left[\frac{c^{\prime} b \sec \alpha}{1+\frac{\tan \phi^{\prime} \tan \alpha}{F_{s}}}+\frac{\tan \emptyset^{\prime}(W \sec \alpha-u b \sec \alpha)}{1+\frac{\tan \phi^{\prime} \tan \alpha}{F_{s}}}\right] \\
& =\frac{1}{\sum W \sin \alpha} \sum\left[\left\{c^{\prime} b+(W-u b) \tan \emptyset^{\prime}\right\} \frac{\sec \alpha}{1+\frac{\tan \phi^{\prime} \tan \alpha}{F_{s}}}\right] \\
& F_{s}=\frac{\sum \frac{1}{m_{a}}\left[c^{\prime} b+(W-u b) \tan \emptyset^{\prime}\right]}{\sum W \sin \alpha}
\end{aligned}
$$

Where $m_{a}=\left(1+\tan \emptyset^{\prime} \tan \alpha / F_{s}\right) \cos \alpha$
Sometimes, the pore pressure $u$ is expressed in terms of pore pressure ratio $r_{u}$. In that case,

$$
u=r_{u} \gamma h=r_{u}(W / b)
$$

The above Eq. gives the factor of safety of the assumed failure surface. As the factor of safety $\left(\mathrm{F}_{\mathrm{s}}\right)$ appears on both the sides, a process of successive approximation is required. A value of $\mathrm{F}_{\mathrm{s}}$ is assumed and the analysis is done. The value computed from the above Eq. is compared with the assumed value. If the two values differ, the process is repeated till convergence. As the convergence is rapid, only 3-4 trials are required. A computer may also be used.

The effective stress analysis is generally done, but the total stress analysis is also possible. The factor of safety determined by Bishop's simplified method is an underestimate and, therefore, it errs on the safe side. The error is generally less than $2 \%$ and not more than $7 \%$ even in an extreme case.

### 2.8. TAYLOR'S METHOD

For slopes made from two different soils the ratio $c_{m} / \gamma H$ has been shown to be the same for each slope provided that the two soils have the same angle of friction. This ratio is known as the 'stability number' and is designated by the symbol, $S_{n}$.

$$
S_{n}=c_{m} / \gamma H
$$

Where $S_{n}=$ stability number
$c_{m}=$ Unit cohesion mobilized (with respect to total stress)
$\gamma=$ Unit weight of soil
and $H=$ Vertical height of the slope
Taylor (1948) prepared two charts relating the stability number to the angle of slope, based on the friction circle method and an analytical approach. The first is for the general case of a $c-\emptyset$ soil with the angle of slope less than $53^{\circ}$, as shown in Fig. 2.9. The second is for a soil with $\emptyset=0$ and a layer of rock or stiff material at a depth $D H$ below the top of the embankment, as shown in Fig. 2.10. Here, $D$ is known as the depth factor; depending upon its value, the slip circle will pass through the toe or will emerge at a distance $n H$ in front of the toe (the value of $n$ may be obtained from the curves). Theoretically, the critical arc in such cases extends to an infinite depth (slope angle being less than $53^{\circ}$ ), however, it is limited to the hard stratum. For $\emptyset=0$ and a slope angle greater than $53^{\circ}$, the first chart is to be used.


Fig. 2.9 Taylor's charts for slop stability (for $\emptyset=0^{0}$ and $\beta<53^{0}$, use Fig. 2.10)


Fig. 2.10 Taylor's chart for slopes with depth limitation ( $\beta>53^{0}$, use Fig.2.9)
(Note: For $\varnothing=0^{\circ}$ and $\beta=90^{\circ}, S_{n}=0.26$. So the maximum unsupported height of a verticalcut in pure clay is $c / \gamma S_{n}$ or $4 c / \gamma$ nearly).

The use of the charts is almost self-explanatory. For example, the first chart may be used in one of the two following ways, depending upon the nature of the problem on hand:

The use of the charts is almost self-explanatory. For example, the first chart may be used in one of the two following ways, depending upon the nature of the problem on hand:

1. If the slope angle and mobilized friction angle are known, the stability number can be obtained. Knowing unit weight and vertical height of the slope, the mobilized cohesion can be got.

The factor of safety may be evaluated as the ratio of the effective cohesion strength to the mobilized unit cohesion.
2. Knowing the height of the slope, unit weight of the earth material constituting the slope and the desired factor of safety, the stability number can be evaluated. The slope angle can be found from the chart against the permissible angle of internal friction.

If the slope is submerged, the effective unit weight $\gamma^{\prime}$ instead of $\gamma$ is to be used.
For the case of sudden drawdown, the saturated unit weight $\gamma_{\text {sat }}$ is to be used for $\gamma$; in addition, a reduced value of $\emptyset, \emptyset_{w}$, should be used, where:

$$
\emptyset_{w}=\left(\gamma^{\prime} / \gamma_{s a t}\right) \times \emptyset
$$

Taylor's charts are based on the assumption of full mobilization of friction, that is, these give the factor of safety with respect to cohesion.

This is all right for a purely cohesive soil; but, in the case of a $c-\emptyset$ soil, where the factor of safety $F_{s}$ with respect to shearing strength is desired, $\emptyset_{m}$ should be used for $\varnothing$ :

$$
\begin{gathered}
\tan \emptyset_{m}=\tan \emptyset / F_{s} \\
\left(\text { Also } \emptyset_{m} \approx \emptyset / F_{s}\right)
\end{gathered}
$$

The charts are not applicable for a purely frictional soil $(c=0)$. The stability then depends only upon the slope angle, irrespective of the height of the slope.

## Assignment-Cum-Tutorial Questions

## A. Questions testing the remembering / understanding level of students

## I) Objective Questions

1. Taylor's stability number curves are used for the analysis of stability of slopes. The angle of shearing resistance used in the chart is the
a. Effective angle
b. Apparent angle
c. Mobilized angle
d. Weighted angle
2. Consider the following statement regarding the Swedish circle method of analyzing stability of slopes.
3. It is general method of analyzing stability of slopes.
4. It satisfies only the overall moment equation of equilibrium.
5. It considers the forces acting on the sides of individual slices
6. It gives a factor a safety which errs on the safe side
of these statement the incorrect statement is/are
a. 3,4
b. $1,4,3$
c. 2,4
d. 3 only
7. Which of the following will have the finite slope
a. Embankment
b. Earth dam
c. Canals
d. All the above
8. In the stability analysis of finite slopes, the Swedish finite circle method assumes that the surface of sliding is
a. An arc of a parabola
b. A straight
c. An arc of a circle
d. An elliptical arc
9. The maximum value of stability number is
a. 0.465
b. 0.261
c. 0.542
d. 1
10. Bishops method of analysis
a. Consider the effect of forces acting on the sides slices
b. Is economical
c. Assumes the slip surface as an arc of a circle
d. All the above
11. Taylor's stability chart are based on the total stresses using the
a. Friction circle method
b. Method of slices
c. $\emptyset_{u}=0$ analysis
d. none of the above
12. In stability analysis, the term mobilized shear strength is referred to as
a. Shear strength
b. Maximum shear stress
c. Applied shear stress
d. none of the above

## II) Descriptive Questions

1. What are the different factor of safety used in the stability of slopes?
2. Explain different types of slope failures?
3. Derive an expression for the factor of safety of an in finite slope in cohesive soil.
4. What is stability number? What is its utility in the analysis of stability of slopes? Discuss the uses of stability charts.
5. Derive an expression for the factor of safety using Swedish circle method.
6. Describe bishop's simplified method. What are its advantages over conventional Swedish circle method?
7. Derive an expression for the factor of safety of an in finite slope in cohesionless soil.

## B. Question testing the ability of students in applying the concepts.

## I) Multiple Choice Questions:

1. Find the factor of safety of a slope of infinite extent having a slope angle $=25^{\circ}$. The slope is made of cohesionless soil with $\emptyset=30^{\circ}$.
2. Taylor's stability number $S^{\prime}$ is
a) $C / F_{c} \gamma H$
b) $\mathrm{C} / \gamma \mathrm{H}$
c) $\mathrm{CH} / \gamma \mathrm{F}_{c}$
d) $C^{2} / \gamma F_{c} H$
3. If $\phi$ is the angle of internal friction of soil and $I$ is the slope of angle, the factor safety of an infinite slope is given by
a) $\tan \phi / \tan \mathrm{i}$
b) $\tan i / \tan \phi$
c) $\cot \phi / \cot i$
d) $\cot i / \cot \phi$
4. The method of slices of Swedish circle method is applicable in
a) Homogeneous soils only
b) Uniform slopes only
c) Stratified soils only
d) Non uniform slopes only
5. A base failure is likely to occur when
a) $\phi>=0^{0} \& \beta<53^{0}$
b) $\phi>=0^{0} \& \beta>53^{0}$
c) $\phi=0^{0} \& \beta<53^{0}$
d) $\phi>0^{0} \& \beta<53^{0}$
6. The method of slices for the stability of slopes
a) Can be used for stratified soils
b) Can be used when seepage occurs and the pore pressure exists within the soil
c) Gives the factor of safety based on moments and not the forces
d) All the above
7. The cohesion and density of a soil are $2 \mathrm{t} / \mathrm{m}^{2}$ and $2 \mathrm{t} / \mathrm{m}^{3}$ respectively. For a factor of safety of 2 and stability number 0.1 , the safe height of slope is
a) 2.5 m
b) 10 m
c) 5 m
d) 50 m

## II) Problems

1. A long natural slope in an overconsolidated clay ( $\mathrm{c}^{\prime}=10 \mathrm{kN} / \mathrm{m}^{2}, \varnothing^{\prime}=25^{0}, \gamma_{\mathrm{sat}}=20 \mathrm{kN} / \mathrm{m}^{3}$ ) is inclined at $10^{0}$ to the horizontal. The water table is at the surface and the seepage is paralleel to the slop. If a plane slip had developed at a depth of 5 m below the surface, determine the factor of safety. Take $\gamma_{\mathrm{w}}=10 \mathrm{kN} / \mathrm{m}^{3}$.
2. A vertical cut is made in a clay deposit ( $\mathrm{c}=30 \mathrm{kN} / \mathrm{m}^{2}, \emptyset=0, \gamma=16 \mathrm{kN} / \mathrm{m}^{3}$ ). Find the maximum height of the cut which can be temporarily supported.
3. What will be the factors of safety with respect to average shearing strength, cohesion and internal friction of a soil, for which the shear strength parameters obtained from the laboratory tests are $\mathrm{c}^{\prime}=32 \mathrm{kN} / \mathrm{m}^{2}$ and $\Phi_{1}=18^{\circ}$; the expected parameters of mobilized shearing resistance are $\mathrm{c}_{\mathrm{m}}=21 \mathrm{kN} / \mathrm{m}^{2}$ and $\Phi^{\prime}=13^{\circ}$ and the average effective pressure on the failure plane is $110 \mathrm{kN} / \mathrm{m}^{2}$. For the same value of mobilized shearing resistance determine the following:
i) Factor of safety with respect to height;
ii). Factor of safety with respect to friction when that with respect to cohesion is unity; and
iii). Factor of safety with respect to strength.
4. The shearing strength parameters of a soil are $c^{\prime}=26.1 \mathrm{kN} / \mathrm{m}^{2} \emptyset^{\prime}=15^{\circ}, c^{\prime}=17.8 \mathrm{kN} / \mathrm{m}^{2}$ Calculate the factor of safety (a) with respect to strength, (b) with respect to cohesion and (c) with respect to friction. The average intergranular pressure $\sigma^{\prime}$ on the failure surface is $102.5 \mathrm{kN} / \mathrm{m}^{2}$.
5. What will be the factors of safety with respect to average shearing strength, cohesion and internal friction of a soil, for which the shear strength parameters obtained from the laboratory tests are $\mathrm{c}^{\prime}=30 \mathrm{kN} / \mathrm{m}^{2}$ and $\Phi_{1}=19^{\circ}$; the expected parameters of mobilized shearing resistance are $\mathrm{c}_{\mathrm{m}}=23 \mathrm{kN} / \mathrm{m}^{2}$ and $\Phi^{\prime}=11^{\circ}$ and the average effective pressure on the failure plane is $123 \mathrm{kN} / \mathrm{m}^{2}$.
6. A slope of overconsolidated clay ( $c^{\prime}=21 \mathrm{kN} / \mathrm{m}^{2}, \varnothing^{\prime}=19^{0}, \gamma_{\mathrm{sat}}=21 \mathrm{kN} / \mathrm{m}^{3}$ ) is inclined at $12^{0}$ to the horizontal. The water table is at the surface and the seepage is paralleel to the slop. If a plane slip had developed at a depth of 6 m below the surface, determine the factor of safety.
7. The following particulars are given for an earth dam of height 12 m . The slope is submerged and the slope angle $\beta=45^{\circ}$.
$\gamma_{b}=8.2 \mathrm{kN} / \mathrm{m}^{3}$
$c^{\prime}=75 \mathrm{kN} / \mathrm{m}^{2}$
$\emptyset^{\prime}=20^{\circ}$
Determine the factor of safety Fs.
8. Find the critical height of an infinite slope having a slope angle of $=25^{\circ}$ The slope is made of cohesive soil. The soil made of clay having $\mathrm{c}^{\prime}=30 \mathrm{kN} / \mathrm{m}^{2}, \varnothing^{\prime}=20^{\circ}, \mathrm{e}=0.65$ and $\mathrm{G}_{\mathrm{s}}=2.7$ and under the following conditions: i) when the soil is dry, ii) when water seeps parallel to the surface of the slope, and iii) when the slope is submerged.

## GEOTECHNICAL ENGINEERING -II

## UNIT -III

## Objective:

To impart the knowledge on lateral earth pressure and stability of retaining walls.

Syllabus: EARTH PRESSURE THEORIES \& RETAINING WALLS<br>Rankines Earth pressure- surface horizontal and inclined, Rankines Earth pressure- Cohesive Soils, Coloumb's Wedge Theory, Rehbann's Graphical Method, Culmann's Graphical Method, Types of retaining walls - gravity and cantilever retaining walls

## Learning Outcomes:

After completion of this unit the student will be able to

- Distinguish Rankine's and Coloumb's earth pressure theories
- Estimate the active earth pressure using culmann's graphical method
- Analyze the stability of gravity retaining walls
- Analyze the stability of cantilever retaining walls


## Learning Material

### 3.1. DIFFERENT TYPES OF LATERAL EARTH PRESSURE

Lateral earth pressures can be grouped into 3 categories, depending upon the movement of the retaining wall with respect to the soil retained. The soil retained is also known as the backfill.
3.1.1. At-rest pressure: The lateral earth pressure is called at-rest pressure when the soil mass is not subjected to any lateral yielding or movement. This case occurs when the retaining wall is firmly fixed at its top and is not allowed to rotate or move laterally. Fig. 3.3 shows the basement retaining walls which are restrained against the movement by the basement slab provided at their tops. Another example of the at-rest pressure is that of a bridge abutment wall which is restrained at its top by the bridge slab. The at-rest condition is also known as the elastic equilibrium, as no part of soil mass has failed and attained the plastic equilibrium.


Fig. 3.3 At-rest pressure
3.1.2. Active pressure: A state of active pressure occurs when the soil mass yields in such a way that it tends to stretch horizontally. It is a state of plastic equilibrium as the entire soil mass is on the verge of failure. A retaining wall when moves away from the backfill, there is a stretching of the soil mass and the active state of earth pressure exists. In Fig. 3.4 (a), the active pressure develops on the right-hand side when the wall moves towards left.
3.1.3. Passive pressure: A state of passive pressure exists when the movement of the wall is such that the soil tends to compress horizontally. It is another extreme of the limiting equilibrium condition. In Fig. 3.1 (b), the passive pressure develops on the left-side of the wall below the ground level, as the soil in this zone is compressed when the movement of the wall is towards right.

The gradual decrease or increase of pressure on the wall with the movement of the wall from at rest condition may be depicted as shown in Fig. 3.2.

(a) Active carth pressure

(b) Passive earth pressure

Fig. 3.1 Wall movement for the development of active and passive earth pressures


Fig. 3.2 Development of active and passive earth pressures

### 3.2. LATERAL EARTH PRESSURE FOR AT REST CONDITION

If the wall is rigid and does not move with the pressure exerted on the wall, the soil behind the wall will be in a state of elastic equilibrium. Consider a prismatic element E in the backfill at depth z shown in Fig. 3.3

(a)

Fig. 3.3 Lateral earth pressure at rest condition
Element E is subjected to the following pressures.
Vertical pressure $=\sigma_{v}=\gamma z$, lateral pressure $=\sigma_{h}$
Where $\gamma$ is the effective unit weight of the soil. If we consider the backfill is homogeneous then both $\sigma_{v}$ and $\sigma_{h}$ increase linearly with depth $z$. In such a case, the ratio of $\sigma_{h}$ and $\sigma_{v}$ remains constant with respect to depth, that is

$$
\frac{\sigma_{h}}{\sigma_{v}}=\frac{\sigma_{h}}{\gamma z}=\text { constant }=K_{0}
$$

Where $K_{0}$ is called the coefficient of earth pressure for the at rest condition or at rest earth pressure coefficient.

The lateral earth pressure $\sigma_{h}$ acting on the wall at any depth z may be expressed as

$$
\sigma_{h}=K_{0} \gamma z
$$

The expression for $\sigma_{h}$ at depth H , the height of the wall, is

$$
\sigma_{h}=K_{0} \gamma H
$$

The distribution of $\sigma_{h}$ on the wall is given in Fig. 3.3 (b). The total pressure $P_{0}$ for the soil for at rest condition is

$$
P_{0}=\frac{1}{2} K_{0} \gamma H^{2}
$$

According to Jaky (1944), a good approximation for $K_{0}$ is given by

$$
K_{o}=1-\sin \emptyset
$$

### 3.3. LATERAL EARTH PRESSURE THEORY

There are two classical earth pressure theories. They are

1. Coulomb's earth pressure theory.
2. Rankine's earth pressure theory.

The first rigorous analysis of the problem of lateral earth pressure was published by Coulomb in (1776). Rankine (1857) proposed a different approach to the problem. These theories propose to estimate the magnitudes of two pressures called active earth pressure and passive earth pressure.

### 3.4. RANKINE'S EARTH PRESSURE THEORY

Rankine considered the equilibrium of a soil element with in a soil mass bounded by a plane surface. The following assumptions were made by Rankine for the derivation of earth pressure.

1. The soil mass is homogeneous and semi-infinite.
2. The soil is dry and cohesionless.
3. The ground surface is plane, which may be horizontal or inclined.
4. The back of the retaining wall is smooth and vertical.
5. The soil element is in a state of plastic equilibrium, i,e., at the verge of failure.

Expressions for the active earth pressure and the passive earth pressure are developed as explained below.
(a) Active Earth Pressure: Let us consider an element of dry soil at a depth z below a level soil surface Fig. 3.4 (a). initially, the element is at-rest conditions, and the horizontal pressure is given by

$$
\sigma_{h}=K_{o} \sigma_{v}
$$

Where $\sigma_{v}$ is the vertical stress at $\mathbf{C}$, and $\sigma_{h}$ is the horizontal stress at $\mathbf{C}$. of course, $\sigma_{v}=\gamma Z$ The stresses $\sigma_{h}$ and $\sigma_{v}$ are, respectively, the minor and major principal stresses, and are indicated by points A and B in the Mohr circle Fig.3.4 (b).

Let us now consider the case when the vertical stress remains constant while the vertical stress is decreased. The point A shifts to position A' and the diameter of the Mohr
circle increases. In the limiting condition, the point A shift to position A" when the Mohr circle (marked (3)) touches the failure envelope. The soil is at the verge of shear failure. It has attained the Rankine active state of plastic equilibrium. The horizontal stress at that state is the active pressure $\left(\mathrm{P}_{\mathrm{a}}\right)$.


Fig.3.4
Fig. 3.5. Shows the Mohr circle when active conditions are developed. Point E represents the active condition. From the figure,


Fig. 3.5. Shows the Mohr circle

$$
\begin{gathered}
P_{a}=O E=O C-C E \\
C E=C D=O C \sin \emptyset^{\prime} \\
P_{a}=O C-O C \sin \emptyset^{\prime}=O C\left(1-\sin \emptyset^{\prime}\right) \\
\sigma_{v}=O B=O C+C B=O C+O C \sin \emptyset^{\prime} \\
\sigma_{v}=o C\left(1+\sin \emptyset^{\prime}\right)
\end{gathered}
$$

From above Eqs. $\frac{P_{a}}{\sigma_{v}}=\frac{1-\sin \phi^{\prime}}{1+\sin \phi^{\prime}}$

$$
\begin{gathered}
P_{a}=\left(\frac{1-\sin \emptyset^{\prime}}{1+\sin \emptyset^{\prime}}\right) \sigma_{v} \\
P_{a}=K_{a} \gamma Z
\end{gathered}
$$

Where $\mathrm{K}_{\mathrm{a}}$ is a coefficient, known as the coefficient of active earth pressure. It is a function of the angle of shearing resistance ( $\varnothing^{\prime}$ ), and is given by

$$
K_{a}=\frac{1-\sin \emptyset^{\prime}}{1+\sin \emptyset^{\prime}}=\tan ^{2}\left(45^{0}-\frac{\emptyset^{\prime}}{2}\right)
$$

(b) Passive Earth Pressure: The passive Rankine state of plastic equilibrium can be explained by considering the element of soil at a point at a depth of Z below the soil surface Fig. 3.6 (a). As the soil is compressed laterally, the horizontal stress $\left(\sigma_{h}\right)$ is increased, whereas the vertical stress $\left(\sigma_{v}\right)$ remains constant. Fig. 3.6 (b) shows the Mohr circles. The circle-1 indicates the in-situ condition, in which point A indicates the horizontal stress and point B, vertical stress. With lateral compressing of the soil, the horizontal stress increases until it reaches a limiting value greater than the vertical stress, indicated by point A" and the Mohr circle (marked(3)) touches the failure envelope. The expression for the passive pressure $\mathrm{P}_{\mathrm{p}}$ can be obtained as follows Fig. 3.7, shows the Mohr circle at failure.


Fig. 3.6


Fig. 3.7 Shows the Mohr circle at failure

$$
\begin{gathered}
P_{p}=O C+C E \\
C E=C D=O C \sin \emptyset^{\prime} \\
P_{a}=O C+O C \sin \emptyset^{\prime}=O C\left(1+\sin \emptyset^{\prime}\right) \\
\sigma_{v}=O B=O C-C B=O C-O C \sin \emptyset^{\prime} \\
\sigma_{v}=o C\left(1-\sin \emptyset^{\prime}\right)
\end{gathered}
$$

From above Eqs. $\frac{P_{P}}{\sigma_{v}}=\frac{1+\sin \phi^{\prime}}{1-\sin \phi^{\prime}}$

$$
\begin{gathered}
P_{P}=\left(\frac{1+\sin \emptyset^{\prime}}{1-\sin \emptyset^{\prime}}\right) \sigma_{v} \\
P_{P}=K_{P} \gamma Z
\end{gathered}
$$

Where $\mathrm{K}_{\mathrm{p}}$ is a coefficient, known as the coefficient of passive earth pressure. It is a function of the angle of shearing resistance ( $\varnothing^{\prime}$ ), and is given by

$$
K_{p}=\frac{1+\sin \emptyset^{\prime}}{1-\sin \emptyset^{\prime}}=\tan ^{2}\left(45^{\circ}+\frac{\emptyset^{\prime}}{2}\right)
$$

### 3.5. RANKIKE'S EARTH PRESSURE WHEN THE SURFACE IS INCLINED

Two stresses are called conjugate stresses when the direction of one stress in parallel to the plane on which the other stress acts. Rankine assumed that the vertical stress on an element of the soil within the inclined backfill and the lateral stress on the vertical plane of the element are conjugate stresses. In other words, he assumed that the lateral stress is parallel to the inclined backfill.

Let us consider an element of soil at depth Z below the soil surface inclined at angle i to horizontal Fig.3.8. the angle $i$ is known as the angle of surcharge. The intensity of vertical stress $\left(\sigma_{v}\right)$ on the element is given by

$$
\begin{gathered}
\sigma_{v}=\frac{\gamma(Z b \cos i)}{b} \\
\sigma_{v}=\gamma Z \cos i
\end{gathered}
$$

The other conjugate stress is the lateral stress $\left(\sigma_{x}\right)$. It may be mentioned that the vertical stress $\sigma_{v}$ is not the principal stress, as ashear stress also exists on the inclined plane at the top of the element. Likewise, the lateral stress $\sigma_{x}$ is also not a principal stress. A relationship between the lateral pressure and the vertical stress can be obtained for the active and passive cases as given below.


Fig. 3.8 Inclined surface
(a) Active Earth Pressure: Fig. 3.9 shows the Mohr circle corresponding to the active limiting conditions. The vertical stress $\sigma_{v}$ is represented by the line OA making an angle i with the horizontal. At any depth, the value of $\sigma_{v}$ is constant and equal to that given by above Eq.
If the lateral extension of the soil is sufficient to induce the state of active plastic equilibrium, the Mohr circle must pass through A and it should be tangential to the failure envelope. The origin of planes P is obtained as the point of intersection of OA with the Mohr circle. The origin of the planes is located by drawing from the point representing a stress (vertical stress, in this case) a line parallel to the plane on which it acts (plane inclined at i , in this case). A vertical line through P cuts the circle ar B below the $\sigma-$ axis. The conjugate stress, which is the active pressure $\left(P_{a}\right)$, is represented by OB. Numerically, the conjugate stress is also equal to OP.


Fig. 3.9

$$
\text { From the figure, } \frac{P_{a}}{\sigma_{v}}=\frac{O B}{O A}=\frac{O P}{O A}=\frac{O D-D P}{O D+D A}
$$

The ratio $P_{a} / \sigma_{v}$ is known as the conjugate stress ratio.

$$
\begin{gathered}
O D=O C \cos i \\
D A=D P=\sqrt{P C^{2}-D C^{2}} \\
P C=F C=O C \sin \emptyset^{\prime} \\
D C=O C \sin i \\
D A=D p=\sqrt{\left(O C \sin \emptyset^{\prime}\right)^{2}-(O C \sin i)^{2}} \\
=O C \sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i} \\
\frac{P_{a}}{\sigma_{v}}=\frac{O C \cos i-O C \sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i}}{O C \cos i+O C \sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i}} \\
\frac{P_{a}}{\sigma_{v}}=\frac{\cos i-\sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i}}{\cos i+\sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i}} \\
\cos i-\sqrt{\left(1-\cos ^{2} \emptyset^{\prime}\right)-\left(1-\cos ^{2} i\right)} \\
\cos i+\sqrt{\left(1-\cos ^{2} \emptyset^{\prime}\right)-\left(1-\cos ^{2} i\right)} \\
\frac{P_{a}}{\sigma_{v}}=\frac{\cos i-\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}{\cos i+\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}} \\
P_{a}=\frac{\cos i-\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}{\cos i+\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}(\gamma Z \cos i) \\
P_{a}=K_{a} \gamma Z
\end{gathered}
$$

Where $K_{a}$ is the coefficient of active pressure, given by

$$
K_{a}=\cos i \times \frac{\cos i-\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}{\cos i+\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}
$$

In must be noted that $P_{a}$ is parallel to the inclined surface.
For the special case, when $\mathrm{i}=0$

$$
K_{a}=\frac{1-\sin \emptyset^{\prime}}{1+\sin \emptyset^{\prime}}
$$

(b) Passive Earth Pressure: This case is similar to the one for the active case with one basic difference that the vertical stress is the smaller of the two conjugate stresses. In Fig. 3.10, OA represents the vertical stress $\left(\sigma_{v}\right)$. The point P shows the origin of planes, and OB represents the passive pressure, From the Fig.,


Fig. 3.10

From the figure, $\frac{P_{p}}{\sigma_{v}}=\frac{O B}{O A}=\frac{O P}{O A}=\frac{O D+D P}{O D-D A}$

$$
\begin{gathered}
O D=O C \cos i \\
D A=D P=\sqrt{A C^{2}-D C^{2}}=\sqrt{F C^{2}-D C^{2}} \\
P C=F C=O C \sin \emptyset^{\prime} \\
D C=O C \sin i \\
D A=D p=\sqrt{\left(O C \sin \emptyset^{\prime}\right)^{2}-(O C \sin i)^{2}} \\
=O C \sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i} \\
\frac{P_{p}}{\sigma_{v}}=\frac{O C \cos i+O C \sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i}}{O C \cos i-O C \sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i}} \\
\frac{P_{p}}{\sigma_{v}}=\frac{\cos i+\sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i}}{\cos i-\sqrt{\sin ^{2} \emptyset^{\prime}-\sin ^{2} i}} \\
=\frac{\cos i+\sqrt{\left(1-\cos ^{2} \emptyset^{\prime}\right)-\left(1-\cos ^{2} i\right)}}{\cos i-\sqrt{\left(1-\cos ^{2} \emptyset^{\prime}\right)-\left(1-\cos ^{2} i\right)}} \\
\frac{P_{p}}{\sigma_{v}}=\frac{\cos i+\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}{\cos i-\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}} \\
P_{p}=\frac{\cos i+\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}{\cos i-\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}(\gamma Z \cos i) \\
P_{p}=K_{p} \gamma Z
\end{gathered}
$$

Where $\mathrm{K}_{\mathrm{a}}$ is the coefficient of active pressure, given by

$$
K_{p}=\cos i \times \frac{\cos i+\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}{\cos i-\sqrt{\cos ^{2} i-\cos ^{2} \emptyset^{\prime}}}
$$

In must be noted that $\mathrm{P}_{\mathrm{a}}$ is parallel to the inclined surface.
For the special case, when $\mathrm{i}=0$

$$
K_{p}=\frac{1+\sin \emptyset^{\prime}}{1-\sin \emptyset^{\prime}}
$$

### 3.6. RANKINE'S EARTH PRESSURE IN COHESIVE SOILS

Rankine's original theory was for cohesionless soils. It was extended by Resal (1910) and Bell (1915) for cohesive soils. The treatment is similar to that for cohesionless soils with one basic different that the failure envelope has a cohesion intercept $\mathrm{C}^{\prime}$, whereas that for cohesionless soils is zero.
(a) Active Case: Fig. 3.11 shows the Mohr circle in which point B indicates the vertical stress and point E shows the active pressure. The failure envelope is tangential to the circle. The relationship between $P_{a}$ and $\sigma_{v}$ can be obtained as under.


Fig. 3.11
From triangle FCD,

$$
\sin \emptyset^{\prime}=\frac{C D}{F C}=\frac{C D}{F O+O C}
$$

$$
\begin{gathered}
\sin \emptyset^{\prime}=\frac{\left(\sigma_{1}-\sigma_{3}\right) / 2}{c^{\prime} \cot \emptyset^{\prime}+\left(\sigma_{1}+\sigma_{3}\right) / 2} \\
\frac{\left(\sigma_{1}-\sigma_{3}\right)}{2}=\frac{\left(\sigma_{1}+\sigma_{3}\right)}{2} \sin \emptyset^{\prime}+c^{\prime} \cos \emptyset^{\prime} \\
\frac{\sigma_{1}}{2}\left(1-\sin \emptyset^{\prime}\right)=\frac{\sigma_{3}}{2}\left(1+\sin \emptyset^{\prime}\right)+c^{\prime} \cos \emptyset^{\prime} \\
\sigma_{3}=\frac{1-\sin \emptyset^{\prime}}{1+\sin \emptyset^{\prime}} \sigma_{1}-\frac{2 c^{\prime} \cos \emptyset^{\prime}}{1+\sin \emptyset^{\prime}} \\
\sigma_{3}=\sigma_{1} \tan ^{2}\left(45^{0}-\emptyset^{\prime} / 2\right)-2 c^{\prime} \tan \left(45^{0}-\emptyset^{\prime} / 2\right)
\end{gathered}
$$

As $\sigma_{3}$ is equal to the active pressure $\left(P_{a}\right)$, and $\sigma_{1}$ is equal to the vertical stress $\sigma_{v}(=\gamma Z)$

$$
\begin{gathered}
P_{a}=\left(\frac{1-\sin \emptyset^{\prime}}{1+\sin \varnothing^{\prime}}\right) \gamma Z-\frac{2 c^{\prime} \cos \emptyset^{\prime}}{1+\sin \emptyset^{\prime}} \\
P_{a}=K_{a} \gamma Z-2 c^{\prime} \sqrt{K_{a}}
\end{gathered}
$$

Where

$$
K_{a}=\frac{1-\sin \varnothing^{\prime}}{1+\sin \varnothing^{\prime}}=\tan ^{2}\left(45^{\circ}-\varnothing^{\prime} / 2\right)
$$

It can be shown that

$$
\frac{\cos \emptyset^{\prime}}{1+\sin \emptyset^{\prime}}=\tan \left(45^{0}-\emptyset^{\prime} / 2\right)=\sqrt{K_{a}}
$$

If $\mathrm{Z}=0$, the active pressure is given by

$$
P_{a}=-2 c^{\prime} \sqrt{K_{a}}
$$

The negative sign shows that the pressure is negative, i.e. it tries to cause a pull on the wall. This tensile stress decreases with an increase in depth, and it becomes zero at a depth $\mathrm{Z}_{\mathrm{c}}$, given by

$$
\begin{gathered}
0=K_{a} \gamma Z_{c}-2 c^{\prime} \sqrt{K_{a}} \\
K_{a}=\frac{2 c^{\prime}}{\gamma \sqrt{K_{a}}}
\end{gathered}
$$

The depth $\mathrm{Z}_{\mathrm{c}}$ is known as the depth of tensile crack. The tensile stress eventually causes a crack to from along the soil-wall interface.

The pressure at the depth H is given by,

$$
P_{a}=K_{a} \gamma H-2 c^{\prime} \sqrt{K_{a}}
$$

## Pressure Distribution

Fig. 3.12 shows the pressure distribution against the retaining wall. The total pressure on the retaining wall of height H is given by integration of above Eq. as


Fig. 3.12 Pressure distribution

$$
\begin{gathered}
P_{a}=\int_{0}^{H}\left(K_{a} \gamma Z-2 c^{\prime} \sqrt{K_{a}}\right) d Z \\
P_{a}=K_{a} \frac{\gamma H^{2}}{2}-2 c^{\prime} \sqrt{K_{a}} H
\end{gathered}
$$

Above eq. is applicable before the formation of crack
After the occurrence of the tensile crack, the force on the wall is caused only by the pressure from $Z=Z_{c}$ to $Z=H$, i.e, tensile stress are neglected.

Thus

$$
P_{a}=\frac{1}{2}\left(H-Z_{c}\right)\left(\gamma H K_{a}-2 c^{\prime} \sqrt{K_{a}}\right)
$$

Substituting the value of Zc

$$
\begin{gathered}
P_{a}=\frac{1}{2}\left(H-\frac{2 c^{\prime}}{\gamma \sqrt{K_{a}}}\right)\left(\gamma H K_{a}-2 c^{\prime} \sqrt{K_{a}}\right) \\
P_{a}=\frac{1}{2} \gamma H^{2} K_{a}-2 c^{\prime} H \sqrt{K_{a}}+\frac{2\left(c^{\prime}\right)^{2}}{\gamma}
\end{gathered}
$$

It acts at a height of $\left(\mathrm{H}-\mathrm{Z}_{\mathrm{c}}\right) / 3$.

For $\emptyset=0$, Above Eq. reduces to

$$
P_{a}=\frac{1}{2} \gamma H^{2}-2 c^{\prime} H+\frac{2\left(c^{\prime}\right)^{2}}{\gamma}
$$

For soils below the water table, the submerged unit weight $\gamma^{\prime}$ and the corresponding values of $\emptyset$ ' and c' should be used.

## (b) Passive Case:

An expression for the passive pressure in a cohesive soil can be determine by referring to Fig. 3.13. The Mohr circle has been drawn such that the point B represents $\sigma_{v}$ and point E as $P_{p}$. In triangle FCD,


Fig. 3.13
From triangle FCD,

$$
\sin \emptyset^{\prime}=\frac{C D}{F C}
$$

$$
\begin{gathered}
\sin \emptyset^{\prime}=\frac{\left(\sigma_{1}-\sigma_{3}\right) / 2}{c^{\prime} \cot \emptyset^{\prime}+\left(\sigma_{1}+\sigma_{3}\right) / 2} \\
\frac{\left(\sigma_{1}-\sigma_{3}\right)}{2}=\frac{\left(\sigma_{1}+\sigma_{3}\right)}{2} \sin \emptyset^{\prime}+c^{\prime} \cos \emptyset^{\prime} \\
\frac{\sigma_{1}}{2}\left(1-\sin \emptyset^{\prime}\right)=\frac{\sigma_{3}}{2}\left(1+\sin \emptyset^{\prime}\right)+c^{\prime} \cos \emptyset^{\prime} \\
\sigma_{1}=\frac{1+\sin \emptyset^{\prime}}{1-\sin \emptyset^{\prime}} \sigma_{3}+\frac{2 c^{\prime} \cos \emptyset^{\prime}}{1-\sin \emptyset^{\prime}} \\
P_{p}=\sigma_{v} \tan ^{2}\left(45^{0}+\emptyset^{\prime} / 2\right)+2 c^{\prime} \tan \left(45^{0}+\emptyset^{\prime} / 2\right) \\
P_{p}=K_{p} \gamma Z+2 c^{\prime} \sqrt{K_{a}}
\end{gathered}
$$

Where $K_{p}=$ coefficient of passive pressure, given by

$$
K_{p}=\frac{1+\sin \emptyset^{\prime}}{1-\sin \emptyset_{\prime}^{\prime}}=\tan ^{2}\left(45^{0}+\emptyset^{\prime} / 2\right)
$$

The failure plane makes an angle of $\left(45^{0}+\varnothing^{\prime} / 2\right)$ with the vertical (major principal plane) and of $\left(45^{0}-\emptyset^{\prime} / 2\right)$ with horizontal (minor principal plane).

## Pressure distribution

Fig. 3.14 shows the pressure distribution obtained from Eq.


Fig. 3.14
When $\mathrm{Z}=0$,

$$
P_{P}=2 c^{\prime} \sqrt{K_{p}}
$$

When $Z=H$,
$P_{P}=\gamma H K_{P}+2 c^{\prime} \sqrt{K_{P}}$
The pressure, unlike active case, in positive throughout the depth. The total pressure on the retaining wall of height H is given by

$$
\begin{gathered}
P_{P}=\frac{1}{2} H\left(\gamma H K_{P)}+2 c^{\prime} \sqrt{K_{P} \times H}\right. \\
P_{P}=K_{p} \gamma H^{2} / 2+2 c^{\prime} H \sqrt{K_{P}}
\end{gathered}
$$

### 3.7. COULOMB'S WEDGE THEORY

Coulomb (1776) developed a method for determination of earth pressure in which he considered the equilibrium of the sliding wedge which is formed when the movement of the retaining wall takes place. As discussed, in the active case, the sliding wedge moves downward and outward relative to the backfill, whereas in the passive case, the sliding wedge moves upwards. The lateral pressure on the wall is equal and opposite to the relative force exerted by the wall in order to keep the sliding wedge in equilibrium. The analysis is a type of limiting equilibrium method:
The following assumptions are made

1. The backfill is dry, cohesionless, homogeneous, isotropic and ideally plastic material.
2. The slip surface is a plane surface which passes through the heel of the wall.
3. The wall surface is rough. The resultant earth pressure on the wall is inclined at an angle $\delta$ to the normal wall, where $\delta$ is the angle of friction between the wall and the backfill.
4. The surface wedge itself acts as a rigid body.

The magnitude of earth pressure is obtained by considering the equilibrium of the sliding wedge as a whole.

In coulomb's theory, a plane failure surface is assumed and the lateral force required to maintain the equilibrium of the wedge is found using the principles of statics. The procedure is repeated for several trial surfaces. The trial surface which gives the largest force for the active case, and the smallest force for the passive case, is the actual failure surface. The method readily accommodates the friction between the wall and the backfill, irregular backfill, sloping wall, surcharge loads, etc. Although the initial theory was for dry, cohesionless soils, it has now been extended to wet soils and cohesive soils as well. The coulomb's theory is more general than the rankine's theory.

### 3.7.1 Coulomb's Active Pressure in Cohesionless Soils:

Fig. 3.15 shows a retaining wall with an inclined back face and a sloping dry granular backfill. In active case the sliding wedge ABD moves downward and the reaction R acts upward and inclined at an angle $\phi$ ' with the normal.


Fig. 3.15
The sliding wedge ABD is in equilibrium under three forces:

1. Weight of the wedge (W).
2. Reaction $R$ on the slip surface.
3. Reaction $\mathrm{P}_{\mathrm{a}}$ from the wall.

It may be noted that at failure conditions the shearing resistance on the failure surface is fully mobilized and the reaction R has the maximum obliquity. It is therefore inclined at an angle $\phi$ ' to the normal to the failure plane, Further $\mathrm{P}_{\mathrm{a}}$ acts upward and it is inclined at an angle $\delta$ with the normal as shown in Fig. 3.15(a).

Fig. 3.15(b) shows the force triangle. As the magnitude of one force and the directions of all three forces are known, the force triangle can be completed. The magnitude of $\mathrm{P}_{\mathrm{a}}$ is determined from the force triangle. The pressure acting on the wall is equal and opposite to Pa.

The procedure is repeated after assuming another failure surface. The surface that gives the maximum value of $\mathrm{P}_{\mathrm{a}}$ is the critical failure plane, and the corresponding force is the active force.

Coulomb's method does not give the point of application of the resultant earth pressure ( $\mathrm{P}_{\mathrm{a}}$ ). The point of application is found to be approximately at the point of intersection E of the back of the retaining wall with a line CE drawn from the centroid C of the failure wedge and parallel to the failure surface. As this procedure is cumbersome, for convenience, the pressure distribution is sometimes assumed to be hydrostatic on the back of the wall, and the resultant pressure $\mathrm{P}_{\mathrm{a}}$ is assumed to act at one-third the height of the wall from the base.

The following points should be carefully noted while using coulomb's theory:

1. For most practical cases, the backfill moves down relative to the wall in the active case and therefore the active force $\mathrm{P}_{\mathrm{a}}$ is inclined at angle $\delta$ below the normal as shown in Fig. 3.15 (a). However, if the wall is supported on a soft, compressible soil, it may settle to such an extent that the movement of the wall will be downward relative movement of the wedge will be upward. In such a case, the force $\mathrm{P}_{\mathrm{a}}$ would be inclined at an angle $\delta$ above the normal to the wall.
2. The angle $\delta$ is the friction angle between the soil and the wall. It may be determined by means of a direct shear test. For concrete walls, $\delta$ is generally taken as $2 / 3 \phi$ '. The value of $\delta$ cannot exceed $\phi^{\prime}$, because in that case the failure will occur in soil.
3. Coulomb's theory assumes the failure surface to be a plane. The actual failure surface is slightly curved. Fortunately for the active case, the error is small, and therefore the failure surface may be assumed to be plane without any significant error.

## Expression for Active Pressure

From the Fig. 3.15 (b), using the law of sines,

$$
\begin{gather*}
\frac{P_{a}}{\sin \left(\alpha-\emptyset^{\prime}\right)}=\frac{W}{\sin \left(180^{0}-\beta+\delta-\alpha+\emptyset^{\prime}\right)} \\
P_{a}=\frac{W \sin \left(\alpha-\phi^{\prime}\right)}{\sin \left(180^{0}-\beta+\delta-\alpha+\emptyset^{\prime}\right)} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \tag{a}
\end{gather*}
$$

Where $P_{a}=$ total active pressure force.
The Weight W of the wedge ABE can be determined from Fig. 3.16 as


Fig. 3.16

$$
W=\frac{1}{2} \mathrm{x} B E \times \mathrm{AG} \mathrm{x} \gamma
$$

Taking $\mathrm{AG}=\mathrm{m}$ and $\mathrm{BE}=\mathrm{L}$, we have $W=\frac{1}{2} \mathrm{x} L \times \mathrm{mx} \gamma$

$$
\begin{gathered}
W=\frac{1}{2}(m \times \gamma)(B G+G E) \\
A B=\frac{H}{\sin \beta}
\end{gathered}
$$

From triangle ABG, $\frac{m}{A B}=\sin \left[180^{\circ}-(\beta+\alpha)=\sin (\beta+\alpha)\right.$

$$
m=A B \sin (\beta+\alpha)=\frac{H \sin (\beta+\alpha)}{\sin \beta}
$$

From triangle $A B G, \frac{B G}{\sin \left(\beta+\alpha-90^{0}\right)}=\frac{m}{\sin (\beta+\alpha)}$

$$
B G=\frac{m \sin \left(\beta+\alpha-90^{\circ}\right)}{\sin (\beta+\alpha)}
$$

From triangle AGE, $\frac{E G}{\sin \left(90^{0}-\alpha+i\right)}=\frac{m}{\sin (\alpha-i)}$

$$
G E=m \frac{\sin \left(90^{0}-\alpha+i\right)}{\sin (\alpha-i)}
$$

Substituting the values of $m, B G$ and GE from Equations

$$
\begin{gathered}
W=\frac{1}{2} \cdot \frac{H \sin (\beta+\alpha)}{\sin \beta} \cdot \gamma \times\left[m \frac{\sin \left(\beta+\alpha-90^{0}\right)}{\sin (\beta+\alpha)}+m \frac{\sin \left(90^{0}-\alpha+i\right)}{\sin (\alpha-i)}\right] \\
W=\frac{1}{2} \frac{H^{2} \gamma \sin (\beta+\alpha)^{2}}{\sin ^{2} \beta}\left[\frac{\sin \left(\beta+\alpha-90^{\circ}\right)}{\sin (\beta+\alpha)}+\frac{\sin \left(90^{0}-\alpha+i\right)}{\sin (\alpha-i)}\right]
\end{gathered}
$$

Substituting the above value of W in Eq. (a)

$$
\begin{gathered}
P_{a}=\frac{1}{2} \frac{H^{2} \gamma \sin (\beta+\alpha)^{2}}{\sin ^{2} \beta} \times \frac{\sin \left(\alpha-\emptyset^{\prime}\right)}{\sin \left(180^{0}-\beta+\delta-\alpha+\emptyset^{\prime}\right)} \mathrm{x} \\
{\left[\frac{\sin \left(\beta+\alpha-90^{0}\right)}{\sin (\beta+\alpha)}+\frac{\sin \left(90^{0}-\alpha+i\right)}{\sin (\alpha-i)}\right]}
\end{gathered}
$$

The active pressure force $P_{a}$ will be a maximum when the failure plane makes an angle $\alpha$ with the horizontal such that $\frac{\partial P_{a}}{\partial \alpha}=0$

Muller-Breslau gave the solution, which is quite complicated. The maximum value of $P_{a}$ thus obtained is coulomb's active force, given by

$$
P_{a}=\frac{1}{2} K_{a} \gamma H^{2}
$$

Where $K_{a}$ is coulomb's active earth pressure coefficient, given by

$$
K_{a}=\frac{\sin ^{2}\left(\beta+\emptyset^{\prime}\right)}{\sin ^{2} \beta \sin (\beta-\delta)\left[1-\sqrt{\frac{\sin \left(\emptyset^{\prime}+\delta\right) \sin \left(\emptyset^{\prime}-i\right)}{\sin (\beta-\delta) \sin (\beta+i)}}\right]^{2}}
$$

The line of action of $P_{a}$ will be at a height of $H / 3$ above the base of the wall and it will be inclined at an angle of $\delta$ to the normal drawn to the back of the wall.

### 3.7.2. Coulomb's Passive Earth Pressure for Cohesionless Soils

The failure surface in coulomb's passive state is assumed to be a plane. Fig. 3.17 shows the case when the passive conditions develop. In this case, the failure wedge moves upwards. The directions of R and $\mathrm{P}_{\mathrm{p}}$ which opposes the movement are also shown. The reaction R acts at $\phi$ ' to the normal in the downward direction and the reaction pressure $\mathrm{P}_{\mathrm{p}}$ acts at an angle $\delta$ to the normal in the downward direction.

The procedure for computing coulomb's passive pressure is similar to one for the active case. However, there is one basic difference. In this case, the critical failure surface is that which gives the minimum value of $\mathrm{P}_{\mathrm{p}}$.

Fig. 3.7 (b) shows the force triangle. The value of $\mathrm{P}_{\mathrm{p}}$ is determined from this triangle. The procedure is repeated after assuming a new trial failure surface. The minimum value of
$\mathrm{P}_{\mathrm{p}}$ is the coulomb passive pressure. Using the procedure similar to that for the active case, it can be shown that the passive pressure is given by


Fig. 3.17

$$
P_{P}=\frac{1}{2} K_{P} \gamma H^{2}
$$

Where $K_{P}$ is coulomb's passive earth pressure coefficient, given by

$$
K_{P}=\frac{\sin ^{2}\left(\beta-\emptyset^{\prime}\right)}{\sin ^{2} \beta \sin (\beta+\delta)\left[1-\sqrt{\frac{\sin \left(\phi^{\prime}+\delta\right) \sin \left(\left(^{\prime}+i\right)\right.}{\sin (\beta+\delta) \sin (\beta+i)}}\right]^{2}}
$$

The resultant passive pressure $P_{p}$ acts at a height of $H / 3$ measured from the bottom of the wall. It would be inclined at an angle to the normal as shown in Fir. 3.17. However, when the retaining wall moves up relative to the soil, the friction angle is measured below the normal and is said to be negative. The negative wall friction produces a value of passive pressure lower than that for the usual positive wall friction.

It is worth noting that the wall friction decreases the active pressure, but it increases the passive pressure. Moreover, the wall friction has a greater influence on the passive pressure than on the active pressure. When exceeds $1 / 3$, coulomb's assumption of plane failure surface is not justified in the passive case. It gives much greater value of Pp compared to that obtained for the actual curved surface. As the passive pressure is generally required to provide the stability to a retaining wall subjected to the active pressure on the other side, the higher value of Pp obtained from the plane failure surface in unconservative or unsafe. For such cases the failure surface should be taken as a logarithmic spiral or a circular.

### 3.8. REHBANN'S CONSTRUCTION FOR ACTIVE PRESSURE

Rehbann (1871) gave a graphical method for the determination of the total active pressure according to coulomb's theory. It is based on poncelet's solution (1840), and is, therefore, also known as Poncelet's method.


Fig. 3.18
Fig. 3.18 shows the graphical construction. The line BD is drawn at an angle $\emptyset^{\prime}$ to the horizontal. The line BL drawn at an angle $\Psi$ with the line BD , is known as the earth pressure line. The angle $\Psi$ is equal to $(\beta-\delta)$.

A semi circle BMD is drawn on BD as diameter. The line AH is drawn parallel to the BL, intersecting the line BD at H . A perpendicular HM is drawn at H , intersecting the semi circle at M.

With B as center and BM as the radius, an arc MF is drawn, intersecting BD at F . The line FE is drawn parallel to BL , intersecting the ground surface at E .

With F as center and FE as radius, an arc is drawn to intersect BD at N . The line BE represents the critical failure plane.

The total pressure $P_{a}$ is given by

$$
\begin{gathered}
P_{a}=\gamma(\text { area of the triangleNEF }) \\
P_{a}=\gamma(1 / 2 \times \mathrm{NF} x)
\end{gathered}
$$

Where $x$ is the perpendicular distance EG between E and BD

## Proof:

The triangle BEF and the force triangle in Fig.3.15 (b) are similar. Therefore,

$$
\begin{align*}
& \frac{P_{a}}{W}=\frac{E F}{B F} \\
& P_{a}=W\left(\frac{E F}{B F}\right) \tag{a}
\end{align*}
$$

From triangle EFG, $\quad \frac{x}{E F}=\sin \Psi$

$$
\begin{equation*}
E F=x \operatorname{cosec} \Psi \tag{b}
\end{equation*}
$$

$$
\begin{gather*}
B F=B D-F D=B D-(G D-G F)=B D-D G+G F \\
B F=B D-x \cot \left(\phi^{\prime}-i\right)+x \cot \Psi \\
B F=L-x \cot \left(\phi^{\prime}-i\right)+x \cot \Psi \ldots \ldots \ldots \ldots \tag{c}
\end{gather*}
$$

Where $L=$ Length of $B D$.

$$
W=\gamma(\text { Volume of triangle } A B E)
$$

Taking Unit length of the wall,

$$
\begin{gather*}
W=\gamma(\text { area of triangle } A B E)=\gamma(\text { area of triangle } A B D-\text { area of triangle } B E D) \\
\mathrm{W}=\gamma\left[\frac{1}{2} \times L \times m-\frac{1}{2} \times L \times x\right]=\frac{1}{2} \gamma L(m-x) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \tag{d}
\end{gather*}
$$

Where $m$ is the perpendicular distance from $A$ to $B D$
From Eq. (a), (b), (c) and (d)

$$
\begin{equation*}
P_{a}=\frac{1}{2} \gamma L(m-x) \times \frac{x \operatorname{cosec} \Psi}{\left[L-x \cot \left(\varnothing^{\prime}-i\right)+x \cot \Psi\right]} . \tag{e}
\end{equation*}
$$

In Eq. (e), substituting

$$
\operatorname{cosec} \Psi=\mathrm{c}
$$

$$
\begin{equation*}
\cot \left(\phi^{\prime}-i\right)-\cot \Psi=\mathrm{d} \tag{f}
\end{equation*}
$$

$$
P_{a}=\frac{1}{2} \gamma L(m-x) \frac{c x}{(L-d x)}
$$

For maximum value of $P_{a}, \frac{\partial P_{a}}{\partial x}=0$

$$
(L-d x)(m c-2 c x)-(m-x) c x(-d)=0
$$

$$
\begin{gathered}
m L-2 x L+d x^{2}=0 \\
m L-x L=x L-d x^{2} \\
\frac{L}{2}(m-x)=\frac{x}{2}(L-d x)
\end{gathered}
$$

Substituting the value of $d$ from Eq.(f) $\frac{L}{2}(m-x)=\frac{x}{2}\left(L-x \cot \left(\emptyset^{\prime}-i\right)+x \cot \Psi\right)$

Using Eq (c)

$$
\frac{m L}{2}-\frac{x L}{2}=\frac{x}{2} \times B F
$$

Area of triangle $A B D-$ Area of triangle $B E D=$ Area of triangle $B E F$

$$
\text { Area of triangle } A B E=\text { Area of triangle } B E F
$$

In other words, for the maximum value of $P_{a}$ the failure plane $B E$ should be such that the triangles $A B E$ and $B E F$ have equal areas.

Let us examine whether Rehbann's construction satisfies the above criterion. The properties of the circle,

$$
B H \times H D=(H M)^{2}
$$

Adding $(B H)^{2}$ to both sides

$$
\begin{gather*}
(B H)^{2}+B H \times H D=(H M)^{2}+(B H)^{2} \\
B H(B H+H D)=(B M)^{2} \\
B H \times B D=(B F)^{2} \\
B H / B F=B F / B D \cdots \cdots \ldots \ldots(i) \tag{i}
\end{gather*}
$$

As AH is parallel to EF , triangles BHJ and BFE are similar.

Therefore,

$$
\begin{equation*}
B H /_{B F}={ }^{B J} /_{B E} . \tag{ii}
\end{equation*}
$$

From (i) and (ii)

$$
B F /{ }_{B D}={ }^{B J} /_{B E}
$$

i.e. $J F$ is parallel to $A D$.

Thus the figure $A J F E$ becomes a parallelogram.

Therefore the perpendicular from point $A$ on the diagonal $J E$ and the perpendicular from point $F$ on the same diagonal would be equal in length. Consequently, the areas of the triangles $A B E$ and $B F E$ which have the same base $B E$ would be equal. It proves that Rehbann's construction satisfies the required criterion.

Expression for total pressure from Rehbann's Construction
From Eq (a), $\quad P_{a}=W \times(E F / B F)$

$$
\begin{aligned}
& =\gamma(\text { area of triangle } A B E) \times E F / B F \\
P_{a} & =\gamma(\text { area of triangle } B F E) \times E F / B F
\end{aligned}
$$

$$
=\gamma\left(\frac{1}{2} \times B F \mathrm{x} x\right) \times E F / B F
$$

$$
\begin{equation*}
P_{a}=\frac{1}{2} \times \gamma \times E F \times x \tag{1}
\end{equation*}
$$

$$
P_{a}=\frac{1}{2} \times \gamma \times N F \times x
$$

Eq.(1) can be written in terms of various angles.
From similar triangles $E F D$ and $A H D$,

$$
\begin{align*}
& \frac{E F}{E D}=\frac{A H}{A D} \\
& (E F)^{2}=\left(\frac{A H \times E D}{A D}\right)^{2} \tag{iii}
\end{align*}
$$

From triangle $H A B$,

$$
\frac{A H}{\sin \left(180^{\circ}-\beta-\emptyset^{\prime}\right)}=\frac{A B}{\sin \Psi}
$$

$$
A H=A B \frac{\sin \left(\beta+\phi^{\prime}\right)}{\sin \Psi}
$$

$$
\frac{B H}{A B}=\frac{\sin \left(\delta+\emptyset^{\prime}\right)}{\sin \Psi}
$$

$$
B H=A B \frac{\sin \left(\delta+\emptyset^{\prime}\right)}{\sin \Psi}
$$

From triangle $A B D, \frac{B D}{A B}=\frac{\sin (\beta+i)}{\sin \left(\phi^{\prime}-i\right)}$

$$
B D=A B \frac{\sin (\beta+i)}{\sin \left(\varnothing^{\prime}-i\right)}
$$

$$
\begin{aligned}
& \frac{A D}{E D}=\frac{A E+E D}{E D}=\frac{A E}{E D}+1=\frac{J F}{E D}+1 \\
& \frac{A D}{E D}=\frac{B F}{B D}+1
\end{aligned}
$$

From Rehbann's construction, $\quad B H \times B D=B F^{2}$

Therefore,

$$
\begin{aligned}
B F & =\sqrt{B H \times B D} \\
\frac{B F}{B D} & =\sqrt{B H / B D}
\end{aligned}
$$

$$
\frac{A D}{E D}=\sqrt{B H / B D}+1
$$

From Eq(iii)

$$
\begin{aligned}
& E F^{2}=(A H)^{2} \times(E D / A D)^{2} \\
& E F^{2}=(A H)^{2} \times \frac{1}{(\sqrt{\mathrm{BH} / \mathrm{BD}}+1)^{2}}
\end{aligned}
$$

Substituting the value of $A H, B H$ and $B D$

$$
E F^{2}=A B^{2}\left[\frac{\sin \left(\beta+\phi^{\prime}\right)}{\sin \Psi}\right]^{2} \times \frac{1}{\left[\sqrt{\frac{\sin \left(\phi^{\prime}+\delta\right)}{\sin \Psi} \cdot \frac{\sin \left(\phi^{\prime}-i\right)}{\sin (\beta+i)}}+1\right]^{2}}
$$

From Eq (1) substituting $A B=H \operatorname{cosec} \beta$ and $x=E F \sin \Psi$

$$
\begin{gathered}
P_{a}=\frac{1}{2} \gamma(H \operatorname{cosec} \beta)^{2} \sin \Psi \mathrm{x}\left[\frac{\sin \left(\beta+\phi^{\prime}\right)}{\sin \Psi}\right]^{2} \times \frac{1}{\left[\sqrt{\frac{\sin \left(\phi^{\prime}+\delta\right) \sin \left(\phi^{\prime}-i\right)}{\sin \Psi \sin (\beta+i)}}+1\right]^{2}} \\
P_{a}=\frac{1}{2} K_{a} \gamma H^{2} \\
K_{a}=\frac{\operatorname{cosec}^{2} \beta \sin ^{2}\left(\beta+\phi^{\prime}\right)}{\sin \Psi\left[\sqrt{\frac{\sin \left(\phi^{\prime}+\delta\right) \sin \left(\phi^{\prime}-i\right)}{\sin \Psi \sin (\beta+\mathrm{i})}}+1\right]^{2}} \\
\beta-\delta, \quad K_{a}=\frac{\sin ^{2}\left(\beta+\phi^{\prime}\right)}{\sin ^{2} \beta \sin (\beta-\delta)\left[1+\sqrt{\frac{\sin \left(\phi^{\prime}+\delta\right) \sin \left(\phi^{\prime}-i\right)}{\sin (\beta-\delta) \sin (\beta+\mathrm{i})}}\right]^{2}}
\end{gathered}
$$

As $\Psi=\beta-\delta$,

### 3.9. CULMANN'S CONSTRUCTION FOR ACTIVE PRESSURE

Rehbann's construction becomes inconvenient when the slope angle i approaches the angle $\varnothing$ '. Culmann (1866) developed a method which is more general than Rehbann's method. It can be used to determine Coulomb's earth pressure for ground surface of any
configuration, for various types of surcharge loads and for layered back fills. Culmann's construction is, in fact, the method of construction of the force triangle in a rotated orientation. The procedure consists of following steps.


Fig. 3.19

1. From point B (Fig. 3.19), a line BD is drawn at an angle $\varnothing$ ' to the horizontal. As the weight of the wedge is plotted along this line, it is also known as the weight line.
2. A line BL is drawn at an angle $\psi=\beta-\delta$, where $\beta$ is the angle which the back face makes with the horizontal and $\delta$ is the angle of friction.
3. A failure surface $B F$ is assumed, and the weight (W) of the failure wedge $A B F$ is computed.
4. The weight $(\mathrm{W})$ of the wedge is plotted along BD such that $\mathrm{BP}=\mathrm{W}$.
5. A line PQ is drawn from point P parallel to BL to intersect the failure surface BF at Q .
6. The length PQ represents the magnitude of $\mathrm{P}_{\mathrm{a}}$ required to maintain equilibrium for the assumed failure plane.
7. Several other failure planes $\mathrm{BF}^{\prime \prime}, \mathrm{BF}^{\prime}, \mathrm{BF}^{\prime \prime}$, etc. are assumed and the procedure repeated. Thus the point Q", Q', Q"", etc. are located.
8. A smooth curve is drawn joining the point Q ", Q ', Q ", etc. the curve is known as culmann's line.
9. A line (shown dotted) is drawn tangential to the culmann line and parallel to BD. Point T is the point of tangency.
10. The magnitude of the largest value ( $\mathrm{P}_{\max }$ ) of $\mathrm{P}_{\mathrm{a}}$ is measured from the tangent point T to the line BD and parallel to BL. It is equal to Coulomb's active pressure $\left(\mathrm{P}_{\mathrm{a}}\right)$.
11. The actual failure plane passes through the point T (shown dotted).

## Effect of Uniform Surcharge or Line Load

Culmann's method can be easily extended to include the effect of uniform surcharge or the line load applied to the backfill. Each such loads that falls within the assumed failure wedge is included by adding it to the weight of the failure wedge.


Fig. 3.20
Fig. 3.20 shows the modification when a line load q' acts on the backfill. The Culmann line upto point Q is similar to the case when there is no line load. However, at point Q , it becomes discontinuous, as suddenly the load of the wedge is increased due to line load $q^{\prime}$. in this case, tangents are drawn to the two segments of the curve, and the value of $\mathrm{P}_{\mathrm{a}}$ is determine. The value of $\left(\mathrm{P}_{\mathrm{a}}+\Delta \mathrm{P}_{\mathrm{a}}\right)$ is measured from the line BD to the maximum point on the Culmann line that includes the line load.

### 3.10. INTRODUCTION OF EARTH-RETAINING STRUCTURES

Structures that are built to retain vertical or nearly vertical earth banks or any other material are called retaining walls. Retaining walls may be constructed of masonry or sheet piles. Some of the purposes for which retaining walls are used are shown in Fig. 3.21. Retaining walls may retain water also. The earth retained may be natural soil or fill.


Fig. 3.21 Use of Retaining walls

### 3.11. TYPES OF EARTH-RETAINING STRUCTURES

Earth-retaining structures may be broadly classified as retaining walls and sheet pile walls. Retaining walls may be further classified as:
(i) Gravity retaining walls -usually of masonry or mass concrete.
(ii) Cantilever walls
(iii) Counterfort walls usually of reinforced concrete.
(iv) Buttress walls

Sheet pile walls may be further classified as cantilever sheet pile walls and anchored sheet pile walls, also called 'bulkheads'.

Gravity walls depend on their weight for stability; walls up to 2 m height are invariably of this type. The other types of retaining walls, as well as sheet-pile walls, are known as 'flexible walls'. All these are shown in Fig. 3.22.
R.C. Cantilever walls have a vertical or inclined stem monolithic with a base slab. These are considered suitable up to a height of 7.5 m . A vertical or inclined stem is used in counterfort walls, supported by the base slab as well as by counterforts with which it is monolithic.

Cantilever sheet pile walls are held in the ground by the passive resistance of the soil both in front of and behind them. Anchored sheet pile wall or bulkhead is fixed at its base
as a cantilever wall but supported by tie-rods near the top, sometimes using two rows of ties and properly anchored to a deadman.


Fig. 3.22 Type of Earth retaining structure

### 3.12. PRINCIPLES OF THE DESIGN OF RETAINING WALLS

Before the actual design, the soil parameters that influence the earth pressure and the bearing capacity of the soil must be evaluated. These include the unite weight of the soil, the angle of shearing resistance, the cohesive intercept and the angle of wall friction. Knowing these parameters, the lateral earth pressure and the bearing capacity of the soil can be
determined. With the earth pressure known, the retaining wall as a whole is checked for stability.


Fig. 3.23
Fig. 3.23 shows a retaining wall with a smooth back face and no surcharge. The active pressure $\mathrm{P}_{\mathrm{a}}$ acts horizontally, as shown. The front face of the wall is subjected to a passive pressure ( $\mathrm{P}_{\mathrm{p}}$ ) below the soil surface. However, it is doubtful whether the full passive resistance would develop. Moreover, often $\mathrm{P}_{\mathrm{p}}$ is small and therefore it may be neglected. This gives more conservative design.

The weight W of the wall and the active pressure Pa have their resultant R which strikes the base at point $D$. there is an equal and opposite reaction $\mathrm{R}^{\prime}$ at the base between the wall and the foundation. For convenience, R' is resolved into the vertical and horizontal components ( $\mathrm{R}_{\mathrm{v}}{ }^{\prime}$ and $\mathrm{R}_{\mathrm{H}}{ }^{\prime}$ ).

From the equilibrium of the system,

$$
R_{V}^{\prime}=W, \text { and } \quad R_{H}^{\prime}=P_{a}
$$

The third equation of equilibrium, namely the moment equation, is used to determine the eccentricity e of the force $R_{V}^{\prime}$ relative to the centre C of the base of the wall. Obviously, by taking moments about the toe,

$$
\begin{gathered}
R_{V}^{\prime} \times \bar{x}=W \times a-P_{a}(H / 3) \\
\bar{x}=\frac{W \times a-P_{a}(H / 3)}{R_{V}^{\prime}}
\end{gathered}
$$

Where $\bar{x}$ is the distance of the point D from the toe.
Thus, eccentricity,

$$
e=b / 2-\bar{x}
$$

Where $\mathrm{b}=$ width of the base.
For a safe design, the following requirements must be satisfied.
(1) No Sliding :

The wall must be safe against sliding. In other words,

$$
\mu R_{V}>R_{H}
$$

Where $R_{V}$ and $R_{H}$ are vertical and horizontal components of R , respectively. The factor of safety against sliding is given by

$$
F_{s}=\frac{\mu R_{V}}{R_{H}}
$$

Where $\mu=$ coefficient of friction between the base of the wall and the soil $(=\tan \delta)$.
A minimum factor of safety of 1.5 against sliding is generally recommended.

## (2) No Overturning :

The wall must be safe against overturning about toe. The factor of safety against overturning is given by

$$
F_{O}=\frac{\sum M_{R}}{\sum M_{O}}
$$

Where $\sum M_{R}=$ sum of resisting moment about toe, and $\sum M_{O}=$ sum of overturning moment about toe.

In Fig. 3.23,

$$
F_{O}=\frac{W \times a}{P_{a} \times H / 3}
$$

The factor of safety against overturning is usually kept between 1.5 to 2.0 .

## (3) No Bearing Capacity Failure :

The pressure caused by $\mathrm{Rv}_{\mathrm{v}}$ at the toe of the wall must not exceed the allowable bearing capacity of the soil.

The pressure distribution at the base is assumed to be linear. The maximum pressure is given by

$$
P_{\max }=\frac{R_{V}}{b}\left(1+\frac{6 e}{b}\right)
$$

The factor of safety against bearing failure is given by

$$
F_{b}=\frac{q_{n a}}{P_{\max }}
$$

Where $q_{n a}=$ allowable bearing pressure.
A factor of safety of 3 is usually specified, provided the settlement is also within the allowable limit.

## (4) No Tension :

There should be no tension at the base of the wall. When the eccentricity (e) is greater than $\mathrm{b} / 6$, tension develops at the heel. Tension is not desirable. The tensile strength of the soil is very small and the tensile crack would develop. The effective base area is reduced. In such a case, the maximum stress is given by

$$
P_{\max }=\frac{4}{3}\left(\frac{R_{V}}{b-2 e}\right)
$$

### 3.13. Gravity retaining wall

As in design of all other structures a trial section is first chosen and analyzed. If the stability yields unsatisfactory results the section is changed and rechecked. Fig. 3.24 shows the general properties of a gravity retaining wall of overall height H . The top width of the stem should be at least 0.3 m for proper placement of concrete in the stem. The depth (D) of the foundation below the soil surface should be at least 0.6 m . The base width of the wall is generally between 0.5 H to 0.7 H ; with an average of $2 \mathrm{H} / 3$.


Fig. 3.24
The earth pressure can be computed using either Rankine's theory or Coulomb's theory. For using Rankine's theory, a vertical line $A B$ is drawn through the heel point $A$. It is assumed that the Rankine active conditions exist along the vertical line $A B$. However, the assumption for the development of Rankine's conditions along $A B$ is theoretically justified only if the shear zone bounded by the line $A C$ is not obtained by the stem of the wall, where $A C$ makes an angle $\eta$ with the vertical, given by

$$
\eta=\left(45^{0}+i / 2\right)-\emptyset^{\prime} / 2-\sin ^{-1}\left(\frac{\sin i}{\sin \emptyset^{\prime}}\right)
$$

Where $i$ is the angle of surcharge

The angle $\alpha$ which the line $A C$ makes with the horizontal is given by

$$
\alpha=\left(45^{0}+\frac{\phi^{\prime}}{2}\right)-\frac{i}{2}+\sin ^{-1}\left(\frac{\sin i}{\sin \emptyset^{\prime}}\right)
$$

When $\mathrm{i}=0$ the value of $\alpha$ is equal to $\left(45^{0}+\frac{\phi \prime}{2}\right)$ Fig. 3.25


Fig. 3.25
While checking the stability, the weight of the soil (Ws) above the heel in the zone $A B C$ should also be taken into consideration, in addition to the earth pressure $\left(P_{a}\right)$ on the vertical plane $A B$ and the weight of the wall $\left(W_{e}\right)$.


Fig. 3.26
Coulomb's theory can also be used for the determination of earth pressure Fig. 3.26. As the coulomb theory gives directly the lateral pressure on the back face $\left(P_{a}\right)$, the forces to be considered are only $P_{a}$ (Coulomb) and the weight of the wall $\left(W_{e}\right)$. In this case, the weight of soil $\left(W_{s}\right)$ is not to be considered separately.

Once the forces acting on the wall have been determined, the stability is checked using the procedure discussed in principles of retaining walls.

For convenience the action of the retaining wall is divided into rectangles and triangles for the computation of weight and the determination of the line of action of the weights.

Semi gravity retaining walls: The base width of the semi gravity retaining walls is slightly smaller than that of a corresponding gravity wall. The rest of the design procedure is the same as that for gravity retaining walls.

### 3.14. CANTILEVER RETAINING WALLS

Fig. 3.27 shows a cantilever retaining wall. The general proportions for an overall height of H are also shown. The top width of the stem is at least 0.3 m . The width of the base slab is kept about $2 H / 3$. The width of the stem at bottom, the thickness of the slab and the length of the projection, each is kept about 0.1 H .


Fig. 3.27

The earth pressure is computed using Rankine's theory on the vertical plane $A B$, provided the shear zone bounded by the line $A C$ is not obstructed by the stem of the wall. The line $A C$ makes an angle $\eta$ with the vertical given by eq above.


Fig. 3.28

Fig. 3.28 shows the forces acting on the wall. The rankine pressure $P_{a}$ acts at an angle i with the horizontal. It is resolved into the vertical and horizontal components $P_{v}$ and $P_{h}$ as shown. The passive pressure $P_{p}$ is also shown, but generally it is neglected. For convenience, the weight of soil $\left(W_{s}\right)$ over the slab is divided into two parts (1) and (2). Likewise, the weight of soil $\left(W_{s}\right)$ over the slab is divided into two parts (1) and (2). Likewise, the weight of the stem is divided into two parts (3) and (4).
(a) Factor of safety against sliding

The factor of safety against sliding may be expressed as

$$
F_{s}=\frac{\sum F_{R}}{\sum F_{d}}
$$

Where $\sum F_{R}=$ sum of the horizontal resisting forces.
$\sum F_{d}=$ sum of the horizontal driving forces.
The above Eq. can be written as

$$
F_{s}=\frac{\left(\sum V\right) \tan \emptyset_{2}+b c_{2}+P_{p}}{P_{h}}
$$

Where $b=$ base width, $\sum V=$ sum of all the vertical forces, $W_{c}, W_{s}$ and $P_{v}$,

$$
\begin{gathered}
P_{v}=P_{a} \sin i \text { and } \quad P_{h}=P_{a} \cos i \\
P_{p}=\text { Passive force in front of the wall } \\
=1 / 2 K_{p 2} \gamma_{2} D^{2}+2 c_{2} \sqrt{K_{p 2}} D
\end{gathered}
$$

Where $\mathrm{c}_{2}, \gamma_{2}$ and $\phi_{2}$ are parameters of the foundation soil.
If the required factor of safety of 1.5 against sliding is not obtained, a base key is generally provided. The key increases the passive resistance to $P_{p}$ ' where

$$
P_{p}^{\prime}=\frac{1}{2} \gamma_{2}\left(D_{1}\right)^{2} K_{p 2}+2 c_{2} D_{1} \sqrt{K_{p 2}}
$$

Where $D_{1}=$ the depth of the bottom of the key wall from soil surface
Generally the base key is constructed just below the stem and some of the main steel of the stem is extended from key.

The friction angle $\phi_{2}$ and $c_{2}$ are generally reduced to about one-half to two thirds of the values for extra safety, as the fall passive resistance is doubtful.
(b) Factor of safety against overturning:

Obtain the factor of safety against overturning,

$$
F_{O}=\frac{\sum M_{R}}{\sum M_{O}}
$$

Where $\sum M_{R}=$ sum of the resisting moments about toe,
$\sum M_{O}=$ sum of the overturning moments about toe.
The only overturning force is $P_{h}$ acting at a height of $H / 3$

$$
M_{O}=P_{h} \times H / 3
$$

The resisting moments $\left(\mathrm{M}_{\mathrm{O}}\right)$ are due to the weights $\mathrm{W}_{1}, \mathrm{~W}_{2}, \mathrm{~W}_{3}, \mathrm{~W}_{4}$ and $\mathrm{W}_{5}$ of the soil and the concrete. The vertical components of pressure $P_{v}$ also helps in resisting moment. Its resisting moment is given by

$$
\begin{gathered}
M_{v}=P_{v} \times b \\
F_{o}=\frac{M_{1}+M_{2}+M_{3}+M_{4}+M_{5}}{P_{h} \times H / 3}
\end{gathered}
$$

Where $M_{1}, M_{2}, \ldots \ldots$ are the moments due to the weights $\mathrm{W}_{1}, \mathrm{~W}_{2} \ldots$. about toe
(c) Factor of safety against bearing capacity failure

The sum of the vertical forces acting on the base is equal to $\sum V$. The horizontal force is $P_{h}$. The resultant force ( $R$ ) is given by

$$
R=\sqrt{\left(\sum V\right)^{2}+\left(P_{h}\right)^{2}}
$$

The net moment of those forces about toe B is given by

$$
\sum M=\sum M_{R}-\sum M_{O}
$$

The distance $x$ ' of the point E , from the toe, where R strikes the base is given by

$$
x^{\prime}=\frac{\sum M}{\sum V}
$$

Hence the eccentricity $e$ of $R$ is given by

$$
e=b / 2-x^{\prime}
$$

If $\mathrm{e}>\mathrm{b} / 6$, the section should be changed, as it indicates tension. The pressure distribution under the base slab is determined as

$$
\begin{aligned}
& P_{\max }=\frac{\sum V}{b}\left(1+\frac{6 e}{b}\right) \\
& P_{\min }=\frac{\sum V}{b}\left(1-\frac{6 e}{b}\right)
\end{aligned}
$$

The factor of safety against bearing capacity failure is given by $F_{b}=\frac{q_{n a}}{p_{\max }}$

## Assignment-Cum-Tutorial Questions

## A. Questions testing the remembering / understanding level of students

## I) Objective Questions

1. Coulomb's theory is applicable for
a) Homogenous soils only
b) Non Homogeneous soils also
c) Smooth retaining walls only
d) Soils which have angle of internal friction only
2. In passive case, the walls moves
a) Towards the backfill
b) away from the backfill
c) No movement at all
d) downwards
3. The state of shear failure accompanying a minimum earth pressure is called the
a) At rest state
b) Active state
c) Passive state
d) None of the above
4. The lateral earth pressure coefficient $K_{a}$ and $K_{p}$ are based on
a) Total stress
b) Neutral stress
c) Effective stress
d) Elastic properties of the soils
5. The coefficient of active earth pressure, $K_{a}$ is
a) $\tan ^{2}(45+\phi / 2)$
b) $1 / \tan ^{2}(45+\phi / 2)$
c) $(1+\sin \phi) /(1-\sin \phi)$
d) None
6. Culmann's construction is more general than
7. The base of the walls are generally designed for $\qquad$

## II) Descriptive Questions

1. Define earth pressure at rest. Show the earth pressure distribution on a retaining wall, assuming the soil is dry?
2. What are the assumptions of Rankine's theory? Derive the expressions for active pressure and passive pressure.
3. Derive an expression for active pressure when the ground surface is inclined?
4. What are the assumptions in Coulomb's theory? Compare Rankine's theory and Coulomb's theory.
5. Describe Rehbann's construction. What is its use?
6. Discuss culmann's method for the determination of active earth pressure.
7. What are different types of retaining walls?
8. Discuss the principles of the design of retaining walls.
9. Determine the active pressure on the wall shown in Fig. 3.1 using Rankine's theory.


Fig. 3.1
10. Determine the stresses at the top and bottom of the cut shown in Fig. 3.2. Also determine the maximum depth of potential crack and the maximum depth of unsupported excavation.


Fig. 3.2

## B. Question testing the ability of students in applying the concepts.

## I) Multiple Choice Questions:

1. As per Rankine's theory, the wall friction is
a) Zero
b) Equal to $\phi$
c) Greater than $\phi$
d) less than $\phi$
2. The lateral earth pressure coefficient of a soil, Ka for active state, Kp for passive state and K 0 for at rest condition, compare as
a) $\mathrm{K}_{0}<\mathrm{K}_{\mathrm{a}}<\mathrm{K}_{\mathrm{p}}$
b) $\mathrm{K}_{\mathrm{a}}<\mathrm{K}_{0}<\mathrm{K}_{\mathrm{p}}$
c) $K_{a}<K_{p}<K_{0}$
d) $\mathrm{K}_{\mathrm{p}}<\mathrm{K}_{0}<\mathrm{K}_{\mathrm{a}}$
3. The active pressure caused by a cohesionless backfill on a smooth vertical retaining wall be reduced by
a) Compacting the backfill
b) providing a surcharge load on backfill
c) Saturating the backfill with water
d) None of the above
4. The yield of a retaining wall required to reach plastic equilibrium in active case is
a) More than that in the passive case
b) less than that in the passive case
c) Equal to that in the passive case
d) None of the above
5. The minimum allowable factor of safety against sliding in case of a cantilever retaining wall is
a) 2.0
b) 3.0
c) 1.5
d) 2.5
6. The Rankine pressure is always $\qquad$ to the wall surface
7. The active pressure, which develops when the wall $\qquad$ from the fill

## II) Problems

1. A gravity retaining wall is shown in Fig.1. Calculate the factor of safety with respect to overturning and sliding given the fallowing data: Wall dimensions: $\mathrm{H}=6 \mathrm{~m}, \mathrm{x}_{1}=$ $0.6 \mathrm{~m}, \mathrm{x}_{2}=2 \mathrm{~m}, \mathrm{x}_{3}=2 \mathrm{~m}, \mathrm{x}_{4}=0.5 \mathrm{~m}, \mathrm{x}_{5}=0.75 \mathrm{~m}, \mathrm{x}_{6}=0.8 \mathrm{~m}, \mathrm{D}=1.5 \mathrm{~m}$. Soil properties: $\gamma_{1}=16.5 \mathrm{kN} / \mathrm{m}^{3}, \Phi_{1}{ }^{\prime}=32^{0}, \gamma_{2}=18 \mathrm{kN} / \mathrm{m}^{3}, \Phi_{2}{ }^{\prime}=22^{0}, \mathrm{c}_{2}{ }^{\prime}=40 \mathrm{kPa}$. Use the Rankine active earth pressure in your calculations.

2. For the cantilever retaining wall shown in Fig.2, let the fallowing data be given: Wall dimensions: $\mathrm{H}=5.4 \mathrm{~m}, \mathrm{x}_{1}=0.45 \mathrm{~m}, \mathrm{x}_{2}=0.75 \mathrm{~m}, \mathrm{x}_{3}=1.20 \mathrm{~m}, \mathrm{x}_{4}=1.80 \mathrm{~m}, \mathrm{x}_{5}=$ $0.85 \mathrm{~m} D=1.20 \mathrm{~m}, \alpha=10^{0}$. Soil properties: $\gamma_{1}=18.5 \mathrm{kN} / \mathrm{m}^{3}, \Phi_{1}{ }^{\prime}=34^{0}, \gamma_{2}=17.8$ $\mathrm{kN} / \mathrm{m}^{3}, \Phi_{2}{ }^{\prime}=18^{0}, \mathrm{c}_{2}{ }^{\prime}=50 \mathrm{kPa}$.

3. For the cantilever retaining wall shown in Fig.2, let the fallowing data be given: Wall dimensions: $\mathrm{H}=6.5 \mathrm{~m}, \mathrm{x}_{1}=0.50 \mathrm{~m}, \mathrm{x}_{2}=1.2 \mathrm{~m}, \mathrm{x}_{3}=1.35 \mathrm{~m}, \mathrm{x}_{4}=2.1 \mathrm{~m}, \mathrm{x}_{5}=1.0 \mathrm{~m}$, $\mathrm{D}=2.0 \mathrm{~m}, \alpha=15^{0}$. Soil properties: $\gamma_{1}=19.5 \mathrm{kN} / \mathrm{m}^{3}, \Phi_{1}{ }^{\prime}=36^{0}, \gamma_{2}=17.0 \mathrm{kN} / \mathrm{m}^{3}, \Phi_{2}{ }^{\prime}=$ $10^{0}, \mathrm{c}_{2}{ }^{\prime}=50 \mathrm{kPa}$. Calculate the factor of safety with respect to overturning, sliding and bearing capacity.
4.Check the stability of the gravity retaining wall shown in Fig. 3. Take allowable soil pressure equal to $600 \mathrm{kN} / \mathrm{m}^{2}$. Use Coulomb's theory


Fig. 3
5. Check the stability of the cantilever retaining wall shown in Fig.4. The allowable soil pressure is $500 \mathrm{kN} / \mathrm{m}^{2}, \varnothing^{\prime}=34^{0}, \delta=25^{0}, \gamma=18 \mathrm{kN} / \mathrm{m}^{3}, \mathrm{i}=15^{0}$.


Fig. 4
6. Check the stability of a cantilever concrete retaining wall having a stem thickness of 0.4 m uniform throughout, 6.0 m height bed block thickness 0.8 m and a projection of 2.5 m on the heel side and 1.5 m on the toe side. The unit weight of the wall material is $25 \mathrm{kN} / \mathrm{m}^{3}$. The soil has a unit weight of $18 \mathrm{kN} / \mathrm{m}^{3}$ and an angle of internal friction of $36^{0}$. Take in to account a uniform surcharge on the ground of $50 \mathrm{kN} / \mathrm{m}^{2}$. The ground level on the toe side is 1.2 m high above the base of the wall.
7. Determine the active pressure on the retaining wall shown in Fig. 5


Fig. 5
8. Determine the active thrust on the retaining wall shown in Fig. 6 using Culmann's method.


Fig. 6
9. A 5 m height retaining wall is shown in Fig. 7. Determine the Rankine active pressure on the wall. (a) Before the formation of the crack. (b) After the formation of the crack.


Fig. 7
10. Determine the coulomb active force on the retaining wall shown in Fig. 8


Fig. 8

## UNIT -IV

## Objective:

To understand load carrying capacity of shallow foundations

## Syllabus: SHALLOW FOUNDATIONS AND SETTLEMENT CRITERIA

Bearing capacity -criteria for determination of bearing capacity-factors influencing bearing capacity- analytical methods to determine bearing capacity- Terzaghi, Meyerhof, Skempton and IS Methods. Safe bearing pressure based on N- value

## Learning Outcomes:

After completion of this unit the student will be able to

- Evaluate the bearing capacity of soil by different methods


## Learning Material

### 4.1 Basic Definitions

1. Ultimate Bering Capacity $\left(\mathbf{q}_{\mathbf{u}}\right)$ : The ultimate bearing capacity is the gross pressure at the base of the foundation at which the soil fails in shear.
2. Net Ultimate Bearing Capacity ( $\mathbf{q}_{\mathbf{n u}}$ ): It is the net increase in pressure at the base of foundation that causes shear failure of the soil. It is equal to the gross pressure minus overburden pressure.

$$
q_{n u}=q_{u}-\gamma D_{f}
$$

where $q_{u}=$ ultimate bearing capacity
$\gamma=$ unit weight of foundation soil
and $D_{f}=$ depth of foundation.
It may be noted that the overburden pressure equal to $\gamma D_{f}$ existed even before the construction of foundation.
3. Net Safe Bearing Capacity ( $\mathbf{q}_{\mathrm{ns}}$ ) : It is the net soil pressure which can be safely applied to the soil considering only shear failure. It is obtained by dividing the net ultimate bearing capacity by a suitable factor of safety. Thus

$$
q_{n s}=\frac{q_{n u}}{F}
$$

Where $\mathrm{F}=$ factor of safety, which is usually taken as 3.0.
4. Gross Safe Bearing Capacity $\left(\mathbf{q}_{s}\right)$ : It is the maximum gross pressure which the soil can carry safely without shear failure. It is equal to the net safe bearing capacity plus the original overburden pressure. Thus
or

$$
\begin{aligned}
& q_{s}=q_{n s}+\gamma D_{f} \\
& \quad q_{s}=\frac{q_{n u}}{F}+\gamma D_{f}
\end{aligned}
$$

5. Net Safe Settlement Pressure ( $\mathbf{q}_{\mathbf{n} \boldsymbol{p}}$ ): It is the net pressure which the soil can carry without exceeding the allowable settlement. The maximum allowable settlement generally varies between 25 mm and 40 mm for individual footings.

The net safe settlement pressure is also known as unit soil pressure or safe bearing pressure.
6. Net Allowable Bearing Pressure ( $\mathbf{q}_{\mathbf{n} \mathbf{a}}$ ): The net allowable bearing pressure is the net bearing pressure which can be used for the design of foundations.

As the requirements for the design of foundation are that there should be no shearing failure and moreover the settlement should also be within the limits, the allowable bearing pressure is the smaller of the net safe bearing capacity ( $\mathrm{q}_{\mathrm{ns}}$ ) and the net safe settlement pressure ( $\mathrm{q}_{\mathrm{np}}$ ). Thus

$$
\begin{aligned}
q_{n a} & =q_{n s} & \text { if } q_{n \rho}>q_{n s} \\
q_{n a} & =q_{n \rho} & \text { if } q_{n s}>q_{n \rho}
\end{aligned}
$$

The net allowable bearing pressure is also known as the allowable soil pressure or allowable bearing pressure or allowable bearing capacity.

### 4.2 TYPES OF SHEAR FAILURE

Vesic (1973) classified the bearing capacity failures into 3 categories

1. General shear failure: Fig. 4.1(a) shows a strip footing resting on the surface of a dense sand or a stiff clay. The figure also shows the load settlement curve for the
footing, where q is the load per unit area and s is the settlement. At a certain load intensity equal to $q_{u}$, the settlement increases suddenly. A shear failure occurs in the soil at that load and the failure surfaces extend to the ground surface. This type of failure is known as general shear failure. A heave on the sides is always observed in general shear failure.
2. Local shear failure: Fig. 4.1(b) shows a strip footing resting on a medium dense sand or on a clay of medium consistency. The figure also shows the load settlement curve. When the load is equal to a certain value $\mathrm{q}_{\mathrm{u}(1)}$, the foundation movement is accompanied by sudden jerks. The failure surfaces gradually extend outwards from the foundation, as shown. However, a considerable movement of the foundation is required for the failure surfaces to extend to the ground surface. The load at which this happens is equal to qu. Beyond this point, an increase of load is accompanied by a large increase in settlement. This type of failure is known as local shear failure. A heave is observed only when there is substantial vertical settlement.
3. Punching shear failure: Fig. 4.1(c) shows a strip footing resting on a loose sand or a soft clay. In this case, the failure surfaces do not extend up to the ground surface. There are jerks in foundation at a load of $q_{u(1)}$. The footing fails at a load of $q_{u}$ at which stage the load - settlement curve becomes steep and practically linear. This type of failure is called the punching shear failure. No heave is observed. There is only vertical movement of the footing.

(b) LOCAL SHEAR FAILURE

(c) PUNCHING SHEAR FAILURE


Fig. 4.1 types of shear failures

### 4.3 TERZAGHI'S BEARING CAPACITY THEORY:

Terzaghi (1943) gave a general theory for the bearing capacity of soils under a strip footing, making the following assumptions.

1. The base of footing is rough.
2. The footing is laid at a shallow depth, i.e. $D_{f} \leq B$
3. The shear strength of the soil above the base of the footing is neglected. The soil above the base is replaced by a uniform surcharge $\gamma D_{f}$.
4. The load on the footing is vertical and is uniformly distributed.
5. The footing is long i.e., $L / B$ ratio is infinite, where $B$ is width and $L$ is the length of the footing.
6. The shear strength of the soil is governed by the Mohr - Coulomb equation.

### 4.3.1 Derivation of Equation:

As the base of the footing is rough, the soil in the wedge ABC immediately beneath the footing is prevented from undergoing any lateral yield (Fig. 4.2). The soil in this wedge (Zone I) remains in a state of elastic equilibrium. It behaves as if it were a part of the footing itself. It is assumed that the angles CAB and CBA are equal to the angle of shearing resistance $\emptyset^{\prime}$ of the soil.


Fig. 4.2 Terzaghi's Analysis

The sloping edges AC and BC of the soil wedge CBA bear against the radial shear zones CBD and CAF (Zone II). The curves CD and CF are arcs of a logarithmic spiral.

Two triangular zones BDE and AFG are the Rankine passive zones (Zones III). An overburden pressure $q=\gamma D_{f}$ acts as a surcharge on the Rankine's passive zones.

The failure zones do not exceed above the horizontal planes passing through the base AB of the footing. In other words, the shearing resistance of the soil located above the base of the footing is neglected, and the effect of the soil is taken equivalent to a surcharge of $\gamma D_{f}$. Because of the assumption, Terzaghi's theory is valid only for shallow foundations ( $D_{f} \leq$ $B)$, in which the term $\gamma D_{f}$ is relatively small.

The loading conditions are similar to that on a retaining wall under passive pressure case. The failure occurs when the downward pressure exerted by loads on the soil adjoining the inclined surfaces CB and CA of the soil wedge is equal to the upward pressure. The downward forces are due to the load $\left(q_{u} \times B\right)$ and the weight of the wedge $\left(1 / 4 \gamma B^{2} \tan \emptyset^{\prime}\right)$. The upward forces are the vertical component of the resultant passive pressure $\left(P_{P}\right)$ and the cohesion ( $c^{\prime}$ ) acting along the inclined surfaces. As the resultant passive pressure is inclined at an angle $\emptyset^{\prime}$ to the normal to the surface of the wedge, it is vertical. Therefore, from the equilibrium equation in the vertical direction

$$
\frac{1}{4} \gamma B^{2} \tan \emptyset^{\prime}+q_{u} \mathrm{x} B=2 P_{P}+2 c^{\prime} \times L_{i} \sin \emptyset^{\prime}
$$

Where $L_{i}=$ length of the inclined surface $\mathrm{CB}\left[=(B / 2) / \cos \emptyset^{\prime}\right]$, therefore

$$
q_{u} \times B=2 P_{P}+B c^{\prime} \tan \emptyset^{\prime}-\frac{1}{4} \gamma B^{2} \tan \emptyset^{\prime}
$$

The resultant passive pressure $\left(P_{P}\right)$ on the surface CB and CA constitutes the following 3 components.

1. Component $\left(P_{P}\right)_{\gamma}$ which is produced by the weight of the shear zone BCDE, assuming the soil as cohesionless ( $\mathrm{c}^{\prime}=0$ ) and neglecting the surcharge q .
2. Component $\left(P_{P}\right)_{c}$ which is produced by the component c ' of the soil, assuming the soil as weightless ( $\gamma=0$ ) and neglecting the surcharge q .
3. Component $\left(P_{P}\right)_{q}$ which is produced by surcharge (q), assuming the soil as cohesionless and weightless ( $c^{\prime}=0, \gamma=0$ ).

The three components $\left(P_{P}\right)_{\gamma},\left(P_{P}\right)_{c}$ and $\left(P_{P}\right)_{q}$ are obtained assuming different surfaces of failures, although their respective failure surfaces are different from the actual failure surface developed for a footing on a soil possessing weight and cohesion and also having a surcharge, the results can be superimposed without introducing much error. Thus, the resultant passive pressure $P_{P}$ is taken equal to the sum of the components $\left(P_{P}\right)_{\gamma},\left(P_{P}\right)_{c}$ and $\left(P_{P}\right)_{q}$.

$$
q_{u} B=2\left[\left(P_{P}\right)_{\gamma}+\left(P_{P}\right)_{c}+\left(P_{P}\right)_{q}\right]+B c^{\prime} \tan \emptyset^{\prime}-\frac{1}{4} \gamma B^{2} \tan \varnothing^{\prime}
$$

Substituting

$$
\begin{gathered}
2\left(P_{P}\right)_{\gamma}-\frac{1}{4} \gamma B^{2} \tan \emptyset^{\prime}=B \times \frac{1}{2} \gamma B N_{\gamma} \\
2\left(P_{P}\right)_{c}+B c^{\prime} \tan \phi^{\prime}=B \times c^{\prime} N_{c} \\
2\left(P_{P}\right)_{q}=B \times \gamma D_{f} N_{q} \\
q_{u} B=B \times c^{\prime} N_{c}+B \times \gamma D_{f} N_{q}+B \times \frac{1}{2} \gamma B N_{\gamma} \\
q_{u}=c N_{c}+\gamma D_{f} N_{q}+\frac{1}{2} \gamma B N_{\gamma} \\
q_{u}=c N_{c}+q_{0} N_{q}+\frac{1}{2} \gamma B N_{\gamma} \quad\left[\text { Since } q_{0}=\text { overburden pressure }=\gamma D_{f}\right]
\end{gathered}
$$

Above Equation is known as Terzaghi's bearing capacity equation. The bearing capacity factors $N_{c}, N_{q}$ and $N_{\gamma}$ are dimensionless numbers, depending upon the angle of shearing resistance ( $\varnothing^{\prime}$ ) of the soil. These are defined by the following equations:

$$
\begin{aligned}
N_{c} & =\cot \phi^{\prime}\left[\frac{a^{2}}{2 \cos ^{2}\left(45+\frac{\phi^{\prime}}{2}\right)}-1\right] \\
N_{q} & =\left[\frac{a^{2}}{2 \cos ^{2}\left(45+\frac{\phi^{\prime}}{2}\right)}\right] \\
a & =e^{\left(3 \pi / 4-\phi^{\prime} / 2\right) \tan \phi^{\prime}} \\
N_{\gamma} & =\frac{1}{2}\left[\frac{K_{p}}{\cos ^{2} \phi^{\prime}}-1\right] \tan \phi^{\prime}
\end{aligned}
$$

Where $K_{p}=$ coefficient of passive earth pressure
Table. 4.1 Terzaghi's Bearing Capacity Factors

| $\varnothing^{\prime}$ | General Shear Failure |  |  | Local Shear Failure |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $N_{c}$ | $N_{q}$ | $N_{\gamma}$ | $N_{c}$ | $N_{q}$ | $N_{\gamma}$ |
| 0 | 5.7 | 1.0 | 0.0 | 5.7 | 1.0 | 0.0 |
| 5 | 7.3 | 1.6 | 0.5 | 6.7 | 1.4 | 0.2 |
| 10 | 9.6 | 2.7 | 1.2 | 8.0 | 1.9 | 0.5 |
| 15 | 12.9 | 4.4 | 2.5 | 9.7 | 2.7 | 0.9 |
| 20 | 17.7 | 7.4 | 5.0 | 11.8 | 3.9 | 1.7 |
| 25 | 25.1 | 12.7 | 9.7 | 14.8 | 5.6 | 3.2 |
| 30 | 37.2 | 22.5 | 19.7 | 19.0 | 8.3 | 5.7 |
| 35 | 57.8 | 41.4 | 42.4 | 25.2 | 12.6 | 10.1 |
| 40 | 95.7 | 81.3 | 100.4 | 34.9 | 20.5 | 18.8 |
| 45 | 172.3 | 173.3 | 297.5 | 51.2 | 35.1 | 37.7 |
| 50 | 347.5 | 415.1 | 1153.2 | 81.3 | 65.6 | 87.1 |

4.3.2 Effect of water table on bearing capacity:

The Ultimate bearing capacity has been developed based on the assumption that the water table is located at a greater depth. If the water table is located close to the foundation, the bearing capacity equation needs modified as below.


Fig. 4.3

## Case1: Water table located above the base of footing (Fig. 4.3 (a))

The effective surcharge is reduced as the effective weight below the water table is equal to the submerged unit. Therefore $q=D_{w} \gamma+a \gamma^{\prime}$

Where $D_{w}=$ depth of the water table below the ground surface $\mathrm{a}=$ height of the water table above the base of the footing

Alternatively, the above equation can be written as, substituting a $=D_{f}-D_{w}$

$$
q=D_{w} \gamma+\left(\mathrm{D}_{\mathrm{f}}-\mathrm{D}_{\mathrm{w}}\right) \gamma^{\prime}
$$

Moreover, the unit weight in the third term of the bearing capacity equation is equal to the submerged unit weight. Thus bearing capacity equation becomes

$$
q_{u}=c^{\prime} N_{c}+\left[\gamma^{\prime} D_{f}+\left(\gamma-\gamma^{\prime}\right) D_{w}\right] N_{q}+0.5 \gamma^{\prime} B N_{\gamma}
$$

If $D_{w}=0, q_{u}=c^{\prime} N_{c}+\left[\gamma^{\prime} D_{f}\right] N_{q}+0.5 \gamma^{\prime} B N_{\gamma}$

If $a=0, q_{u}=c^{\prime} N_{c}+\left[\gamma D_{f}\right] N_{q}+0.5 \gamma^{\prime} B N_{\gamma}$

Case2: Water table located at a depth $b$ below base (Fig. 4.3 (b))

If the water table is located at the level of the base of footing or below it, the surcharge term is not affected. However, the unit weight in the third term of bearing capacity equation is modified as

$$
\gamma^{\prime} \text { becomes } \gamma^{\prime}+\frac{b}{B}\left(\gamma-\gamma^{\prime}\right)
$$

$\mathrm{B}=$ base width of the footing
$\mathrm{b}=$ depth of the water table below the base.
Therefore,

$$
q_{u}=c^{\prime} N_{c}+\gamma D_{f} N_{q}+0.5\left[\gamma^{\prime}+\frac{b}{B}\left(\gamma-\gamma^{\prime}\right)\right] B N_{\gamma}
$$

When $\mathrm{b}=0$, water table is at the base of the footing $\rightarrow q_{u}=c^{\prime} N_{c}+\left[\gamma D_{f}\right] N_{q}+0.5 \gamma^{\prime} B N_{\gamma}$ When $\mathrm{b}=\mathrm{B}$, water table as at a depth of B from the base

$$
q_{u}=c^{\prime} N_{c}+\left[\gamma D_{f}\right] N_{q}+0.5 \gamma B N_{\gamma}
$$

Hence, when the ground water table is located at a depth b equal to or greater than B, there is no effect on the ultimate bearing capacity.

## General Expression

Taking the submerged unit as roughly one-half of the bulk unit weight, the general equation for the ultimate bearing capacity can be written from the above conditions as

$$
q_{u}=c^{\prime} N_{c}+\gamma^{\prime} D_{f} N_{q} W_{q}+0.5 \gamma B N_{\gamma} W_{\gamma}
$$

Where $W_{q}$ is water table correction factor for the second term

$$
W_{q}=1-0.5 a / D_{f} \leq 1
$$

Where $W_{\gamma}$ is water table correction factor for the third term

$$
W_{\gamma}=0.5+0.5 b / B \leq 1
$$

It may be noted that both the corrections vary linearly.

### 4.3.3 Bearing Capacity of Square and Circular Footing

The actual bearing capacity is derived for the strip footing. The deformations under a strip footing are two dimensional. It is known as a plane strain case in theory of elasticity. On the other hand, the deformations of soil under a square or a circular footing are three dimensional. A rigorous analytical solution for a three dimensional case is extremely difficult.

Based on experimental results, Terzaghi gave the following equations for the ultimate bearing capacity for square and circular shallow footings.
a) Square footing

$$
q_{u}=1.2 c^{\prime} N_{c}+\gamma D_{f} N_{q}+0.4 \gamma B N_{\gamma}
$$

Where B is the dimension of each side of footing
b) Circular Footing

$$
q_{u}=1.2 c^{\prime} N_{c}+\gamma D_{f} N_{q}+0.3 \gamma B N_{\gamma}
$$

Where B is the diameter of the footing
The bearing capacity factors $\boldsymbol{N}_{\boldsymbol{c}}, \boldsymbol{N}_{\boldsymbol{q}}$ and $\boldsymbol{N}_{\boldsymbol{\gamma}}$ are the same as that for the strip footing.
NOTE: Terzaghi (1943) took the shape factor as 1.3, which was later changed by Terzaghi and Peck (1967) to 1.2. Both values are used in practice.
For rectangular footing, the following equation is generally used.

$$
q_{u}=c^{\prime} N_{c}(1+0.2 B / L)+\gamma D_{f} N_{q}+\frac{1}{2} \gamma B N_{\gamma}(1-0.2 B / L)
$$

$\mathrm{L}=$ length of the footing.

### 4.4. MEYERHOF'S BEARING CAPACITY THEORY

Meyerhof (1951) gave a general theory of bearing capacity for a strip footing for a strip footing at any depth. Meyerhof considered the failure mechanism similar to that assumed by Terzaghi, but extended the failure surface above foundation level. Thus the shearing strength of the soil above the footing base was also accounted for in the analysis. The curved rupture surfaces are more general than those assumed by Terzaghi.


Fig. 4.4 Meyerhof's Analysis
The right half of the Fig. 4.4 shows the failure surfaces assumed by Meyerhof. The zone ABC is the elastic zone but the angle which the inclined surfaces AC and BC make with horizontal was varied between $\phi^{\prime}$ and $\left(45+\phi^{\prime} / 2\right)$. The zone BCD is the zone of the radial shear. The zone BDEF is the zone of mixed shear in which shear varies between radial shear
and plane shear. The surface BE is known as equivalent free surface. It makes an angle $\beta$ with the horizontal.

The resultant effect of the wedge BEF of soil is represented by the normal stress ( $\mathrm{q}_{0}$ ) and the shear stress ( $\tau_{0}$ ) on the surface BE. The angle $\beta$ increases with an increase in depth $D_{f}$ and is equal to $90^{\circ}$ for deep foundations. The parameters $\beta, \mathrm{q}_{0}$ and $\tau_{0}$ are known as foundation depth parameters.

Meyerhof gave the following equation for the ultimate bearing capacity of strip footings.

$$
q_{u}=c^{\prime} N_{c}+q_{0} N_{q}+0.5 \gamma B N_{\gamma}
$$

Where $N_{c}, N_{q}$ and $N_{\gamma}$ are the general bearing capacity factors of meyerhof's theory. These factors depend upon the roughness of base, depth of footing and the shape of the footing, in addition to the angle of shearing resistance $\phi$ '. Meyerhof also gave charts for $N_{c}, N_{q}$ and $N_{\gamma}$ for shallow strip footings.

As the equivalent free surface cannot be directly located, the normal stress $\mathrm{q}_{0}$ is determined by a semi-graphical method, which is quite cumbersome. However for shallow footings, $\mathrm{q}_{0}=\gamma \mathrm{D}_{\mathrm{f}}$.

The ultimate bearing capacity given by Meyerhof's theory is close to the experimental values. For shallow footings, the value lies in between the general shear value and the local shear value of Terzaghi's analysis. However, for deep footings, Meyerhof's analysis gives values much greater than Terzaghi's analysis. The main advantage of Meyerhof's theory is that it can be used for deep foundations and for footings on slopes.

## a) Cohesionless soils

For cohesionless soils ( $\mathrm{c}=0$ ), the ultimate bearing capacity is given by a sample equation suggested by Meyerhof as

$$
q_{u}=0.5 \gamma B N_{\gamma q}
$$

Where $N_{\gamma q}$ is known as the resultant bearing capacity factor
It depends upon the coefficient of earth pressure (K) within the failure zone, the $\mathrm{D}_{\mathrm{f}} / \mathrm{B}$ ratio, the angle of shearing resistance $\phi$ ' and the angle of friction $\delta$ between the vertical shaft of the foundation and the surrounding soil. The value of K depends upon a number of factors, such as the characteristics of the soil, the stress history of the ground, the method of construction and physical characteristics of the foundation. The value of K usually lies between the active and passive earth pressure coefficients. The value of $N_{\gamma q}$ are reduced to half the values.

## b) Cohesive Soils

For cohesive soils ( $\phi=0$ ) Meyerhof's simplified equations is

$$
q_{u}=c N_{c q}+\gamma D_{f}
$$

Where $N_{c q}$ is the bearing capacity factor, depending upon the $\mathrm{D}_{\mathrm{f}} / \mathrm{B}$ ratio of the footing and on the adhesion on the sides of footing. For example, for a $\mathrm{D}_{\mathrm{f}} / \mathrm{B}$ ratio of $2.0, N_{c q}$ has a maximum value of 8.30 when the adhesion is zero, and a maximum value of 8.8 when the adhesion is equal to the cohesion (c) of the soil.

The above mentioned equations are for the ultimate bearing capacity of strip footings. The equations can also be used for rectangular, square and circular footings in conjunction with the empirical shape factors given by Meyerhof. The values of $N_{\gamma q}$ and $N_{c q}$ may be obtained from the original paper.


Fig. 4.5 Meyerhof's charts

### 4.5. IS CODE METHOD

IS: 6403-1981 gives the equation for the net ultimate bearing capacity.

$$
q_{n u}=c N_{c} s_{c} d_{c} i_{c}+q\left(N_{q}-1\right) s_{q} d_{q} i_{q}+0.5 B \gamma N_{\gamma} s_{\gamma} d_{\gamma} i_{\gamma} W^{\prime}
$$

Where $\mathrm{q}=$ effective pressure at the base.
The factor $\mathrm{W}^{\prime}$ is taken into account the effect of the water table. If the water table is at or below a depth of $\left(D_{\mathrm{f}}+B\right)$, measured from the ground surface, $\mathrm{W}^{\prime}=1.0$. if water table is likely to rise to the base of the footing or above, the value of $W^{\prime}$ is taken as 0.50 ..

If the water table is located at a depth D below the ground surface, such that $\mathrm{D}_{\mathrm{f}}<\mathrm{D}<(\mathrm{Df}+\mathrm{B})$,

The value of W' is obtained by linear interpolation. A little reflection would show that W' is the same as the factor $\mathrm{W}_{\gamma}$. The factor $\mathrm{W}_{\mathrm{q}}$ is indirectly accounted for by taking q as the effective surcharge in above equation.

The bearing capacity factors $\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}$ and $\mathrm{N}_{\gamma}$ are given in table 4.2. the shape factors given in table 4.3 are used. The depth factors are given below.

$$
\begin{gathered}
d_{c}=1+0.2\left(\frac{D_{f}}{B}\right) \tan \left(45^{0}+\emptyset^{\prime} / 2\right) \\
d_{q}=d_{\gamma}=1.0 \quad \text { for } \emptyset^{\prime}<10^{0} \\
d_{q}=d_{\gamma}=1+0.1\left(\frac{D_{f}}{B}\right) \tan \left(45^{\circ}+\emptyset^{\prime} / 2\right) \quad \text { for } \emptyset^{\prime}>10^{0}
\end{gathered}
$$

The inclination factors given below are used.

$$
\begin{gathered}
i_{c}=i_{q}=\left(1-\alpha^{0} / 90^{0}\right)^{2} \\
i_{\gamma}=\left(1-\alpha^{0} / \varnothing^{\prime}\right)^{2}
\end{gathered}
$$

Where $\alpha^{0}$ is the inclination of the load with vertical.
Table 4.2 Bearing Capacity Factors

| $\varnothing^{\prime}$ | $0^{0}$ | $5^{0}$ | $10^{0}$ | $15^{0}$ | $20^{0}$ | $25^{0}$ | $30^{0}$ | $35^{0}$ | $40^{0}$ | $45^{0}$ | $50^{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~N}_{\mathrm{c}}$ | 5.14 | 6.49 | 8.35 | 10.98 | 14.83 | 20.72 | 30.14 | 46.12 | 75.31 | 133.88 | 266.89 |
| $\mathrm{~N}_{\mathrm{q}}$ | 1.0 | 1.57 | 2.47 | 3.94 | 6.40 | 10.66 | 18.40 | 33.30 | 64.20 | 134.88 | 319.07 |
| $\mathrm{~N}_{\gamma}$ | 0.0 | 0.45 | 1.22 | 2.65 | 5.39 | 10.88 | 22.40 | 48.03 | 109.41 | 271.76 | 762.89 |

Table 4.3

| Shape of Footing | $\mathrm{S}_{\mathrm{c}}$ | $\mathrm{S}_{\mathrm{q}}$ | $\mathrm{S}_{\gamma}$ |
| :---: | :---: | :---: | :---: |
| Continuous Footing (strip) | 1.0 | 1.0 | 1.0 |
| Rectangular Footing | $1+0.2 \mathrm{~B} / \mathrm{L}$ | $1+0.2 \mathrm{~B} / \mathrm{L}$ | $1-0.4 \mathrm{~B} / \mathrm{L}$ |
| Square Footing | 1.3 | 1.2 | 0.8 |
| Circular Footing | 1.3 | 1.2 | 0.6 |

### 4.5.1 IS Code method for Cohesive Soil

IS : 6403-1981 givens the following equation for the net ultimate bearing capacity of the footing immediately after construction on a cohesive soil. The equation is obtained by substituting $\mathrm{N}_{\mathrm{q}}=1.0$ and $\mathrm{N}_{\gamma}=0.0$.

$$
q_{n u}=c N_{c} s_{c} d_{c} i_{c}
$$

Where $\mathrm{N}_{\mathrm{c}}=5.14$.

The value of $\mathrm{C}_{\mathrm{u}}$ is obtained from unconfined compression strength test or it can be derived from the static cone test. The static cone test gives the point resistance $\left(q_{c}\right)$. for normally consolidated clays, the point resistance $\mathrm{q}_{\mathrm{c}}$ is generally less than $2000 \mathrm{kN} / \mathrm{m}^{2}$ and the value of the undrained cohesion $\mathrm{C}_{\mathrm{u}}$ varies between $\mathrm{q}_{\mathrm{d}} / 18$ to $\mathrm{q}_{\mathrm{d}} / 15$. For over-consolidated clays, the point resistance is generally greater than $2000 \mathrm{kN} / \mathrm{m}^{2}$, and the values of Cu varies between $\mathrm{q}_{\mathrm{d}} / 26$ to $\mathrm{q}_{\mathrm{d}} / 22$.

### 4.6. SAFE BEARING CAPACITY BASED ON N-VALUE

### 4.6.1 Peck method:

Terzaghi and Peck (1967) gave charts for the safe bearing pressures including a total settlement of 25 mm and a differential settlement of 19 mm for different sizes of footing. Peck et al (1974) revised the Terzaghi and Peck curves to take into consideration the later research, and gave the following equation for the safe settlement pressure.

$$
q_{n} \rho=0.41 C_{w} N s
$$

Where $q_{n} \rho=$ safe settlement pressure $\left(\mathrm{kN} / \mathrm{m}^{2}\right)$
$\mathrm{N}=$ average penetration number, corrected for overburden pressure and Dilatancy.
$\mathrm{s}=$ settlement $(\mathrm{mm}), \mathrm{C}_{\mathrm{w}}=$ water table correction factor
For a settlement of 40 mm ,

$$
q_{n} \rho=16.4 C_{w} N
$$

Fig. 4.6 gives a chart for $\mathrm{s}=40$ and $\mathrm{C}_{\mathrm{w}}=1.0$


Fig. 4.6
For a settlement of 25 mm ,

$$
q_{n} \rho=10.25 C_{w} N
$$

The Water table correction is determined using the relation,

$$
C_{w}=0.5+0.5 D_{w} /\left(D_{f}+B\right)
$$

Where $D_{w}=$ depth of water table below the ground surface, $D_{f}=$ depth of footing, $B=$ width

### 4.6.2 Teng's Equation:

Teng (1962) expressed the charts given by Terzaghi and Peck (1948) in the form of the following formulas. Allowance was made for an increase in pressure with depth by introducing a depth factor.

For a settlement of 25 mm ,

$$
q_{n} \rho=35.0(N-3)\left[\frac{B+0.3}{2 B}\right]^{2} W_{r} R_{d}
$$

Where $q_{n} \rho=$ safe settlement pressure $\left(\mathrm{kN} / \mathrm{m}^{2}\right), \mathrm{N}=$ SPT Number, $\mathrm{B}=$ width of footing $(\mathrm{m})$,
$\mathrm{W} \gamma=$ water table correction factor $=0.5+0.5 b / B \leq 1$

$$
\mathrm{R}_{\mathrm{d}}=\text { depth correction factor }=\left[1+\frac{D_{f}}{B} \leq 2.0\right]
$$

The above equation can be written in general form as

$$
q_{n} \rho=1.40(N-3)\left[\frac{B+0.3}{2 B}\right]^{2} W_{\gamma} R_{d} s
$$

Where $\mathrm{s}=$ tolerable settlement (mm)

### 4.6.3 Meyerhof's equation:

Meyerhof proposed equations which are slightly different from Teng's equations. According to him, for a settlement of 25 mm ,

$$
q_{n} \rho=12.2 N W_{r} R_{d} \text { for } B \leq 1.2 m
$$

and

$$
q_{n} \rho=8.1 N\left[\frac{B+0.3}{B}\right]^{2} W_{\gamma} R_{d} \text { for } B \geq 1.2 m
$$

where all the terms are the same as in Teng's equation, except $R_{d}$ which is given by

$$
R_{d}=\left[1+0.33 D_{f} / B \leq 1.33\right]
$$

### 4.6.4 Bowle's Equation:

Bowles (1977) suggested that the net allowable pressure given by Meyerhof's equation can be safely increased by $50 \%$. Thus for a settlement of 25 mm

$$
\begin{gathered}
q_{n} \rho=18.3 N W_{\gamma} R_{d} \text { for } B \leq 1.2 m \\
q_{n} \rho=12.2 N\left[\frac{B+0.3}{B}\right]^{2} W_{\gamma} R_{d} \text { for } B \geq 1.2 m
\end{gathered}
$$

### 4.6.5 IS: 6403 - $\mathbf{1 9 7 1}$ Equation:

IS: 6403 - 1971 gives the following equation, which is similar to Teng's equation. For a settlement of 40 mm

$$
q_{n} \rho=55.4(N-3)\left[\frac{B+0.3}{2 B}\right]^{2} W_{\gamma}
$$

The depth factor is not considered.
Fig. 4.7 gives the allowable soil pressure for a settlement of 25 mm (Terzaghi and peck, 1967). Fig. 4.8 gives the allowable soil pressure for a settlement of 40 mm .


Fig. 4.7


Fig. 4.8

## GEOTECHNICAL ENGINEERING -II

## UNIT - V

## Objective:

To understand load carrying capacity of deep foundations.

## Syllabus: PILE FOUNDATION

Classification of piles - Load carrying capacity of piles based on static pile formulaeDynamic pile formulae- Pile load tests - Load carrying capacity of pile groups insands and clays.

## Learning Outcomes:

After completion of this unit the student will be able to

- Analyze the load carrying capacity of deep foundation.


## Learning Material

### 5.1 NECESSITY OF PILE FOUNDATION

Pile foundations are used in the following conditions:

1. When the strata at or just below the ground surface is highly compressible and very weak to support the load transmitted by the structure.
2. When the plan of the structure is irregular relative to its outline and load distribution. It would cause non uniform settlement if a shallow foundation. A pile foundation is required to reduce differential settlement.
3. Pile foundations are required for the transmission of structural loads through deep water to a firm stratum.
4. Pile foundations are used to resist horizontal forces in addition to support the vertical loads in earth retaining structures and tall structures that are subjected to horizontal forces due to wind and earthquake.
5. Piles are required when the soil conditions are such that a wash cut, erosion or scour of soil may occur from underneath a shallow foundation.
6. Piles are used for the foundations of some structures, such as transmission towers, off shore platforms, which are subjected to uplift.
7. Incase of expansive soils, such as black cotton soil, which swell or shrink as the water content changes, piles are used to transfer the load below the active zone.
8. Collapsible soils, such as loess, have a breakdown of structure accompanied by a sudden decrease in void ratio when there is an increase in water content. Piles are used to transfer the load beyond the zone of possible moisture changes in such soils.

### 5.2 CLASSIFICATION OF PILES

Piles can be classified according to

1. Material used
2. The mode of transfer of load
3. The method of construction
4. The use
5. The displacement of soil

### 5.2.1 Classification based on material used:

i) Steel piles: Steel piles are generally either in the form of thick pipes or rolled steel H sections. Pipe steel piles are driven into the ground with their ends open or closed. Piles are provided with a driving point or shoe at the lower end. Epoxy coatings applied in the factory during manufacture of pipes to reduce corrosion of the steel piles. Sometimes, concrete encasement at site is done as a protection against corrosion. To take into account the corrosion, an additional thickness of the steel section is usually recommended.
ii) Concrete piles: Cement concrete is used in the construction of the concrete piles. Concrete piles are either precast or cast in-situ. Precast concrete piles are prepared in a factory or a casting yard. The reinforcement is provided to resist handling and driving stresses. Precast piles can also be pre-stressed using high strength steel pretensioned cables. A cast in-situ pile is constructed by making a hole in the ground and then filling it with concrete. A cast in-situ pile may be cased or uncased. A cased pile is constructed by driving a steel casing into the ground and filling it with concrete. An uncased pile is constructed by driving the casing to the desired depth and gradually withdrawing casing when fresh concrete is filled. An uncased pile may have a pedestal.
iii) Timber piles: Timber piles are made from tree trunks after proper trimming. The timber used should be straight, sound and free from defects. Steel shoes are provided to prevent damage during driving. To avoid damage to the top of the pile, a metal band or a cap is provided. Splicing of timber piles is done using a pipe sleeve or metal straps and bolts. The length of the pipe sleeve should be at least five times the diameter of the pile. Timber piles below the water table have generally long life. However, above the water table, these are attacked by insects. The life of the timber
pile can be increased by preservatives such as creosote oils. Timber piles should not be used in marine environment where these are attacked by various organisms.
iv) Composite piles: A composite pile is made up of the two materials. A composite pile may consist of the lower portion of steel and the upper portion of cast in-situ concrete. A composite pile may also have the lower portion of timber below the permanent water table and the upper portion of concrete. As it is difficult to provide a proper joint between two dissimilar materials, composite piles are rarely used in practice.

### 5.2.2 Classification based on mode of transfer of loads:

i) End Bearing Piles: End bearing piles transmit the loads through their bottom tips. Such pies acts as columns and transmit the load through a weak material to a firm stratum below. If bed rock is located within a reasonable depth, piles can be extended to the rock. The ultimate bearing capacity of the pile depends upon the bearing capacity of rock. If instead of bed rock, a fairly compact and hard stratum of soil exists at a reasonable depth, piles can be extended a few meters into the hard stratum. End Bearing Piles are also known as Point Bearing Piles

The ultimate load carried by the pile is equal to the load carried by the point or bottom end.
ii) Friction Piles: Friction piles do not reach the hard stratum. These piles transfer the load through skin friction between the embedded surface of the pile and the surrounding soil. Friction piles are used when a hard stratum does not exist at a reasonable depth. The ultimate load carried by the pile is equal to the load transmitted by the skin friction.

The friction piles are also known as floating piles, as these do not reach the hard stratum.
iii) Combined End bearing and Friction piles: These piles transfer loads by a combination of end bearing at the bottom of the pile and friction along the surface of the pile shaft. The ultimate load carried by the pile is equal to the sum of the load carried by the pile load and the load carried by the skin friction.

### 5.2.3 Classification based on method of installation:

i) Driven Piles: These piles are driven into the soil by applying blows of a heavy hammer on their tops.
ii) Driven and cast-in-situ piles: These piles are formed by driving a casing with a closed bottom end into the soil. The casing is later filled with concrete. The casing may or may not withdrawn.
iii) Bored and cast-in-situ piles: These piles are formed by excavating a hole into the ground and then filling it with concrete.
iv) Screw piles: These Piles are screwed into the soil.
v) Jacked Piles: These piles are jacked into the soil by applying a downward force with the help of a hydraulic jack.

### 5.2.4 Classification based on Use:

i) Load Bearing Piles: These Piles are used to transfer the load of the structure to a suitable stratum by end bearing, by friction or by both.
ii) Compaction piles: These piles are driven into the loose granular soils to increase the relative density. The bearing capacity of the soil is increased due to densification caused by vibrations.
iii) Tension piles: These Piles are in tension. These piles are used to anchor down structures subjected to hydrostatic uplift forces.
iv) Sheet piles: Sheet piles from a continuous wall or bulkhead which is used for retaining earth or water.
v) Fender piles: Fender piles are sheet piles which are used to protect water front structures from impact of ships and vessels.
vi) Anchor piles: These piles are used to provide anchorage for anchored sheet piles. These piles provide resistance against horizontal pull for a sheet pile wall.

### 5.2.5 Classification based on displacement of soil:

i) Displacement Piles: All driven piles are displacement piles as the soil is displaced laterally when the pile is installed. The soil get densified. The installation may cause heaving of the surrounding ground. Precast concrete pile and closed end pipe piles are high displacement piles. Steel H piles are low displacement piles.
ii) Non displacement piles: Bored piles are non displacement piles. As the soil is removed when the hole is bored, there is no displacement of the soil during installation. The installation of these piles causes very little change in the stresses in the surrounding soil.

### 5.3 LOAD CARRYING CAPACITY OF PILES

Like a shallow foundation, a pile foundation should also be safe against shear failure and settlements should be within the permissible limits. So for estimating the load carrying capacity of a pile foundation can be grouped in 4 categories.

### 5.3.1STATIC METHODS:

The static methods give the ultimate capacity of an individual pile depends upon the characteristics of soil. The ultimate capacity is given by

$$
Q_{u}=Q_{p}+Q_{s} \ldots \ldots \ldots \ldots . \mathrm{i}
$$

$Q_{u}=$ ultimate failure load, $Q_{p}=$ point or base or tip resistance of the pile (Fig.5.1).
$Q_{s}=$ shaft resistance developed by friction between the soil and the pile shaft


Fig. 5.1

## Static methods for driven piles in sand:

The ultimate capacity of a single pile driven into sand is obtained by using Eqi

$$
\begin{gathered}
\text { Where } Q_{p}=q_{p} A_{p} \\
Q_{s}=f_{s} A_{s}
\end{gathered}
$$

Determination of $Q_{p}$ : The ultimate bearing capacity of the soil at the pile tip can be computed from the bearing capacity equation similar to that for a shallow foundation

$$
q_{p}=q^{\prime} N_{q}+0.4 \gamma B N_{\gamma} \ldots \ldots \ldots . . \mathrm{ii}
$$

$q^{\prime}=$ effective vertical pressure at the pile tip, $\mathrm{B}=$ pile tip width $\gamma=$ unit weight of the soil in the zone of the pile tip
$N_{q}$ and $N_{\gamma}=$ bearing capacity factors for deep foundations

In driven piles, the second term of eqii is generally small and is, therefore neglected. Thus $q_{p}=q^{\prime} N_{q}$


Fig. 5.2


Fig. 5.3

In case of driven piles, it has been established that the effective vertical pressure $(\bar{q})$ at the pile tip increases with only until a certain depth of penetration, known as critical depth $\left(D_{c}\right)$. Below the critical depth, the effective vertical pressure remains essentially constant Fig. 5.2 (a). The critical depth depends upon the angle of shearing resistance of the soil and the width of the pile (Fig.5.2). Its value can be roughly taken as 10B for loose sand and 20B for dense sand.

The bearing capacity factor $\mathrm{N}_{\mathrm{q}}$ based on theoretical analysis. These values vary over a wide range because of different assumptions made in defining the shear zone near the pile tip. Fig. 5.3 shows the value of $\mathrm{N}_{\mathrm{q}}$ given by various investigators and that given by IS:2911. The values given by Berezontzev are quite dependable and are generally used.

In the derivation of the value of $\mathrm{N}_{\mathrm{q}}$, it has been assumed that the soil above the pile tip is similar to the soil below the pile tip. If the pile penetrates a compact stratum only slightly and the soil above the tip is loose, it would be more appropriate to use the value of $\mathrm{N}_{\mathrm{q}}$ for shallow foundations.

If the pile is of relatively large diameter, the second term in eqii becomes significant. The value of $N_{\gamma}$ can be conservatively taken as the $N_{\gamma}$ value used for shallow foundation.

Determination of $\boldsymbol{Q}_{s}$ : The frictional resistance $Q_{s}$ is obtained from after estimating the unit skin friction. The unit skin friction for a straight-sided pile depends upon the soil pressure and acting normal to the pile surface and the coefficient of friction between the soil and the pile material (Fig. 5.4).


Fig. 5.4
The soil pressure normal to the vertical pile surface is horizontal pressure and is related to the effective vertical soil pressure as

$$
\sigma_{h}=K \sigma_{v}
$$

Where $\mathrm{K}=$ earth pressure coefficient $\sigma_{v}=$ effective vertical pressure at that depth
Thus unit skin friction acting at any depth can be written as

$$
f_{s}=\sigma_{v} \tan \delta \quad \text { or } \quad f_{s}=K \sigma_{v} \tan \delta
$$

Where $\tan \delta=$ coefficient of friction between sand and the pile material
Selection of suitable values of K and $\delta$ requires good engineering judgment. Tomilson (1975) gave the values of K and $\delta$, as given in the following table 5.1.

Table 5.1

| Pile Material | $\delta$ | K <br> (loose sand) | K <br> (dense sand) |
| :---: | :---: | :---: | :---: |
| Steel | $20^{0}$ | 0.50 | 1.0 |
| Concrete | $0.75 \emptyset$ | 1.0 | 2.0 |
| Timber | $0.67 \emptyset$ | 1.5 | 4.0 |

$$
Q_{s}=\sum_{i=1}^{n} K\left(\sigma_{v}\right)_{i} \tan \delta\left(A_{s}\right)_{i}
$$

Where $\mathrm{n}=$ number of layers in which the pile is installed.
$\left(\sigma_{v}\right)_{i}=$ effective normal stress in the $\mathrm{i}^{\text {th }}$ layer
$\left(A_{s}\right)_{i}=$ surface area of the pile in the $\mathrm{i}^{\text {th }}$ layer
Assuming the linear variation of $\sigma_{v}$

$$
Q_{s}=K \tan \delta A_{p}\left(q_{0} D\right)
$$

$A_{p}=$ Perimeter of the pile. $q_{0}=$ average effective pressure $=\frac{\gamma^{\prime} D}{2}=\frac{q^{\prime}}{2}$
$D=$ depth of the pile.

The ultimate load for the pile can be written as

$$
Q_{u}=q^{\prime} N_{q} A_{p}+\sum_{i=1}^{n} K\left(\sigma_{v}\right)_{i} \tan \delta\left(A_{s}\right)_{i}
$$

## Static method for driven piles in saturated clay:

$Q_{u}=Q_{p}+Q_{s}$ can be used for the determination of the load carrying capacity of driven piles in saturated clay. The point resistance can be expressed as

$$
Q_{p}=q_{p} A_{p}
$$

Where $q_{p}$ is the unit point resistance, equal to the ultimate bearing capacity of the soil.
For cohesive soils ( $\varnothing=0$ ), the ultimate bearing capacity is found from the following equation, which is similar to that for a shallow foundation.

$$
q_{u}=c N_{c}+q N_{q}
$$

As $N_{q}=1.0$ for $\emptyset=0$, the above equation becomes

$$
\begin{gathered}
q_{u}=c N_{c}+q \\
Q_{p}(\text { Gross })=\left(c N_{c}+q\right) A_{p} \\
Q_{p}(\text { net })=\left(c N_{c}\right) A_{p}
\end{gathered}
$$

In above equation, c is the cohesion of the clay in the zone surrounding the pile tip, and $N_{c}$ is the bearing capacity factor for the deep foundation.

The value of $N_{c}$ depends upon the $\mathrm{D} / \mathrm{B}$ ratio and it varies from 6 to 9 . A value of $N_{c}$ is reduced to the values proposed by Skempton.

The skin resistance of the pile can be expressed as $Q_{s}=c_{a} A_{s}$
Where $c_{a}=$ unit adhesion developed between clay and pile shaft.

The unit adhesion is related to the unit cohesion by the relation $c_{a}=\alpha c^{\prime}$

Where $\alpha$ is unit adhesion factor and $c^{\prime}$ is the average cohesion along the shaft length.
The value of $\alpha$ depends upon the consistency of the clay. For normally consolidated clays, the value of $\alpha$ is taken as unity. According to IS: 2911-1979, the value of $\alpha$ can be taken as unity for soils having soft to very soft consistency. Fig. 5.5 shows the variation of $\alpha$ with the undrained cohesion c . It may be noted that for normally consolidated clays, with c less than about $50 \mathrm{kN} / \mathrm{m}^{2}$, the value of $\alpha$ is equal to unity.


Fig. 5.4

As c increases, the value of $\alpha$ decreases. For over consolidated stiff to hard clays, its value is usually taken as 0.3 . for tapered piles, the value of $\alpha$ is generally $20 \%$ greater than that for a straight pile.

For very long piles ( $\mathrm{D} \geq 25 \mathrm{~m}$ ), the above method for estimating the skin friction is very conservative. For such soils, the unit skin friction also depends upon the effective overburden pressure. According to Vijayvergiya and Focht (1972), the average unit skin friction can be expressed as $f_{s}=\lambda\left(\sigma_{v}^{\prime}+2 c\right)$

Where $\lambda=$ friction capacity factor, $\sigma_{v}^{\prime}=$ mean effective vertical stress for the embankment length, $\mathrm{c}=$ undrained cohesion.

The value of $\lambda$ can be obtained from Fig. 5.5 (McClelland, 1974)


Fig. 5.5

Once the unit skin friction has been estimated, the shaft resistance is determined from $Q_{s}=f_{s} A_{s}$

For cohesive soils, the ultimate load can be determined by adding the point resistance and the shaft resistance.

$$
Q_{u}=c N_{c} A_{p}+\alpha c^{\prime} A_{s}
$$

## Static Method for Bored Piles:

a) Bored piles in sand: The following equation can be used

$$
Q_{u}=q^{\prime} N_{q} A_{p}+\sum_{i=1}^{n} K\left(\sigma_{v}\right)_{i} \tan \delta\left(A_{s}\right)_{i}
$$

$\sigma_{v}^{\prime}=$ effective vertical pressure, limited to a maximum value given by the critical depth.
$K=$ lateral earth pressure coefficient for bored foundation
$\tan \delta=$ coefficient of friction between sand and concrete

$$
K=1-\sin \varnothing
$$

The value of K generally varies between 0.3 and 0.75 . An average value of 0.5 is adopted.
b) Bored piles in clay: The following equation can be used

$$
Q_{u}=c N_{c} A_{p}+\alpha c^{\prime} A_{s}
$$

Where $\mathrm{A}_{\mathrm{s}}=$ area of shaft that is effective in developing skin friction.
$\alpha$ depends upon the pile type and the method of drilling.

$$
\begin{aligned}
\alpha & =0.5 \text { straight shafts excavated Dry } \\
& =0.3 \text { when drilled with slurry } \\
& =0.15 \text { to } 0.3 \text { for belled shafts }
\end{aligned}
$$

### 5.4 Dynamic Formula:

The dynamic formulae are based on the assumption that the kinetic energy delivered by the hammer during driving operation is equal to the work done on the pile. Thus

$$
W h \eta_{h}=R \times S
$$

Where $\mathrm{W}=$ weight of hammer $(\mathrm{kN})$, $\mathrm{h}=$ height of ram drop $(\mathrm{cm}), \eta_{h}=$ efficiency of pile hammer, $\mathrm{R}=$ pile resistance $(\mathrm{kN})=\mathrm{Qu}$, and $\mathrm{S}=$ pile penetration per blow $(\mathrm{cm})$

In above equation, no allowance has been made for the loss of energy during driving operation, loss caused by elastic contraction of the pile, soil, pile cap, cushion and due to the inertia of the pile. Some energy is also lost due to generation of heat. Various formulae have been proposed, which basically differ only in the methods for accounting of the energy losses, as described below.

### 5.4.1 Engineering News Record Formula:

According to Engineering News Record (ENR) formula (1888) the ultimate load is given by

$$
Q_{u}=\frac{W h \eta_{h}}{S+C}
$$

Where $S=$ penetration of pile hammer blow, It is generally based on the average penetration obtained from the last few blows $(\mathrm{cm}), \mathrm{C}=$ constant $=2.54$ for drop hammer

$$
=0.254 \text { for steam hammer }
$$

In above equation, the product $\mathrm{W} \times \mathrm{h}$ can be replaced by the rated energy of hammer $E_{n}$ in $\mathrm{kN}-\mathrm{cm}$. Thus

$$
Q_{u}=\frac{E_{n} \eta_{h}}{S+C}
$$

The efficiency $\eta_{h}$ of the drop hammer is generally between 0.7 and 0.9 , and that for a single acting and a double acting hammer is between 0.75 and 0.85 . For diesel hammer, it usually lies between 0.80 and 0.90 .

A factor of safety of 6 is usually recommended. However, the pile load tests reveal that the actual factor of safety varies between $2 / 3$ and 30 . The formula is therefore, not dependable. Modified Formula: The Engineering News Record formula has been modified recently. In the modified formula, the energy losses in the hammer system and that due to impact are considered. According to this formula,

$$
Q_{u}=\frac{W h \eta_{h}}{S+C}\left[\frac{W+e^{2} P}{W+P}\right]
$$

Where $\mathrm{P}=$ weight of the pile, $\mathrm{e}=$ Coefficient of restitution, $\eta_{h}=$ hammer efficiency
The hammer efficiency depends upon various factors, such as pile driving equipment, driving procedure, type of pile and the ground conditions. For drop hammers, it is usually taken between 0.75 and 1.0 ; for single hammers between 0.75 and 0.85 ; for double acting or differential hammer 0.85 and for diesel hammers 0.85 to 1.00 .

The representative values of the coefficient of restitution (e) are as under.
Broomed timber pile

$$
=0.0
$$

Good timber pile
Driving cap with timber dolly on steel pile $=0.3$
Driving cap with plastic dolly on steel pile $=0.5$
Helmet with composite plastic dolly and packing on RCC pile $=0.4$

### 5.4.2 Hiley Formula:

Hiley $(1925,1930)$ gave a formula which takes into account various losses.

$$
Q_{u}=\frac{W h \eta_{h} \eta_{b}}{S+C / 2}
$$

$\eta_{h}=$ efficiency of hammer blow, $\mathrm{h}=$ height of the free fall of the ram or hammer $(\mathrm{cm})$, $\mathrm{S}=$ final set or penetration per blow, $\mathrm{C}=$ sum of temporary elastic compression of the pile, dolly, packings, and ground $\left(=C_{1}+C_{2}+C_{3}\right), C_{1}$ temporary compression of dolly and packing ( $=1.77 \mathrm{R} / \mathrm{A}$, when the driving without dolly $=9.05 \mathrm{R} / \mathrm{A}$, when the driving is with short dolly), $\mathrm{C}_{2}=$ temporary compression of the pile ( $=0.657 \mathrm{RD} / \mathrm{A}$ ), $\mathrm{C}_{3}=$ temporary compression of ground $(=3.55 \mathrm{R} / \mathrm{A}), \mathrm{D}=$ length of the pile, $\mathrm{A}=$ cross sectional area, of the pile, $\mathrm{R}=$ pile resistance (tones).
The efficiency of hammer blow $\eta_{b}$ depends upon the weight of hammer (W), weight of the pile, anvil and helmet follower ( P ) and the coefficient of restitution (e).
a) For W. $>\mathrm{eP}$,

$$
\eta_{b}=\frac{W+e^{2} P}{W+P}
$$

b) For $\mathrm{W}<\mathrm{eP}$,

$$
\eta_{b}=\frac{W+e^{2} P}{W+P} \cdot\left(\frac{W-e P}{W+P}\right)^{2}
$$

The coefficient of restitution (e) varies from zero for a deteriorated condition of the head of pile to 0.5 for a steel ram of double acting hammer striking on steel anvil and driving a reinforced concrete pile. For a C.I ram of a single acting or drop hammer striking on the head of RCC Pile, $\mathrm{e}=0.4$ and that striking on a well conditioned driving cap and helmet with hard wood on RCC pile, e=0.25 (IS: 2911-1979).

### 5.4.3 Danish Formula:

According to Danish formula (1929),

$$
\begin{gathered}
Q_{u}=\frac{W h \eta_{h}}{S+1 / 2 S_{0}} \\
S_{0}=\left[\frac{2 \eta_{h}(W h D)}{A E}\right]^{1 / 2}
\end{gathered}
$$

In which $\mathrm{S}_{0}=$ elastic compression of pile, $\mathrm{D}=$ length of pile, $\mathrm{A}=$ cross sectional area, $\mathrm{E}=$ modulus of elasticity of pile material.
The allowable load is found by taking a factor of safety of 3 to 4 .
For determining the final set (S) per blow, take $\mathrm{Q}_{\mathrm{u}}=3 \mathrm{Q}_{\mathrm{a}}$

$$
S=\left(\frac{W h \eta_{h}}{3 Q_{a}}\right)-\frac{1}{2} S_{0}
$$

Where $\mathrm{Q}_{\mathrm{a}}=$ allowable load.

### 5.5 PILE LOAD TEST

The most reliable method for determining the load carrying capacity of a pile is the pile load test. The set-up generally consist of two anchor piles provided with an anchor girder or a reaction girder at their top (Fig. 5.6).the test pile is installed between anchor piles in the manner in which the foundation piles are to be installed. The test pile should be at least 3 B or 2.5 m clear from the anchor pile.


Fig. 5.6

The load is applied through a hydraulic jack resting on the reaction girder. The measurement of pile movement is taken with respect to a fixed reference mark. The test is conducted after a rest period of 3 days after the installation in sandy soils and a period of one month in silts and soft clays. The load is applied in equal increment of about $20 \%$ of the allowable load. Settlements should be recorded with the three dial gauges. Each stage of the loading is maintained till the rate of movement of the pile top is not more than 0.1 mm per hour in sandy soils and 0.02 mm per hour in case of clayey soils or a maximum of two hours(IS:2911-1979).Under each load increment, settlements are observed at $0.5,1,2,4,8,12,16,20,60$ minutes. The loading should be continued up to twice the safe load or the load at which the total settlement reaches a specified value. The load is removed in the sane decrements at 1 hour interval and the final rebound is recorded 24 hours after the entire load has been removed.


Fig. 5.7 Load settlement curve
The Fig. 5.7 shows a typical load-settlement curve (firm line) for loading as well as unloading obtained from a pile load test. For any given load, the net pile settlement $\left(\mathrm{S}_{\mathrm{n}}\right)$ is given by

$$
\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{t}}-\mathrm{S}_{\mathrm{e}}
$$

Where $\mathrm{S}_{\mathrm{n}}=$ total settlement (gross settlement), $\mathrm{S}_{\mathrm{e}}=$ elastic settlement (rebound)
The Fig. 5.7 also shows the net settlement (chain dotted line).
Fig. 5.8 shows two load-net settlement curves obtained from a pile load tests on two different soils. At the ultimate load (QU), the load net-settlement curve (2) or there is a sharp break as in the curve (1), as shown in figure. The safe load is usually taken as one-half of the ultimate load.

According to IS: 2911, the safe load is taken as one-half of the load at which the total settlement is equal to 10 percent of the pile diameter ( 7.5 percent in case of under-reamed piles) or two-thirds of the final load at which the total settlement is 12 mm , whichever is less.

According to another criterion, the safe load is taken as one-half to two-thirds of the load which gives a net settlement of 6 mm .


Fig. 5.7
The limiting settlement criteria are also sometimes specified. Under the load twice the safe load, the net settlement should not be more than 20 mm or the gross settlement should not be more than 25 mm .

The test described above is known as initial test. It is carried out on a test pile to determine the ultimate load capacity and hence the safe load. The pile loads test described in this section is a type of load-controlled test, in which the load is applied in steps. The test is also known as slow maintained test.

### 5.6 GROUP ACTION OF PILES

A pile is not used singularly beneath a column or a wall, because it is extremely difficult to derive the pile absolutely vertical and to place the foundation exactly over its centre line. If eccentric loading results, the connection between the pile and the column may break or the pile may fail structurally because of bending stresses. In actual practice, structural loads are supported by several piles acting as a group. For columns, a minimum of three piles in a triangular pattern are used. For walls, piles are installed in a staggered arrangement on both sides of its centre line. The loads are usually transferred to the pile group through a reinforced concrete slab, structurally tied to the pile tops such that the piles act as one unit. The slab is known as pile cap. The load acts on the pile cap which distributes the load to the piles (Fig.5.8).

The load carrying capacity of a pile group is not necessarily equal to the sum of the capacity of the individual piles. Estimation of the load-carrying capacity of a pile group is a complicated problem. When the piles are spaced a sufficient distance apart, the group capacity may approach the sum of individual capacities. On the other hand, if the [piles are
closely spaced, and this may reduce the load-carrying capacity of the piles (Fig. 5.9).For such a case, the capacity is limited by the group action.


Fig. 5.8


Fig. 5.9
The efficiency $\left(\eta_{g}\right)$ of a group of piles is defined as the ratio of the ultimate load of the group to the sum of individual ultimate loads.

$$
\eta_{g}=\frac{Q_{g(u)} / N}{Q_{u}} \times 100
$$

Where $Q_{g(u)}=$ ultimate load of the group, $Q_{u}=$ ultimate load of the individual pile,
$\mathrm{N}=$ number of piles in the group.

Thus the group's efficiency is equal to the ratio of the average load per pile in the group at which the failures occurs to the ultimate load of a comparable single pile.

The group efficiency depends upon the spacing of the piles. Ideally, the spacing should be such that the efficiency is $100 \%$.Generally, and the centre to centre spacing is kept between 2.5B and 3.5 B , where B is the diameter of the pile.

### 5.7 PILE GROUPS IN SAND AND GRAVEL

For piles driven in loose and medium dense cohesion less soils, the group efficiency is high. The soil around and between the piles is compared due to vibration caused during the driving operation. For better results, it is essential to start driving the piles at the centre and then work outward.

The piles and the soil between them move together as a unit when subjected to loads. The group acts as a pier foundation having a base equal to the gross plan area contained between the piles.

### 5.7.1 End-bearing piles

For driven piles bearing on dense, compact sand with spacing equal to or greater than 3B, the group capacity is generally taken equal to the sum of individual capacity. Thus

$$
Q_{g}=N Q_{u}
$$

In this case, the load taken by the group is much greater $\left(\eta_{g}>100 \%\right)$ than the sum of the individual capacities, and the piles fail as individual piles.

For spacing less than 3B, the group capacity is found for the block of piles group.

### 5.7.2 Friction piles

The group efficiency of friction piles in sand is obtained from the following expression:

$$
\eta_{g}=\frac{Q_{g(u)}}{N Q_{u}} \times 100=\frac{f_{s}\left(\rho_{g} D\right)}{N f_{s}(\rho D)} \times 100
$$

Where $\rho_{g}=$ perimeter of the block, $\rho=$ perimeter of the individual pile, $\mathrm{D}=$ length of pile, $f_{s}=$ unit friction resistance

If the centre-to-centre spacing is large, the group efficiency $\left(\eta_{g}\right)$ may be more than $100 \%$. The piles will behave as individual piles, and the group capacity is obtained from

$$
Q_{g}=N Q_{u}
$$

If $\eta_{g}$ is less than $100 \%$,

$$
Q_{g}=\eta_{g} \frac{\left(N Q_{u}\right)}{100}
$$

The group efficiency can also be obtained from the Converse-Lebarre equation given below.

$$
\eta_{g}=1-\left[\frac{(n-1) m+(m-1) n}{m n}\right] \frac{\theta}{90}
$$

Where $\mathrm{m}=$ number of rows of piles, $\mathrm{n}=$ number of piles in a row, $\theta=\tan ^{-1}(\mathrm{~B} / \mathrm{s}), \mathrm{B}=$ diameter of pile, $\mathrm{s}=$ spacing of pile, centre-to-centre, $\eta_{g}=$ group efficiency (expressed as a ratio).

## Bored piles

For bored piles in sand at conventional spacing of $3 B$, the group capacity is taken as $2 / 3$ to $3 / 4$ times the sum of individual capacities for both the end bearing and the friction piles. Thus

$$
Q_{g(u)}=(2 / 3 \text { to } 3 / 4)\left(N Q_{u}\right)
$$

In bored piles there is limited densification of the sand surrounding the pile group. Consequently, the efficiency is lower.

### 5.8 PILE GROUPS IN CLAY

As the pile group acts as a block, its ultimate capacity is determined by adding the base resistance and the shaft resistance of the block. The capacity of the block having closely spaced piles $(\mathrm{S} \leq 3 \mathrm{~B})$ is often limited by the behaviour of the group acting as a block. The group capacity of the block is given by

$$
Q_{g(U)}=q_{p}\left(A_{g}\right)+\alpha c\left(\rho_{g} D\right)
$$

Where $q_{p}=$ unit point resistance $\left(\mathrm{N}_{\mathrm{c}}=9.0\right), \mathrm{A}_{\mathrm{g}}=$ base area of the block, $\rho_{g}=$ perimeter of the block, $\mathrm{D}=$ depth of the block, $\alpha=$ adhesion factor ( $=1.0$ for soft clays), $\mathrm{c}=$ undrained cohesion.

As discussed earlier, the individual pile capacity is given by

$$
Q_{(U)}=q_{p}\left(A_{p}\right)+\alpha c(\rho D)
$$

The group capacity considering the piles as individual piles is given by

$$
Q_{g(u)}=N Q_{u}
$$

The lower of the two values, given by above Eqsis the actual capacity.

### 5.9 SETTLEMENT OF PILE GROUPS

The settlement of a pile group is due to elastic shortening of piles and due to the settlement of the soil supporting the piles. It is assumed that the pile group acts as a single large deep foundation, such as a pier or a mat. The total load assumed to act

At a depth equal to two-thirds the pile length in the case of friction piles (Fig. 10 (a)). In the case of end-bearing piles, the total load is assumed to act at the pile tips (Fig. 10 (b)). In the case of combined action, the frictional component is assumed to act at $2 / 3 \mathrm{D}$ and bearing component at the tip.

For determination of the settlements the compression characteristics of the soil are required. For clayey soils, the characteristics are determined from laboratory tests on undisturbed samples. For cohesionless soils, the characteristics are obtained from empirical correlations developed from in-situ penetration tests.

(a) FRICTION PILES

(b) END BEARING PILES

Fig. 5.10

## Assignment-Cum-Tutorial Questions

## A. Questions testing the remembering / understanding level of students

## I) Objective Questions

1. The minimum spacing of piles which derive their capacity only by friction should not be less than $\qquad$ times the diameter of the pile shaft.
2. For settlements evaluation of friction pile group in clay, the load is assumed to be applied as a uniform load at a depth of about $\qquad$ the length of the piles.
3. The pile capacity in foundation is controlled by $\qquad$
4. The minimum number of piles needed in a pile group to support a column is. $\qquad$
5. Group efficiency of the friction piles in clay is
a) Exactly $100 \%$
b) $>100 \%$
c) $<100 \%$
d) almost $100 \%$

## II) Descriptive Questions

1. What are the conditions where a pile foundation is more suitable than a shallow foundation?
2. Describe various types of pile foundation.
3. How would you estimate the load carrying capacity of a pile in cohesive and coheionless soils?
4. Discuss various dynamic formula. What are their limitations?
5. How would you estimate the group capacity of piles in sand and clay?
6. Describe pile load test?

## B. Question testing the ability of students in applying the concepts.

## I) Multiple Choice Questions:

1. The failure of a pile foundation is due to
a) General Shear
b) Local Shear
c) Mixed Shear
d) Punching Shear
2. Dynamic formulae cannot be used to obtain carrying capacity in
a) Saturated Sands b) dry sands
c) saturated Clays
d) dry gravels
3. Negative skin friction occurs when
a) An upward drag exists in the pile
b) The surrounding soil settles more than the pile
c) The pile passes continuously through a firm soil
d)The driving operation begins
4. Negative skin friction on pile is dominant in
a) Piles resting on hard sands
b) Friction Piles in sands
c) Friction piles in soft clays
d) friction per bearing piles
5. Close spacing of piles in dense sand will cause
a) Objectionable upheaval
b) Increased compaction
c) Increased pile capacity
d) Decreased the pile capacity

## II) Problems

1. A concrete pile, 30 cm diameter, is driven into a medium dense sand ( $\emptyset=35^{\circ}, \gamma=21$ $\mathrm{kN} / \mathrm{m}^{3}, \mathrm{~K}=1.0, \tan \delta=0.70$ ) for a depth of 8 m . Estimate the safe load, taking a factor of safety of 2.50 .
2. A 30 cm diameter concrete pile is driven into a homogeneous consolidated clay deposit $\left(\mathrm{C}_{\mathrm{u}}=40 \mathrm{kN} / \mathrm{m}^{2}, \alpha=0.7\right)$. if the embedded length is 10 m , estimate the safe load (F.S. $=$ 2.5).
3. A concrete pile, 40 cm diameter, is driven into a medium dense sand ( $\varnothing=30^{0}, \gamma=20$ $\mathrm{kN} / \mathrm{m}^{3}, \mathrm{~K}=1.2, \tan \delta=0.70$ ) for a depth of 10 m . Estimate the safe load, taking a factor of safety of 2.50 .
4. A 40 cm diameter concrete pile is driven into a homogeneous consolidated clay deposit $\left(\mathrm{C}_{\mathrm{u}}=52 \mathrm{kN} / \mathrm{m}^{2}, \alpha=0.75\right)$. if the embedded length is 12 m , estimate the safe load (F.S. $=$ 2.5).
5. A pile group consists of 9 friction piles of 30 cm diameter and 10 m length driven in clay $\left(\mathrm{C}_{\mathrm{u}}=100 \mathrm{kN} / \mathrm{m}^{2}, \gamma=20 \mathrm{kN} / \mathrm{m}^{3}\right)$, as shown in Fig. 1. Determine the safe load for the group ( $\mathrm{FoS}=3, \alpha=0.6$ ).


Fig. 1
6. Determine the safe load of the pile, 30 cm diameter, is driven into a medium dense sand ( $\emptyset=35^{0}, \gamma=21 \mathrm{kN} / \mathrm{m}^{3}, \mathrm{~K}=1.0, \tan \delta=0.70$ ) for a depth of 8 m . Estimate the safe load, taking a factor of safety of 2.50 . If the water table raises to 2 m below the ground surface. Take $\gamma_{\mathrm{w}}=10 \mathrm{kN} / \mathrm{m}^{3}$.
7. A pile group consists of 9 friction piles of 30 cm diameter and 15 m length driven in clay $\left(\mathrm{C}_{\mathrm{u}}=50 \mathrm{kN} / \mathrm{m}^{2}, \gamma=21 \mathrm{kN} / \mathrm{m}^{3}\right)$, as shown in Fig. 1. Determine the safe load for the group ( $\mathrm{FoS}=3, \alpha=0.75$ ).
8. A group of friction piles of 30 cm diameter is subjected to a net load of 2000 kN , as shown in Fig. 2. Estimate the consolidation settlement.


Fig. 2

## GEOTECHNICAL ENGINEERING-II

## UNIT-VI

## Objective:

To understand load carrying capacity of shallow and deep foundations.

## Syllabus: Well Foundations

Different shapes of Well Foundation, Forces acting on Well Foundation, Design of individual component, Construction and sinking of well foundations, Tilts and shifts.

## Learning outcomes:

After completion of this unit the student will be able to

- Estimating the forces acting on well foundations
- Designing the individual components of the well
- How sinking operation was done on well foundation
- How to rectify tilts and shifts in well foundations


## Learning material

### 6.1 Introduction of Well Foundation

A Well foundation is similar to the open Caisson. Well Foundation can be constructed on the dry bed or after making a sand island. At locations where the depth of water is greater than 5 m to 6 m and the velocity of water is high, wells can be fabricated on the river bank and then floated to the final position and grounded. Great care is to be exercised while grounding a well to ensure that its position is correct. Once the well has touched the bed, sand bags are deposited around it to prevent scour. The well may sink into the river bed by 50 to 60 cm under its own weight. Further sinking operation is similar to the sinking of wells on dry bed. The well is sunk into the ground to the desired level by excavating through the dredged holes.

Figure shows the section of a well. A strong cutting edge is provided to facilitate sinking. The tapered portion of the well above the cutting edge is known as well curb. The walls of the well are known a steining. Steining is made of brick masonry, stone masonry, plain or reinforced concrete. As Steining later becomes an integral part of the structure, it should be properly designed for the imposed loads. Further, it should be heavy enough to overcome frictional resistance during sinking.


Fig 1 Parts of a Well Foundation
After the well has been sunk to the final position, the bottom plug is formed by concreting. The bottom plug serves as the base of the well. The well is filled with sand partly or completely. At the top of the well, a top plug is formed by concreting. A RCC well cap is provided at the top to transmit the load of the super structure to the well.

### 6.2 Different Shapes of well foundations:

Different shapes of wells that are commonly used are shown in the Figure.

Single circular

Twin circular

Dumb well

Double D

Twin hexagonal

Twin octagonal

Rectangular

1. The most commonly used shape is circular, as it has high structural strength and is convenient in sinking the chances of tilting also minimum in this shape. The shape is quite suitable for piers of the single line railway bridges and the double lane road bridges. However, when the piers are excessively long, the circular shape becomes uneconomical. The maximum diameter of circular wells is generally limited to 9 m .
2. Double D Wells are generally used for the piers and abutments of bridges which are too long to be accommodated on a circular well of 9 m diameter. The wells of this shape can also be sunk easily. However, considerable bending moments are caused in the Steining due to the difference in pressure between the outside and inside of the well. Further, the square corners at the partition wall offer greater resistance to sinking.
3. Double octagonal wells are better than the double D wells in many respects. The square corners are eliminated and bending stresses are considerably reduced. However, they offer greater resistance than double D wells against sinking on account of increased surface area. Moreover, the construction is more difficult.
4. Twin circular wells are two independent wells placed very close to each other and having a common well cap. The wells are sunk simultaneously. These wells are suitable where the length of the pier is considerable, which cannot be accommodated on a double D or Double octagonal well. Twin circular wells are advantageous when the depth of the sinking is small and the bearing capacity of the soil is high. The disadvantage of twin circular wells is that there is a possibility of the relative settlement of the two wells even if a heavy RCC top cap is provided unless the wells are founded on an incompressible soil.
5. Rectangular Wells are generally used for bridge foundation having depths upto 7-8 m . For large foundations, double rectangular wells are used. For piers and abutments of very large size, rectangular wells with multiple dredge holes are used. Bending stresses in the Steining are very high in rectangular wells.

### 6.3 Forces acting on well foundation

- Dead loads
- Live loads
- Impact loads
- Wind loads
- Water pressure
- Longitudinal forces
- Centrifugal forces
- Buoyant forces
- Earth pressure
- Temperature stresses
- Seismic forces


### 6.4 Design of individual components of the well

1. Cutting Edge: The cutting edge should have a sharp angle for cutting through the soil. It should be strong enough as that it does not bend when penetrating through a soil containing boulders. A sharp vertical edge having an angle of 300 with the vertical or having a slope of one horizontal to two vertical is generally used. However, if the sharp edges are likely to be damaged, a cutting edge with a stub nose is used. The cutting edge should be properly anchored to the well curb.
2. Well Curb: Figure shows the curb of a well. Curbs are generally made of reinforced concrete. During sinking operation, the curb cuts through the soil. The figure shows the forces acting on the curb when the well has penetrated to a consideration depth below the scour level.

$D_{0}$ - external diameter
$D_{1}$ - internal diameter
Force acting on curb
Forces acting tangentially to the bevel surface
$Q=\mu P$
Where $\mu$ is coefficient of friction between soil and concrete of the curb, $\mathrm{P}=$ forces acting normal to the bevel surface.
Resolving vertically, $\mu P \sin \theta+P \cos \theta=N$
$P=\frac{N}{(\mu \sin \theta+\cos \theta)}$
Where $\mathrm{N}=$ vertical force on the curb, $\theta=$ angle which the bevel edge makes with the horizontal.
Resolving horizontally, $P \sin \theta-\mu P \cos \theta=H$
Where $\mathrm{H}=$ horizontal force on the curb per m .
Substituting P ,
$H=\frac{N(\sin \theta-\mu \cos \theta)}{(\mu \sin \theta+\cos \theta)}$
Hoop tension, $\mathrm{T}=\mathrm{Hxd} / 2$, where d is diameter
$T=0.5 N\left(\frac{\sin \theta-\mu \cos \theta}{\mu \sin \theta+\cos \theta}\right) d$
Sometimes, sand blow may cause sudden descend of the well sinking and an increase in the hoop tension. To account for such an eventually, the hoop tension is increased by $50 \%$. Thus
$T=0.75 N\left(\frac{\sin \theta-\mu \cos \theta}{\mu \sin \theta+\cos \theta}\right) d$
Suitable reinforcement should be provided to resist the hoop tension T developed.
When the cutting edge is not able to move downward due to the reaction developed at the curb and the bottom plug, the hoop tension developed is given by
$T=\left(\frac{q d^{2}}{8 r}\right) \frac{d}{2}$
Where $\mathrm{q}=$ pressure at the base $=\frac{\text { total weight }}{\text { area of plug }}$
$r=$ vertical height of the imaginary inverted arch.
In case of granular soils, the hoop tension is relieved by active earth pressure around the curb. The net hoop tension is given by
$T=\frac{d}{4}\left[\frac{q d^{2}}{4 r}-\left(p_{1}+p_{2}\right) b\right]$
$p_{1}=\frac{1}{2} K_{a} \gamma^{\prime} D^{2}$
$p_{2}=\frac{1}{2} K_{a} \gamma^{\prime}(D-b)^{2}$
In which $\mathrm{b}=$ height of the curb, $\mathrm{D}=$ depth of the curb below the scour level.
At the junction of the curb and Steining, a moment $\mathrm{M}_{0}$ develops due to the horizontal force H caused by bevelled action, given by $M_{0}=H \times b / 2$
Suitable reinforcement is provided at the inner corner to take care of this moment and is anchored into the Steining.
IRC: 21 - 1972 recommends a minimum reinforcement of $72 \mathrm{~kg} / \mathrm{m}^{3}$ in a well curb. The reinforcement should be properly arranged.
The slope of the inner face of the curb should be such as to push forward easily. The angle with the vertical should preferably be not more than $30^{\circ}$ in ordinary soil and $45^{0}$ for sandy soils.
3. Well Steining: The thickness of the Steining should be adequate for the stresses developed during sinking and after installation. The thickness of the Steining may be obtained from the following equation:
$\pi B(H-h) f=\pi(B-t) \gamma_{c} t H+P$
Where $\mathrm{H}=$ depth to which the well has progressed, $\mathrm{h}=$ height at which it has got suspended, $\mathrm{B}=$ diameter $=$ weight added for sinking.
The design of Steining reinforcement depends upon the skin friction and the unit weight of the well. It is usual practice to provide reinforcement of about 5 to $6 \mathrm{~kg} / \mathrm{m}^{3}$ of the brick and concrete Steining. About $75 \%$ of the total reinforcement is in the form of vertical reinforcement and $25 \%$ in the shape of the laterals or hoop rings. The vertical reinforcement is spread near both faces. The laterals should be checked for the moment developed due to the eccentric kentledge and half the weight of the well at an eccentricity of one fourth the width of well in any direction. This condition is generally critical when the well has sunk to about half the designed depth.
The thickness of the Steining is usually fixed empirically. For railway bridges in India, it is generally taken as one fourth of the outside diameter. For road bridges, it is kept as one eight of the outside diameter if it is in brick masonry and one tenth of the outside diameter if in cement concrete. However, the thickness is increased by 12 cm
per 3 m of depth after the first 3 m of Steining in brick masonry and 15 cm per 6 m of depth after the first 6 m for cement concrete.
A thumb rule commonly used is
$t=K(B / 8+H / 100)$
Where $\mathrm{B}=$ external diameter of well, $\mathrm{H}=$ depth below low water level,
$\mathrm{K}=\mathrm{a}$ constant $=1.0$ for sandy soils
$=1.1$ for soft clay and
$=1.25$ for hard clay and boulders
4. Bottom plug: The bottom plug should be strong enough to transmit the load to the soil below. It is designed as a thick plate subjected to unit bearing pressure under the maximum vertical load.
The bottom plug is given the shape of an electric bulb to produce an arch action, to reduce hoop tension in the curb and to provide larger base area. About $10 \%$ extra cement is added because some cement is washed away on account of water. The water in the well must be still and its normal level. Bottom plugging should always be done in one continuous operation.
While founding the well on rock, it should be properly anchored by taking it 25 cm to 30 cm deep into the rock bed. Adequate dowel bars should be provided.
5. Well Cap: The bottom of the well cap is generally kept at the low water level. It is designed as a slab resting on the well. The Well cap may be extended as cantilevers to accommodate piers of slightly larger size than that of the well.

If the width of the pier is greater than that size of the dredge hole, it is assumed that the weight of a cone of concrete having an apex angle of $60^{\circ}$ is carried by the slab and the remaining load is transmitted to the Steining. The well cap should have a minimum reinforcement of about $80 \mathrm{~kg} / \mathrm{m}^{3}$.
6. Top plug: The function of the top plug is to transmit the load of the pier to the well Steining. If a well cap is provided, there is no need of a top plug. However, it is generally provided as an extra safety precaution. Offsets are provided at the top of the Steining to provide bearing to the plug. Cement concrete (1:2:4) is used for the construction of the top plug.
7. Sand Filling: The main purpose of sand filling is to provide stability to the well by increasing its weight and to reduce the tensile stresses caused at the base by bending moment. However, sand cannot be depended upon for transmitting the weight of the pier to the bottom plug and therefore, it does not contribute towards the structural strength of the well.

On the Indian railways, the practice is to do the sand filling upto the top plug. Some of the highway engineers recommend that the sand filling should be done upto the lowest scour level. The actual depth of the filling should be fixed considering the requirement of the dead weight for the stability.

### 6.5 Sinking of the Wells:

The sinking operation consists of the following steps:

1. Laying the well curb: If the river bed is dry, the cutting edge over which the well curb is to be built is placed at the required position after excavating the river bed to about 15 cm . When the depth of water is up to 5 m , a sand island should be large enough to accommodate the well with adequate working space all around. In the case of depth of water more than 5 m , it is generally more economical to build the curb on dry ground at the river bank and float it to the site.

It is useful to insert wooden sleepers below the cutting edge at regular intervals to distribute the load evenly over the ground. The shuttering of the well curb is then erected. The outer shuttering is generally made of steel and the inner shuttering of brick masonry. The reinforcement of the curb is then placed in proper position such that the vertical bars project about 2 m above the top of the curb. Concreting of the curb is done in one continuous operation. The curb concrete is allowed to set at least for one week before the shuttering is stripped off. The sleepers are also then removed.
2. Well Steining: After sinking the well curb, the Steining is raised by about 1.5 m at a time and its sinking done after allowing at least 24 hrs for setting. The Steining is built using straight edges, preferably of angle iron. The lower portion of the straight edges is kept butted with the Steining of the earlier stages. The Steining must not be built in plumb at any intermediate stage when the well has tilted to one side. Once the well has acquired a grip of about 6 m in ground, the Steining can be raised by about 3 m at a time. The height of Steining built at any stage should be such that the well does not lose stability.
3. Sinking Process: Sinking process is begun after having cast the curb and the first stage of Steining and allowing enough time for curing. The well is sunk by excavating material from inside the curb manually or mechanically. When the depth of water inside the well is up to 1 m , dredging can be done manually. However, beyond this depth of water, excavation is done with the help of jhams. Jham is tied to a rope moving or pulley. It is pulled by the men. Every time, a diver dives and pushes the Jham into the soil and comes up. The Jham is then pulled out. In an improved version, the Jham has been replaced an automatic grab operated by diesel winches. Straight chisels are used for breaking hard material so that is can be taken out by grab. Under cutting chisels are used to loosen the material which lies under the Steining. Explosives are used for sinking through rock.
As the well sinks, the friction on the sides increases. To accelerate the process of sinking, additional loading, known as kentledge, is applied on the well. Kentledge is generally in the form of sand bags placed on a suitable platform erected on its top such that it does not interfere with the excavation. Sometimes, even kentledge is not sufficient to sink the well. In such cases, the frictional resistance developed on its outer periphery is reduced by forcing jets of water on the outer face. However, this method is effective only in the case of wells sunk in sandy strata.
In some cases, pumping out the water from inside the well is effective in well sinking. However, this method should be discouraged in early stages when the depth is shallow. It is not desirable to resort to pumping out water unless the well has gone
deep enough or has passed through clayey strata so that the chances of tilt and shift are reduced. Dewatering is not allowed after the well has sunk to about 10 m . After this stage, the sinking is done by usual methods of grabbing, chiselling, applying kentledge or blasting.
Great precaution is necessary if dewatering of the well is done when it is at a shallow depth to avoid blowing of sand from under the cutting edge. If blowing of sand occurs, it results in the loss of time and labour in removing the sand. It also presents danger to the men working inside, as the well may get filled up to a height of a few meters if the blow is large. The well may also tilt suddenly. Scrap gunny bags and grass bundles are placed around the periphery of the well on the outside into the funnel formed. It blocks the passage through which the blow of sand is taking place.

### 6.6 Measures for rectification of tilts and shifts:

The well should be sunk straight and at the correct position. However, it is not easy task to achieve this objective. Sometimes, the well tilts on one side or it shifts away from the desired position. The following precautions must be taken to avoid tilts and shifts.

1. The outer surface of the well curb and Steining should be regular and smooth.
2. The diameter of the curb should be kept about 4 to 8 cm larger than the outer diameter of Steining and the well should be symmetrically placed.
3. The cutting edge should be of uniform thickness and sharpness.
4. Dredging should be done uniformly on all sides in a circular well and in both pockets of a twin well.

Tilts and shifts must be carefully checked and properly recorded. The correct measurement of the tilt is an important filed observation required during well sinking. It is not possible to specify the permissible limits of tilts and shifts. Each case should be examined individually. IS: $3955-1967$ recommends that tilt should generally be limited to 1 in 60 . The shift should be restricted to one percent of depth sunk. In case they exceed the above limits, the following measures are taken for their rectification:

1. Regulation of grabbing: To rectify the tilt, the higher side grabbed more by regulating the dredging. However, this method is effective in initial stages of sinking. In case of tilted wells, dredging does not work satisfactorily. In such a case, a hole is made in the Steining on the higher side and the rope of the grab is pulled through the hook. Thus dredging with hooking is done. Alternatively, the well may be dewatered, if possible, and open excavation may be carried out on the higher side.

2. Eccentric loading: To provide greater sinking effort on the higher side, eccentric loading is applied by adjusting the kentledge. A suitable platform is constructed on the higher side for this purpose. As the sinking progresses, heavier kentledge with greater eccentricity would be required to rectify the tilt.

3. Water Jetting: If the water jets are applied on the outer face of the well on the higher side, the side friction is reduced and the tilt is rectified.
4. Excavation under cutting edge: A tilted well in a hard clayey stratum does not straighten due to unbroken hard stratum on the higher side. If dewatering of the well is possible, open excavation is done under the cutting edge. In case dewatering is not possible, divers can be sent to loosen the strata.
5. Inserting wooden sleeper under the cutting edge: Sometimes wooden sleepers are inserted temporarily below the cutting edge on the lower side to avoid further tilt. Alternatively, a hook is inserted below the cutting edge on the lower side and pulled with a wire rope and kept strained.
6. Pulling the well: In early stages of sinking, pulling the well towards the higher side, by placing one or more steel ropes around the well with vertical sleepers
packed in between to distribute the pressure over larger areas of well Steining, is quite effective.
7. Strutting the well: The well is strutted on its tilted side with suitable logs of wood to prevent further tilt. The well Steining is provided with sleepers to distribute the load from the strut. The other ends of the logs rest against a firm and non-yielding base having driven piles.
8. Pushing the wells by jacks: The tilt can be rectified by pushing the well with a suitable arrangement through mechanical or hydraulic jacks. Figure shows a tilted well being pushed by a jack resting against the vertically sunk well.


## Assignment-Cum-Tutorial Questions

## A. Questions testing the remembering / understanding level of students

## I) Objective Questions

1. Well foundations are commonly used as foundation for the $\qquad$ .
2. In case of well foundation, the IS code recommends that tilt and shift of well should respectively be $\qquad$
3. A well foundation is a type of $\qquad$
4. The grip length below the maximum scour level for the railway bridges is usually
5. The thickness of Steining for railway bridges is usually kept as $\qquad$ of outside diameter.
6. To provide greater sinking effort eccentric loading is applied on the $\qquad$ side.
7. The tilt can be rectified by pushing the well with a suitable arrangement through
$\qquad$ jacks.
8. The water jets are applied on the $\qquad$ face of the well.

## II) Descriptive Questions

1. What are the design features of top plug
2. Discuss the situations where a well foundation is more suitable than other type of foundations.
3. What are the forces acting on a well foundation?
4. What are different shapes of wells?
5. What are the components of a well foundation
6. What are the design features of a cutting edge
7. What are the precautions are to be taken to avoid tilts and shifts.
8. What is the effect of water pressure on well foundations?

## B. Question testing the ability of students in applying the concepts.

## I) Multiple Choice Questions:

1. In some well foundation, the following is not provided
a) RCC Well Cap
b) Top plug
c) Bottom plug
d) Curb
2. the most commonly used shape of a well foundation is
a)Double D Well
b) Circular Wells c)Double Octagonal Well
d) Rectangular Wells
3. Bending stress are more in the $\qquad$ type of well foundation.
a)Circular Wells b) Twin Circular Wells c)Double D Wells d) Rectangular Wells
4. Twin circular well foundations are advantageous in which of the following case
a) When the depth of sinking is small and bearing capacity is high
b) When the depth of sinking is high and bearing capacity is high
c) When the depth of sinking is high and bearing capacity is small
d) None of the above
5. Sequence of the components of well foundations from bottom to top
a) Bottom plug, Steining, Well Curb, Well Cap
b) Bottom plug, Steining, Top Plug, Well Cap
c) Bottom Plug, Well Curb, Well Cap, Steining
d) None of the above
6. The diameter of the curb should be kept larger than the outer diameter of Steining
a) 4 to 8 cm
b) 10 to 12 cm c) 12 to 13 cm
d) 5 to 8 cm
7. The cutting edge should be
a) Sharp b) thin c) soft d) smooth
8. Some of the highway engineers recommend that the sand filling should be done
a) Up to the lowest scour level. b) At the lower level c)at the middle level d)up to top portion

## II) Descriptive \& Problems:

1. Explain an open caisson (well foundation) with a neat sketch showing all the component parts.
2. Discuss the method of construction of well foundation and its sinking into the ground?
3. What are the tilts and shift? What are the remedial measures to control these?
4. Discuss the various forces acting on a well foundation.
5. What are the cares to be taken, if there is a difference in level of sinking of well during the construction?
6. Sketch a completed well foundation for a bridge pier indicate various components and their functions
7. What are the merits and demerits of the well foundation?
8. What are the different shapes of wells? Discuss the characteristics of each type.
9. What are the design features of a well curb?
