HANDOUT FOR ENGINEERING PHYSICS

Subject : Engineering Physics

Class : I B.Tech. - II SEM   Year : 2019-19

Branch : Civil Engineering   Credits : 03

1. **Course Objectives**
   - To solve oscillating systems problems.
   - To analyse crystal parameters to investigate crystal structures.
   - To apply principles of optics for engineering applications.

2. **Course Outcomes**

Upon successful completion of the course, the students will be able to

1. derive expression for oscillations (SHM).
2. assess the main characteristics of sound propagation.
3. recognize crystal structures and X-Ray crystallography.
4. relate basic knowledge of NDT to carry out inspection in accordance with the established procedures.
5. apply principles of interference, diffraction and polarization and LASERS to engineering situations.

3. Program Outcomes

a. An ability to apply knowledge of mathematics, science and engineering principles to civil engineering problems.
b. An ability to analyze design and conduct experiments and interpret the resulting data.
c. An ability to design a system, component or process to meet desired goals in civil engineering applications.
d. An ability to function on multi disciplinary teams.
e. An ability to identify, formulate and solve challenging engineering problems.
f. An understanding of professional and ethical responsibility.
g. An ability to communicate effectively through verbal, written and drawing presentations.
h. An ability to understand the impact of engineering solutions in a global, economical and social context with a commitment on environmental and safety issues.
i. An ability to recognize the need of engaging in lifelong learning and acquiring further knowledge in specialized areas.
j. Ability to excel in competitive examinations, advanced studies and become a successful engineer in construction industry.
k. An ability to use the techniques, skills and modern engineering tools and software for engineering design and practices.
l. The understanding of basic finance & management techniques and construction practices including work procurement and legal issues.

4. Mapping of Course Outcomes with Program Outcomes

	a	b	c	d	e	f	g	h	i	j	k	l
CO1	H	H							H			
CO2	H	M							H			
CO3			H						H			
CO4	H		H						H			
CO5	H	H							H			

5. Mapping of the course with Program Outcomes

	a	b	c	d	e	f	g	h	i	j	k	l
Subject Name												

6. Gaps Identified In The Course
   None
7. Contents Beyond Syllabus

   None

8. Mapping of Contents Beyond Syllabus with Program Outcomes

	a	b	c	d	e	f	g	h	i	j	k	l
CBS 1												
CBS 2												

9. Prescribed Text Books
   - RK Gaur & SLGupta, Engineering Physics(Edition 2011), Dhanapat rai publications
   - Dr. M.N. Avadhanulu, Dr. P.G.Kshirsagar Engineering Physics(9^{th} Edition), S.Chand
10. Reference books
   - D.K.Bhattacharya, Poonam Tandon, Engineering Physics, Oxford University Press.
   - Charles Kittel, Introduction to solid state physics, Wiley India Pvt. Ltd.
   - B.B. Laud, Laser and Non-Linear Optics, New Age international publishers
   - P.K. Palanisamy, Engineering Physics, SciTech publications.

11. URLs And Other E-Learning Resources
    http://dae.nic.in/?q=node/179
    www.hyperphysics.phy-astr.gsu.edu/HBASE/hframe.html
    http://www.virginia.edu/bohr/mse209/

12. Brief History And Current Developments In The Subject Area

- **"Physics is the subject, dealing with Nature & Natural Phenomena"**
  Every concept that we study in Physics is a consequence of Natural Observation. For example shadow formation led to the concept of Light Nature. A sound knowledge of engineering physics is essential for the engineering student to reach new heights of life.
- **Physics and Technology**
  The technological development of any society is very closely related with the applications of physics. Steam engines and the detailed study of heat and thermodynamics were the

initiators of the industrial revolution. The development of transistors and development of computers were the initiators of IT revolution. We see the applications of physics in every walk of life. The variation of shades of the painted building with temperature is a result of the advancements in nano-particles. The noise absorbers, developments in the acoustic architecture, aseismic designs etc., are due to analysis of vibrations. The testing of materials for presence of flaw in the structure developed without actual destruction shows the successful utilization of ultrasonics for testing. The structural analysis of materials helps in optimum utilization of the materials. The advances in optics resulted in anti-reflective coatings, stress detection in the model structures etc.

13. Pre-Requisites

Basic Knowledge of Mathematics, Fundamentals in Physics

14. Lecture Schedule / Lesson Plan

S.No	TOPIC	No. of Periods	No. of Tutorials
	Unit I: Simple Harmonic Motion		
1	Simple harmonic motion, displacement, amplitude, period	1	1(T1)
2	Frequency, Phase, Wavelength	1	
3	Equation for SHM, undamped harmonic oscillator	1	
4	Energy of undamped harmonic oscillator	1	
5	Equation of motion of damped harmonic oscillator	1	
6	Under damped, critically damped, over damped conditions, Energy of damped harmonic oscillator	1	
7	Power dissipation, relaxation time, Quality factor, logarithmic decrement in amplitude	1	
8	Forced vibrations- transient and steady state conditions	1	
9	Under damped, critically damped, over damped conditions	1	1(T2)
10	Amplitude resonance	1	
11	Problems	1	
	Unit II Acoustics		
12	Reverberation time- Sabine's formula	1	1(T3)
13	Determination of absorption coefficient-absorption properties of surfaces	1	
14	Measurement of absorption in a room,	1	
15	Acoustic noise	1	
16	Factors effecting acoustics of buildings	1	1(T4)
17	Acoustic design of a hall, Acoustic quieting	1	
18	Methods for acoustic quieting, Sound proofing	1	
19	**Problems**	1	
	Unit III Crystal Structures		1(T5)
20	Space lattice – Basis – Unit Cell – Lattice parameters	1	
21	Bravais lattices – Crystal systems	1	
22	Structures and packing fractions of SC	1	
23	Structures and packing fractions of BCC and FCC	1	
24	Directions and planes in crystals – Miller indices	1	1(T6)
25	Separation between successive (h k l) planes	1	
26	Bragg's law	1	
27	Laue method, Powder diffraction method	1	
28	Problems	1	
	Unit IV Non-Destructive testing using ultrasonics		

29	Basic principles of ultrasonic testing	1	1(T7)
30	Piezoelectric transducers	1	
31	Couplant and inspection standards	1	
32	Pulse-echo technique	1	
33	Flaw detector	1	
34	Different types of scans, Applications	1	
35	Problems	1	
36	**UNIT – V: Physical Optics**		
37	Interference in thin films	1	1(T8)
38	Newton's rings – path difference	1	
39	Conditions for bright and dark rings, Fraunhofer Diffraction	1	
40	Fraunhofer diffraction at single slit	1	1(T9)
41	Intensity distribution due to single slit	1	
42	Fraunhofer diffraction due to grating	1	
43	Resolving power of grating, Polarization	1	
44	Double refraction, Quarter wave plate and half wave plate	1	
45	Problems	1	
	UNIT – VI: Laser		
46	Characteristics of lasers	1	1(T10)
47	Spontaneous and Stimulated emission of radiation – Einstein's coefficients	1	
48	Requirements of lasers, three energy level system	1	
49	Four energy level systems, Ruby Laser	1	
50	Helium Neon laser	1	1(T11)
51	CO_2 laser, Applications	1	
52	Problems	1	
	Total	**52**	**11**

ENGINEERING PHYSICS

UNIT-I
SIMPLE HARMONIC MOTION

Objective:

- Understand periodic motion of particles of medium and restoring forces proportional to displacement

Syllabus:

Simple harmonic motion-Displacement-Amplitude-period-frequency-phase-wavelength - equation for simple harmonic motion- free and forced -Theoretical analysis(a) free vibrations, (b)Damped vibrations, (c) Forced vibrations – Resonance.

Outcomes:

At the end of the unit, the students are able to

1. understand the basic definitions of simple harmonic motion
2. analyse the undamped, damped and forced harmonic motions.
3. Understand the concept of resonance.

Classification of Vibrations

Vibrations can be classified in several ways. Some important classifications are

1. Free and Forced vibrations
2. Un-damped and Damped vibrations
3. Deterministic and Random vibrations
4. Steady and Transient vibrations
5. Linear and Non-linear vibrations
6. Rectilinear and Torsional Vibrations

We concentrate on free and forced vibrations. The specific cases considered are un-damped and damped vibrations.

`Free Vibrations:

The system oscillates on its own after initial disturbance

Forced vibrations:

The system oscillates under external influence

Un-damped Vibrations:

The vibrations in which no energy is lost or dissipated

Damped vibrations:

The vibrations, that involve loss of energy due to friction and other resistances.

We consider simple harmonic motion. In simple harmonic motion, the following cases are considered.

a. Free un-damped vibrations
b. Free damped vibrations
c. Forced vibrations

Simple Harmonic Motion

Definition: The motion of a body to and fro about a fixed point is called simple harmonic motion.

Some features of simple harmonic motion are

1. The motion is periodic
2. When displaced from the fixed point or mean position, a restoring force acts on the particle, tending to bring it to the mean position.
3. Restoring force is directly proportional to its displacement.

Displacement:

The displacement of simple harmonic oscillator is given by

$$\boldsymbol{x = A\sin(\omega t + \phi)}$$

where A is amplitude, ω is angular frequency and ϕ is the phase

Amplitude: The maximum displacement or deformation of a vibrating system from its mean position is called amplitude.

Time period: The time taken for one complete oscillation.

For a simple harmonic oscillator,

Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

where m is mass and k is elastic constant.

The time period does not depend on amplitude of the oscillation.

Frequency: The number of oscillations per second

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where m is mass and k is elastic constant

The frequency of oscillations is independent of amplitude of oscillations.

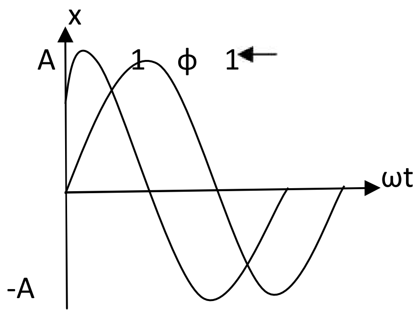
Phase:

Displacement,

$$\boldsymbol{x = A\sin(\omega t + \phi)}$$

The angle (ωt+ϕ) is called phase of the oscillation.

ϕ is called phase angle. It is useful in comparing the motions of two bodies



Velocity: The velocity 'v' of the oscillating particle,

$$v = \frac{dx}{dt} = \frac{d}{dt}(A\sin(\omega t+\phi))$$

$$= \omega A\cos(\omega t+\phi)$$

$$= \omega\sqrt{a^2 - x^2}$$

At mean position, x=0

$v_{max} = \omega a$

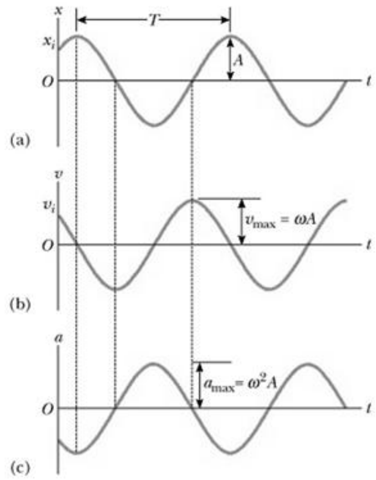
Velocity is zero at the extreme positions.

Acceleration:

The acceleration of an oscillating particle,

$$a = \frac{d^2x}{dt^2} = -\omega^2\sin(\omega t+\phi)$$

$$= -\omega^2 x$$



Equations of motion of linear Harmonic Oscillator

Let 'm' be the mass of particle, x be the displacement of the object from its equilibrium position and k be the linear restoring force constant.

Then equation of motion in one dimension is

$$m\frac{d^2x}{dt^2} = -kx. \quad \text{--- 1}$$

Let the solution of eqn 1 be

$$x = a\, e^{\alpha t} \text{ ---2}$$

where a and α are constants

From eqn 1

$$\frac{d^2x}{dt^2} = \frac{-k}{m} x = -\omega^2 x \text{---3}$$

where $\omega^2 = \frac{k}{m}$ --- 4

From eqn 2

$$\frac{dx}{dt} = a\alpha\, e^{\alpha t}$$

$$\frac{d^2x}{dt^2} = a\alpha^2 e^{\alpha t} \longrightarrow 5$$

Subs eqn 5 and 2 in eqn 3

$a\alpha^2 e^{\alpha t} = -\omega^2\ ae^{\alpha t}$

$(\alpha^2 + \omega^2)\, ae^{\alpha t} = 0$

$\Rightarrow (\alpha^2 + \omega^2)\, x = 0$

$\Rightarrow (\alpha^2 + \omega^2) = 0$

$\alpha^2 = -\omega^2$

$\Rightarrow \alpha = \pm i\omega$

Hence $\alpha = +i\omega$ and $\alpha = -i\omega$

The general solution for eqn 1 is

$x = x_1 + x_2$

$= a_1 e^{i\omega t} + a_2 e^{-i\omega t}$

$= (a_1+a_2)\cos\omega t + i\,(a_1-a_2)\sin\omega t$

Put $(a_1+a_2) = A\sin\theta$

And $(a_1 - a_2) = A\cos\theta$

Then $x = A\sin\theta\ \cos\omega t + A\cos\theta \sin\omega t$

$= A\sin(wt + \theta) \longrightarrow 6$

Equation 6, represents the displacement in simple harmonic oscillator, A is the amplitude ω is the angular frequency and θ is the phase of the oscillator.

Example: Simple pendulum, physical pendulum, mass attached to a spring etc.

Three important conditions for SHM:

In case of oscillators, the following three conditions should be satisfied for simple harmonic oscillator. They are

1. There should be a position of equilibrium
2. There should not be any energy loses

3. The acceleration of the object is directly proportional to displacement, but it is in opposite direction.

Energy of simple harmonic oscillator (Undamped or free)

A simple harmonic oscillator possesses both kinetic energy (KE) and potential energy (PE).

→ KE is by virtue of inertia (mass)

→ PE is due to the displacement

There is continuous transfer between the two energies KE and PE, but the total energy of the system remains constant at a given instant of time. At equilibrium position PE=0, at extreme positions, KE=0.

$$\text{Kinetic energy, } E_k = \tfrac{1}{2} mv^2$$
$$= \tfrac{1}{2} m\omega^2 A^2 \cos^2(\omega t+\phi)$$
$$= \tfrac{1}{2} m\omega^2 A^2 (1- \sin^2(\omega t+\phi))$$
$$= \tfrac{1}{2} m\omega^2 (A^2 - A^2 \sin^2(wt+\phi))$$
$$= \tfrac{1}{2} m\omega^2 (A^2 - x^2)$$
$$= \tfrac{1}{2} k (A^2 - x^2)$$

where elastic constant, $k= m\omega^2$ -- 1

Potential energy, U is

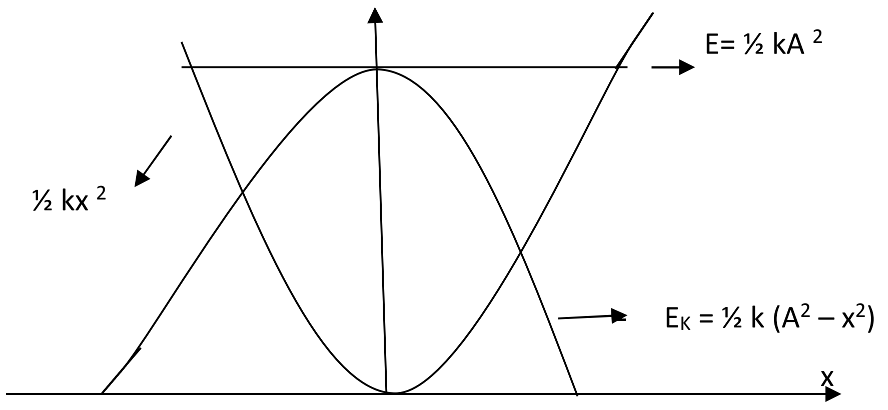
$$U= -\int^{x} Fdx$$
$$U= -\int_0^{x} kdx = \tfrac{1}{2} kx^2 \quad \text{-- 2}$$

Total Energy,

$$E= KE + PE$$
$$= \tfrac{1}{2} k (A^2 - x^2) + \tfrac{1}{2} kx^2$$
$$= \tfrac{1}{2} kA^2 \text{ Constant}$$

The total energy is constant for simple harmonic oscillator. It is proportional to the square of amplitude.

→ It is independent of time



Damped Harmonic Oscillator:

Undamped harmonic oscillator is an ideal situation. Frictional forces play an important role in the vibrations. In normal conditions, there is a gradual decease in the amplitude of the vibrations and they vanish. This kind of motion is said to be damped. The vibrations are damped by friction and the oscillator is called damped harmonic oscillator.

Damping:

The phenomenon of decay in amplitude of oscillations is known as damping

Ex: a pendulum immersed in water.

In addition to restoring force, damping force exists in opposite direction to velocity. The following are the two forces acting on damped harmonic oscillator.

i) Restoring force,

Restoring force α - x

F= - k x; k is linear restoring force

ii) Frictional force, the damping in amplitude of oscillations is due to the changes in velocity, hence

Damping force = $-\gamma \frac{dx}{dt}$

where γ is positive constant that depends the medium and shape of the body.

Hence, $F = -kx - \gamma\frac{dx}{dt}$ --- 1

From Newton's second law,

F=ma

$= m\frac{d^2x}{dt^2}$ --- 2

From 1 & 2

$m\frac{d^2x}{dt^2} = -kx - \gamma\frac{dx}{dt}$

$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$

$\frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \omega^2_o x = 0$ --- 3

where $\omega_0{}^2 = \frac{k}{m}$ --- 4

and ω_o is the natural frequency of the system.

Solution to equation 3 is

$x = Ae^{-\gamma t/2m}\cos^2(\omega t+\phi)$ --- 5

$\frac{dx}{dt} = -\frac{\gamma}{2m}Ae^{-\gamma t/2m}\cos(\omega t+\phi) - \omega Ae^{-\gamma t/2m}\sin(\omega t+\phi)$--- 6

$\frac{d^2x}{dt^2} = (\frac{\gamma^2}{4m^2} - \omega^2)\, A\, e^{-\gamma t/2m}\cos(\omega t+\phi) + \frac{\omega\gamma}{m}A\, e^{-\gamma t/2m}\sin(\omega t+\phi)$ --- 7

Substitute eqn 5, 6 and 7 in eqn 3

On simplification,

$= (\frac{\gamma^2}{4m^2} - \omega^2 - \frac{\gamma^2}{4m^2} + \omega_0{}^2)\, Ae^{-\gamma t/2m}\cos(\omega t+\phi) = 0$

$\Rightarrow -\frac{\gamma^2}{4m^2} - \omega^2 + \omega_0{}^2 = 0$

$\Rightarrow \omega^2 = \omega_0{}^2 - \frac{\gamma^2}{4m^2}$

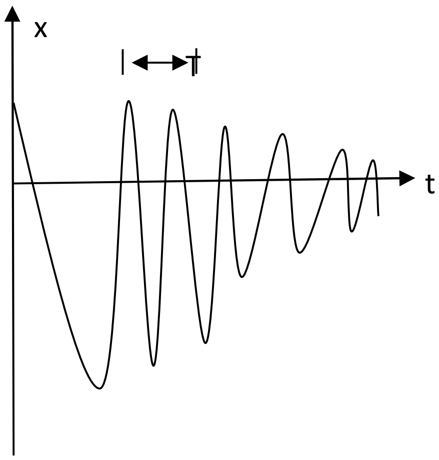
Angular frequency of damped oscillations is

$\omega = \sqrt{\omega_0{}^2 - \frac{\gamma^2}{4m^2}}$ -- 8

Case 1: Weak damping

When $\omega_0 > \frac{\gamma}{2m}$

Frequency of oscillator is real and positive quantity. It is similar to undamped simple harmonic oscillator. The difference is the amplitude of oscillations decreases exponentially as $Ae^{-\gamma t/2m}$ but the frequency remains constant Example: - Motion of pendulum in air, oscillations in LCR circuit, ballistic galvanometer.



Case 2: Heavy damping or over damping:

When $\gamma > 2m\omega_o$, ω is imaginary

The solution to eqn 3 ◯ is of the form

$$x(t) = C_1 e^{-\gamma t} + C_2 e^{\gamma t}$$

where C_1 and C_2 are constants. There are no oscillations. The amplitude decreases exponentially with time and dies out finally.

Application : The Phenomenon is used in the door closing mechanism.

Case 3:

Critical Damping

When $\gamma = 2m\omega_o$, $\omega = 0$

There is no oscillation, the system approaches equilibrium as fast as possible

Ex: in ballistic galvanometer.

Application: In shock absorbers of motorbike

Digression:

A shock absorber contains a spring in a scaled container with liquid. They decrease the effect of joints of a bumpy trail.

Energy of damped harmonic oscillator.

In underdamped harmonic oscillator, the amplitude of a system decays exponentially in time.

$$KE = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$$
$$= \frac{1}{2} m A^2 e^{-\gamma t/2m} \left\{\frac{\gamma r}{2m} \cos(\omega t+\phi) - \omega \sin(\omega t+\phi)\right\}^2$$

We consider the time average condition we assume that the amplitude remains nearly constant in one oscillation

$$\langle KE\rangle = \frac{1}{4} m A^2 e^{-\gamma t/2m} \left\{\frac{\gamma r^2}{4m^2} + \omega^2\right\}$$

Since $\langle \sin^2\theta\rangle = \langle \cos^2\theta\rangle = \frac{1}{2}$

$\langle \cos\theta \sin\theta\rangle = 0$

For underdamped oscillator, $\omega \cong \omega o$

where ω_o is the natural frequency of the system and $\frac{\gamma^2}{4m^2} << \omega_o^2$

Hence $\langle KE\rangle = \frac{1}{4} m\omega_o^2 A^2 e^{-\gamma t/2m}$ --- 1

Potential Energy,

$$\langle U\rangle = \frac{1}{2} m\omega_o^2 A^2 \langle e^{-\gamma t/2m} \cos^2(\omega t+\theta)\rangle$$
$$= \frac{1}{2} m \omega_o^2 A^2 e^{-\gamma t/2m} (\tfrac{1}{2})$$
$$= \frac{1}{4} m \omega_o^2 A^2 e^{-\gamma t/2m} \quad --- \ 2$$

Hence total Energy,

$E = KE+PE$

$= \frac{1}{4} m\omega_o^2 A^2 e^{-\gamma t/2m} + \frac{1}{4} m \omega_o^2 A^2 e^{-\gamma t/2m}$

$= \frac{1}{2} m\omega_o^2 A^2 e^{-\gamma t/2m}$ --- 3

The energy of the oscillator decreases with time.

Power Dissipation :

The rate at which the energy is lost is defined as power dissipation, P

$$P = -\frac{dE}{dt}, \quad \text{but } E = \frac{1}{2} m\omega_o^2 A^2 e^{-\gamma t/2m}$$

$$= -\frac{d}{dt}\left(\frac{1}{2} m\omega_o^2 A^2 e^{-\gamma t/2m}\right)$$

$$= \frac{1}{2} m\omega_o^2 A^2 e^{-\gamma t/2m}\left(-\frac{\gamma}{m}\right)$$

$$= \frac{\gamma}{m} E$$

Logarithmic decrement :

In the damped harmonic oscillator, the amplitude of the oscillations decreases logarithmically.

Amplitude, $A = A_o e^{-\gamma t/2m}$

Let A_1, A_2, A_3, ------------------ be the amplitudes at

Time t= T, 2T, 3T, ……….. respectively.

Where T is time period.

$A_1 = A_o e^{-\gamma t/2m}$

$A_2 = A_0 e^{-\gamma t/2m}$

$A_3 = A_o e^{-\gamma t/2m}$

…………………………………………….

$$\frac{A_0}{A_1} = \frac{A_1}{A_2} = \frac{A_2}{A_3} = \ldots\ldots\ldots = e^{-\gamma t/2m}$$

$$= e^{\lambda}$$

Where 'λ' is known as logarithmic decrement

$$\lambda = \log_e \frac{Ao}{A1} = \text{Log}_e \frac{A1}{A2} = \ldots\ldots\ldots\ldots$$

Logarithmaic decrement:

The natural logarithm of the ration between two successive maximum amplitudes what are separated by one period.

Relaxation Time:

The time taken for the total energy to decay to (1/e) of its original value

We known, $E = 1/2\, m\, \omega_o^2 A^2 e^{-\gamma t/2m}$ -- 1

At t=0, $E = \frac{1}{2} m\omega_o^2 A^2 = \frac{1}{2} kA^2 = E_o$ { since, $k = m\omega_o^2$ }

Hence $E = E_o e^{-\gamma t/2m}$

at $t = \tau$, $E = E_o/e$

Hence $e^{-1} = e^{-\gamma \tau/2m}$

$$\frac{\gamma\tau}{m} = 1 \Rightarrow \tau = \frac{m}{\gamma}$$

Quality Factor:

The quality factor is defined as 2Π times the energy stored in the system to the energy lost per period.

Energy stored in system, = E

Energy dissipated per period, $= \frac{\gamma E}{m} T = \frac{E}{\tau} T$

Where relaxation time, $\tau = \frac{m}{\gamma}$

$$\text{Hence } Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per period}}$$

$$= 2\pi \frac{E\,\tau}{E\,T}$$

$$= \frac{2\pi}{T}\tau = \omega\,\tau$$

Hence, higher the value of Q, higher could be the value of relaxation time, τ which implies that lower the damping.

Forced Vibrations

The energy of a damped oscillator decreases in time as a result of dissipative force. The oscillations sustain under external force. The resulting oscillations of the system are forced vibrations

Forced Oscillations:

The oscillations of a system, under external force are called forced oscillations.

On applications of external force two stages are observed.

1. Transient Period
2. Steady State Motion

Transient Period: The period in which the free oscillations of the system die out.

Steady State Motion: The state when a system/particle performs vibrations or oscillations with exactly the frequency of external force.

In mathematical analysis, we consider the steady state motion.

There are two frequencies in the case of forced oscillations, they are

1. Natural frequency (ω_o): The frequency of the considered object
2. Driving frequency (ω_f): The frequency of the external force.

The restoring force on the system,

$F_1 = - k x$ -- (1)

Where K is linear restoring force damping force on the system, T

$F_2 = - \gamma \frac{dx}{dt}$ -- (2)

Where r is damping coefficient

Driving force, $F_3 = F_o \cos\omega t$ -- 3

Hence total force on the system,

$F= F_1+F_2+F_3$

But from Newton's second law

$F= ma = m \frac{d^2x}{dt^2}$

Hence $m \frac{d^2x}{dt^2} = -\gamma \frac{dx}{dt} - kx + F_o \cos\omega t$

$\frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \frac{k}{m} x= F_o \cos\omega t$ -- 4

Let solution to LHS be

$x(t) = Ae^{i\propto t}$

Then $\frac{dx}{dt} = i\propto Ae^{i\propto t}$

$\frac{d^2x}{dt^2} = -\propto^2 Ae^{i\propto t}$

$-\propto^2 Ae^{i\propto t} + \frac{i\propto t}{m} Ae^{i\propto t} + \omega_0^2 Ae^{i\propto t} = \frac{Fo}{m} e^{i\omega t}$

$(-\propto^2 + \frac{i\propto t}{m} + \omega_0^2) Ae^{i\propto t} = \frac{Fo}{m} e^{i\omega t}$ -- 5

From Eqn 5

$\propto = \omega$ [the system oscillates with same frequency as the frequency of external force] and $A = \frac{F_0}{m} \frac{1}{(-\omega^2+\frac{i\omega\gamma}{m}+\omega_0^2)}$

$$= \frac{F_0}{m} \frac{1}{(\omega_0^2-\omega^2)+\frac{i\omega\gamma}{m})}$$

On rationalizing

$$A_o \, Cos\, \phi = \frac{F_0}{m} \frac{(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2} \quad ---- \quad 6$$

$$A_o \sin\phi = \frac{F_0}{m} \frac{\left(\frac{\omega\gamma}{m}\right)}{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2} \quad ---- \quad 7$$

Hence amplitude,

$$A_o^2 = \frac{F_0^2}{m^2} \frac{1}{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2}$$

$$A_o = \frac{F_0}{m} \frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2}} \quad --- \quad 8$$

Phase,

$$\tan\phi = \frac{-\frac{\gamma\omega}{m}}{(\omega_0^2-\omega^2)} \quad --- \quad 9$$

$$\phi = \tan^{-1} \frac{\frac{\gamma\omega}{m}}{\omega^2-\omega_0^2} \quad --- \quad 10$$

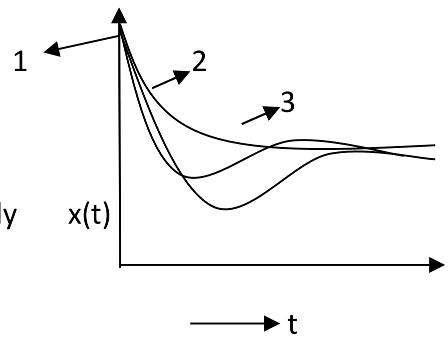
Equation 8 gives the amplitude where as 9 gives the phase of the displacement.

Hence the displacement can be written as

$X = A_0 e^{-i\phi} e^{i\omega t}$

$= Ae^{i(\omega t-\phi)}$

Where amplitude A_o and phase are given by eqs 8 and 9 respectively



From figure,

1 Underdamped oscillator
2 Critically damped oscillator
3 Over damped Oscillator

Case 1 When driving frequency is low,

$\omega << \omega_o$

Amplitude, $A_o = \frac{F_0}{m} \frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2}}$

$\cong = \frac{F_0}{m\omega_0^2} = F/K = \text{constant}$

Phase, $\theta = \tan^{-1} (\frac{\gamma}{m\omega_0}) = \tan^{-1}(o) = 0$

→The amplitude of the vibration is independent of frequency of force. It depends on the applied force and force constant, ‘K’.

→ The driving force and displacement are in phase.

Case: II When $\omega=\omega_o$

$$A_o = \frac{F_0}{m}\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2}}$$

For $\omega=\omega_o$

$$(A_o)_{max} = \frac{F_0}{m}\cdot\frac{m}{\gamma\omega_0} = \frac{F_0}{\gamma\omega_0}$$

$$\theta = \tan^{-1}\left(\frac{\gamma\omega/m}{0}\right) = \frac{\Pi}{2}$$

The amplitude of the vibration depends on the damping coefficient. If r is small, amplitude is large.

The displacement lags behind the force by ($\Pi/2$).

Case: III

Where $\omega >> \omega_0$, the applied frequency is greater than the natural frequency of the oscillator.

$$A_o = \frac{F_0}{m}\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2}}$$

$$\cong \frac{F_0}{m\omega^2}$$

$$\text{Phase, } \theta = \tan^{-1}\left[\frac{\frac{\gamma\omega}{m}}{(\omega_0^2-\omega^2)}\right]$$

$$= \tan^{-1}\left[\frac{\frac{\gamma\omega}{m}}{-\omega^2}\right] = \tan^{-1}(-0)$$

$$= \Pi$$

Hence the amplitude 'A' goes on decreasing and phase difference tends to 'Π'

Resonance:

When the frequency of the driving force is near the natural frequency, ω_o of the oscillating system, the oscillation amplitude becomes very large. This phenomenon is called resonance.

Definition: The increase in amplitude of vibration near the natural frequency is called resonance

Resonance frequency: The frequency at which resonance is observed

Following features can be observed at resonance frequency,

1. the energy transfer between applied force to the oscillator is maximum.
2. the external force and the velocity of the particle are in phase.

For forced vibrations,

$$\text{Amplitude, } A = \frac{F_0}{m}\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2}} \quad --- \quad 1$$

$$\tan\phi = \frac{-\frac{\gamma\omega}{m}}{(\omega_0^2-\omega^2)} \quad --- \quad 2$$

For amplitude to be maximum,

$$(\omega_0^2-\omega^2)^2 + \left(\frac{\gamma\omega}{m}\right)^2 \text{ is minimum}$$

$$\frac{d}{dw}\left((\omega_0^2-\omega^2)^2 + \left(\frac{\gamma\omega}{m}\right)^2\right) = 0$$

$$2(\omega_0^2-\omega^2)(-2\omega) + 2\frac{\gamma\omega}{m}\frac{\gamma}{m} = 0$$

$$(\omega_0^2 - \omega^2)^2 = \frac{\gamma^2}{2m^2}$$

$$\omega = \sqrt{\left(\omega_0^2 - \frac{\gamma^2}{2m^2}\right)} \qquad \text{---} \qquad 3$$

$$(\omega_0^2 - \omega^2)^2 + \left(\frac{\gamma\omega}{m}\right)^2 = \frac{\gamma^2}{m^2}\,\omega_0^2 - \frac{\gamma^4}{4m^4}$$

$$= \frac{\gamma^2}{m^2}\left(\omega_0^2 - \frac{\gamma^2}{4m^2}\right) \qquad \text{---} \qquad ④$$

Hence ① becomes,

$$A_{max} = \frac{Fo}{m \cdot \frac{\gamma}{m}\sqrt{\left(\omega_0^2 - \frac{\gamma^2}{4m^2}\right)}}$$

$$= \frac{Fo}{\gamma\sqrt{\left(\omega_0^2 - \frac{\gamma^2}{4m^2}\right)}}$$

At low damping,

$$A_{max} = \frac{F_0}{\omega_0 \gamma}$$

Hence $A_{max} \rightarrow \propto$ as $\gamma \rightarrow o$

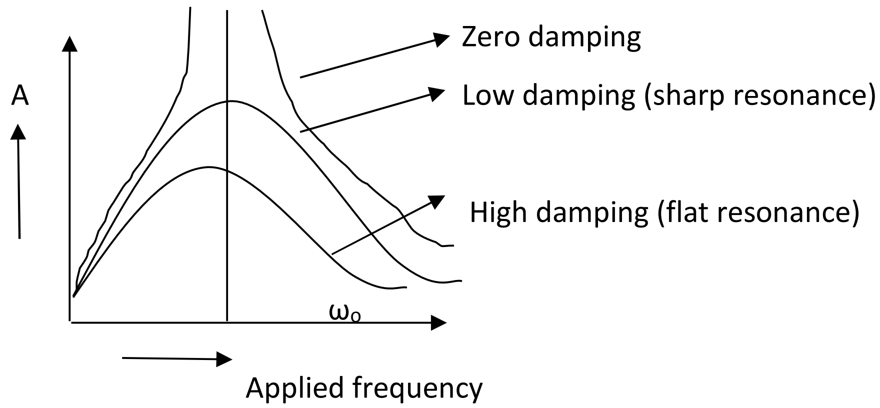


Figure: Variation of Amplitude with frequency

The following observations can be made from the amplitude – frequency curve.

1. When $\omega < \omega_o$, the amplitude is nearly the same for all values of damping

   As frequency increases

   a) For zero damping, the amplitude is infinite at $\omega = \omega_o$
   b) For lower damping, the frequency of maximum amplitude shifts towards lower frequency
   c) For higher damping, the amplitude decreases and becomes flat. The frequency at which the resonance occurs shifts to further lower frequencies.
   d) The lower the damping force, the sharper the resonance and the higher the damping force the flatter the resonance.

Assignment –cum- tutorial questions

SECTION-A

Objective questions

1. Acceleration of a particle is maximum when its displacement is maximum for ( )
   a) SHM b) an oscillatory motion c) a Linear motion d) all of those
2. For a particle, to execute simple harmonic motion, the force should be ( )
   a) Constant b) proportional and opposite to the displacement
   c) opposite to the displacement d) proportional to its displacement
3. For a linear harmonic oscillator, the driving forces are superposed ( )
   a) linearly b) quadratic ally
   c)cannot be superposed d) are neglected
4. In simple harmonic, motion the particle is ( )
   a) always accelerated b) alternately accelerated and restarted
   c) always retarded d) neither accelerated nor retarded
5. Expression for Hooke's law is
6. A particle executes linear simple harmonic motion with an amplitude of 2 cm. When the particle is at 1 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is ( )
   (a) $1/2\pi\sqrt{3}$ (b) $2\pi\sqrt{3}$ (c) $2\pi/\sqrt{3}$ (d) $\sqrt{3}/2\pi$ (e) $\sqrt{3}/\pi$
7. In the simple harmonic motion, if the displacement from equilibrium position is zero ( )
   i. potential energy is double than kinetic energy
   ii. potential energy is half of kinetic energy
   iii. potential and kinetic energy are equal
   iv. potential and kinetic energy are different
8. In the simple harmonic motion, there is a constant relation between mass displacement and ( )
   a) Frequency b)Acceleration c) Velocity d|) None
9. The effect of the frictional force on an oscillatory motion is that the velocity ( )
   a) increases exponentially with time
   b) decreases exponentially with time
   c) is unaffected by the passage of time
   d) first increases and then decreases as time passes
10. The transient period is the time for which the ( )
    a)force oscillations die out and steady state motion is established
    b)free oscillations and steady state motion take place together
    c) force oscillations phase out
    d) steady state motion phases out
11. For a higher damping factor, the resonance ( )
    a) is very high b)is flatter c) is unaffected d)varies linearly

SECTION-B

DESCRIPTIVE QUESTIONS

Short Answer Questions:

1. What are Damped Oscillations?
2. Distinguish between free vibrations and forced vibrations.
3. What is resonance?
4. How is critical damping useful in a vibrating system?
5. Write briefly on forced oscillations.
6. Find the power dissipated in a damped harmonic oscillator.
7. Define simple harmonic motion.
8. Explain the characteristics of SHM?

Long Answer Questions

1. Establish the equation of motion of a simple harmonic oscillator and solve it. Derive the expressions for its velocity, period and frequency.
2. Deduce the expression for total energy of simple harmonic oscillator. Show that the total energy remains independent of time and displacement
3. What are damped oscillations? Solve the differential equation of a damped harmonic oscillator and discuss specifically the case when it is under damped.
4. Solve the differential equation of a damped harmonic oscillator. Investigate the conditions under which the oscillations are said to be underdamped, over damped and critically damped.
5. Define quality factor for damped harmonic oscillator and obtain the expression in terms of relaxation time.
6. What are forced oscillations? Derive and solve the differential equation of a driven harmonic oscillator.
7. What are forced oscillations? Derive the expression for the amplitude of forced oscillations. Explain resonance.
8. Discuss the phase relationship between the amplitude of a forced oscillator and the driving force.

Questions testing the ability of students in applying the concepts

1. A travelling wave on a string has a frequency of 20HZ and a wavelength of 30cm. Its amplitude is 1mm. Write the equation of wave in SI units
2. A wave along a string has the following equation (x in metres and in seconds) Y= 0.01 sin (20t – 2.0x) m. Find the
   a) Amplitude b) Frequently c) speed and d) wavelength\
3. A particle executing SHM has a maximum amplitude A and maximum energy E. Find its displacement at the instant its KE is 3E/4
4. The total energy of a particle executing SHM is 0.256J. Time period is 2 seconds. The displacement of the particle at (T/4)s is $8/\sqrt{2}$ cm. Calculate the amplitude and mass of the particle.
5. The frequency of a tuning for K is 300 Hz. If its quality factor Q is 5×10^4, find the time after which its energy becomes (1/10) of its initial value.
6. The Q-factor of an oscillator is 500. Find its initial energy if its amplitude is 0.01m. Also calculate the energy lost in first cycle. Given mw^2=100 N/m

7. The frequency of a tuning fork is 300Hz. If its quality factor Q is $5x10^4$, find the time after which its energy becomes (1/10) of its initial value.
8. The amplitude of a second pendulum falls to half initial value in 150s. Calculate the Q- factor.

Questions testing the analyzing / evaluating ability of students

1. Write the equations of motion for simple pendulum and derive the expression for the time period.
2. Analyse the motion of a block of mass 'M' attached to a spring of constant 'K'. Derive the expression for time period.

Text books referred:

1. Dr. M.N. Avadhanulu, Dr. P.G.Kshirsagar Engineering Physics(9th Edition), S.Chand
2. D.K. Bhattacharya, Poonam Tandon, Engineering Physics, Oxford University Press

Engineering Physics
Unit 2 Acoustics

Objective:

- Assess the main characteristics of sound propagation in closed structures like buildings, auditoriums etc.

Syllabus:

Reverberation time and absorption coefficient of a hall - Sabine's formula - Acoustics of buildings - Factors affecting the acoustics of buildings - Principles to be observed in the acoustical design of an auditorium.

Outcomes:

At the end of the unit, the students are able to

1. understand the basic definitions of sound.
2. analyse the propagation of sound in closed room and derive Sabine’s empirical formula.
3. understand the concepts involved in the design of auditoriums and buildings.

Acoustics deals with origin, propagation and hearing of sound. Architectural acoustics deals with design of buildings, auditoriums, musical halls, lecture halls, recording rooms etc.

Intensity /loudness of sound: The intensity of sound is the amount of sound energy flowing through unit area of a section kept perpendicular to the direction of propagation of sound. It is the sensation perceived by ear.

$$I \propto P^2$$

where P is the pressure amplitude.

Sound intensity level- decibel:

Relative intensity of sound is of practical significance compared to absolute intensity of sound wave.

Threshold of hearing: The lowest of intensity of sound at 1 kHz to which human ear can respond is $I_o = 10^{-12}\, W/_{m^2}$. This is known as threshold of hearing. It is chosen as zero or standard intensity. It is taken as reference for measurement of sound.

The ratio of the intensity of sound wave to threshold intensity of hearing is defined as the intensity level of sound.

Let I and I_0 represent the intensities of two sounds of a particular frequency and L and L_0 be the corresponding measures of loudness.

$$\text{Then } L_1 = k\ logI$$

$$L_0 = k\ logI_0$$

The intensity level of sound, $L = \log\frac{I}{I_0}\ \ bels.$

Reverberation and Time of Reverberation

A sound produced inside a hall will propagate in all directions. Sound waves incident on the surfaces of walls, floor, ceiling and furniture inside a hall, will be multiply reflected. As the source of sound is turned off, the listener hears the sound with gradually reducing intensity for some time due to the persistence of sound by multiple reflections at different places in the room. A listener inside the hall will receive the sound waves directly from the source, as well as the reflected waves.

Reverberation: *The persistence of audible sound even after the source of sound is turned off is called Reverberation.*

Reverberation time: *The time taken by the sound intensity to fall to one millionth* $\left(\frac{1}{10^6}\right)$ *of its initial intensity i.e., the intensity just before the source of sound is turned off is called Reverberation time.*

SABINE'S FORMULA FOR REVERBERATION TIME:

Sabine's formula is derived from reverberation theory. The reverberation theory explains the nature of growth and decay of sound energy in an enclosure. The following assumptions are made in the theory.

1. The sound is distributed uniformly in the enclosure.
2. Sound travels uniformly in all directions.
3. Absorption of sound by air is neglected.

Consider a big hall and let a source of sound be filling the hall uniformly with sound. Each elemental volume of the hall acts as a source of sound energy.

Energy emitted from a volume 'dV' in the hall:

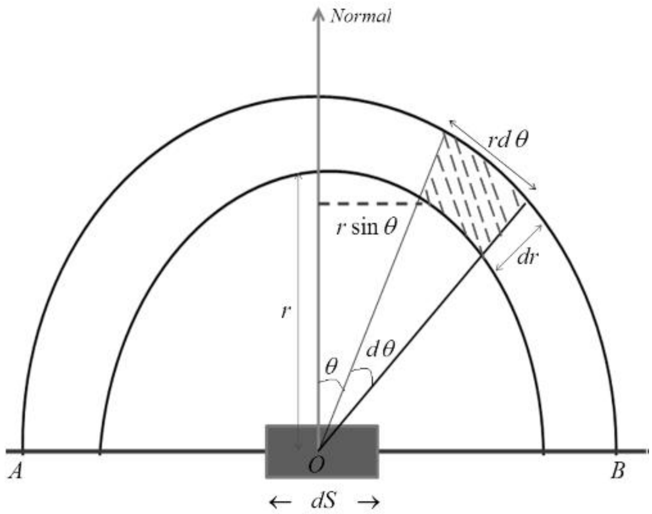


Figure 1: The sound emitted from small element on to ‘dS’.

Consider a volume, dV, in the hall. Let the sound energy emitted from this volume be incident on the surface of a plane wall, AB. From the centre of ‘dS’, draw a normal. From the same centre, draw two circles with radii r and r+dr in the plane containing the normal. Let the radii make angles θ and θ+dθ with the normal.

For the shaded portion,

Length of the arc = r dθ

Radial length = dr

Surface area of the element = r dθ dr

Let the elemental area be rotated through a small angle, dϕ as shown in figure 2.

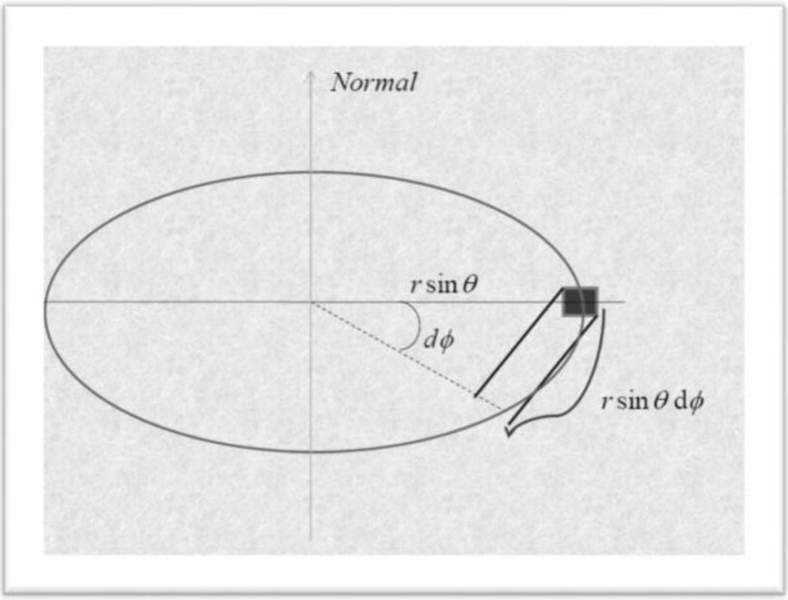


Figure 2: The rotation of the volume element about the normal

Volume of the element, $dV = (r\, d\theta\, dr)\,(r\sin\theta\, d\phi) = r^2 \sin\theta\, d\theta\, dr\, d\phi$

Let E be the sound energy per unit volume, then the energy present in the volume element at any moment = E r^2 sinθ dθ dr dϕ

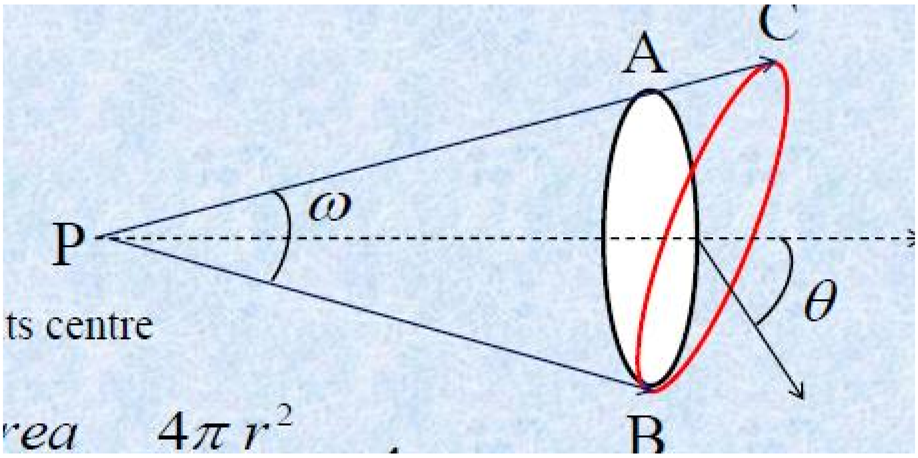


Figure3: Solid angle subtended by 'dS' at 'dV'

The solid angle subtended by 'dS' at 'dV' is

$$d\omega = \frac{dS\, cos\theta}{r^2}$$

The amount of energy that reaches dS from dV

$$= \frac{E\ dV}{4\pi}\,\frac{dS\ cos\theta}{r^2}$$

$$= \frac{E\ ds\ sin\theta\ cos\theta\ d\theta\ d\phi\ dr}{4\pi}$$

The total energy received by 'dS' in one second from the entire volume,

$$\text{Total energy,} = \frac{E\ dV}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{v} sin\theta\ cos\theta\ d\theta\ d\phi\ dr$$

$$= \frac{E\ dS}{4\pi}\ v\ 2\pi\int_{\theta=0}^{\pi/2} sin\theta\ cos\theta\ d\theta$$

$$= \frac{E\ v\ dS}{4}$$

Let 'α' be the absorption coefficient of the surface of the wall AB, then the sound energy absorbed per second by the surface element *dS* is

$$dW_A = \frac{Ev\alpha\ dS}{4}$$

Therefore, the total energy absorbed by all the wall surfaces in the hall is

$$W_A = \frac{Ev}{4}\sum \alpha\ ds$$

$$W_A = \frac{Ev}{4}\ A$$

where 'A' is the total absorption of all the surfaces.

Build up of sound in a Hall:

Let P be the power of sound source, V be the total volume of the hall and E be the energy density at any instant,

Total energy in the hall at a particular instant, =EV

$$Rate\ of\ growth\ of\ energy\ in\ the\ hall = \frac{d}{dt}(EV) = V\frac{d}{dt}(E)$$

At any instant,

$$\begin{pmatrix} Rate\ of\ growth\ of \\ energy\ in\ the\ hall \end{pmatrix} = \begin{pmatrix} Rate\ of\ supply\ of\ energy \\ from\ the\ source \end{pmatrix} - \begin{pmatrix} Rate\ of\ absorption\ of\ all \\ surfaces\ in\ the\ hall \end{pmatrix}$$

$$V\frac{d}{dt}(E) = P - \frac{E\upsilon A}{4}$$

$$\frac{d}{dt}(E) + \frac{\upsilon A}{4V}E = \frac{P}{V}$$

Put $\frac{\upsilon A}{4V} = \alpha$, then

$$\frac{d}{dt}(E) + \alpha E = \frac{4P}{\upsilon A}\alpha$$

Multiply on both sides with $e^{\alpha t}$ on both sides of the above equation, then

$$\left(\frac{d}{dt}(E) + \alpha E\right)e^{\alpha t} = \left(\frac{4P}{\upsilon A}\alpha\right)e^{\alpha t}$$

$$\frac{d}{dt}[Ee^{\alpha t}] = \frac{4P}{\upsilon A}\alpha e^{\alpha t}$$

$$integrating\ on\ both\ the\ sides, we\ get$$

$$Ee^{\alpha t} = \frac{4P}{\upsilon A}\alpha e^{\alpha t} + K \text{ ---------- (1)}$$

where 'K' is constant of integration. It is obtained from boundary conditions.

Growth of energy density:

At t=0, E=0

Then from equation (1),

$$K = -\frac{4P}{\upsilon A}$$

Equation (1) becomes

$$E = \frac{4P}{\upsilon A}(1 - e^{-\alpha t})$$

$$put\ E_m = \frac{4P}{\upsilon A}$$

$$then\ E = E_m(1 - e^{-\alpha t}) \text{ ------------ (2)}$$

from equation (2), the sound energy increases exponentially with time, till it attains the steady state value at t=∞

Decay of sound energy in the Hall:

Switch off the source of sound after the energy reached a steady state value. Consider that instant as , t=0, then P=0 and $E=E_m$. We know energy density

$$Ee^{\alpha t} = \frac{4P}{vA}\alpha e^{\alpha t} + K \text{ ---------- (1)}$$

From the above conditions, $K = Ee^{\alpha t}$ --------- (2)

But $|K| = \frac{4P}{vA} = E_{m,}$ equation (2) becomes,

$$E = E_m e^{-\alpha t} \text{------- (3)}$$

From equation (3), it can be concluded that the sound energy decreases exponentially with time after the source of sound is switched off.

Deduction of Sabine's formula:

From definition, at reverberation time, t=T, $\frac{E}{E_m} = 10^{-6}$ ------- (1)

We know that the decay in the sound energy after the source is switched off is given by

$$E = E_m e^{-\alpha t} \text{-------- (2)}$$

From equations (1) and (2), $e^{-\alpha t} = 10^{-6}$

$$\Rightarrow e^{\alpha T} = 10^6$$

Taking log on both sides, we get

$$\alpha T = 6\, log_e 10 = 6 * 2.303 \text{ ----------- (3)}$$

We know, $\frac{vA}{4V} = \alpha$, equation (3) becomes,

$$\frac{vA}{4V}T = 6\ x\ 2.303 \text{ -------- (4)}$$

$$\Rightarrow T = \frac{4\ x\ 6\ x\ 2.303\ x\ V}{vA}$$

Take, $v = 344\ m/S$ then $T = \frac{4\ x\ 6\ x\ 2.303\ x\ V}{344\ x\ A} = \frac{0.161\ V}{A}$ ---------- (5)

Equation (5) is identical to the equation proposed by Sabine for reverberation time.

Limitations of Sabine's formula:

1. The formula does not give correct result if $\alpha > 0.2$

2. When α=1, then T should be zero. But according to Sabine's formula, T =0.161 V seconds.
3. In practice, the sound does not distribute uniformly in the room due to absorption from air.

DETERMINATION OF ABSORPTION COEFFICIENT:

Method 1:

Let T_1 be the reverberation time without any material in the room. Then

$$T_1 = \frac{0.161\,V}{\sum_1^n \alpha_n S_n} = \frac{0.161\,V}{A}$$

where $A = \sum \alpha_n S_n$ denotes the absorption due to walls, flooring and ceiling of the empty room.

Place a material with area S, and absorption coefficient α'. The reverberation time,

$$T_1 = \frac{0.161\,V}{A + \alpha' S}$$

Then $$\frac{1}{T_2} - \frac{1}{T_1} = \frac{\alpha' S}{0.161V}$$

$$therefore, \alpha' = \frac{0.161V}{S}\left(\frac{1}{T_2} - \frac{1}{T_1}\right) \text{ ----- (1)}$$

Absorption coefficient of the material can be calculated from the above equation.

Method 2:

If the material is already fixed to the walls or ceiling of the room, then the following procedure is used.

Let the powers of the two sources be P_1 and P_2 respectively.

The steady state energy density of source P_1, $E_1 = \frac{4P_1}{\upsilon A}$

The steady state energy density of source P_2, $E_2 = \frac{4P_2}{\upsilon A}$

Let T_1 and T_2 be the respective times of decay of energy density to the inaudibility level E_0. Then,

$$E_0 = \frac{4P_1}{\upsilon A}\, e^{-\alpha T_1}$$

$$E_0 = \frac{4P_2}{\upsilon A}\, e^{-\alpha T_2}$$

$$therefore, \quad \frac{P_2}{P_1} = e^{\alpha(T_2 - T_1)}$$

$$\alpha = \frac{log_e P_2 - log_e P_1}{(T_2 - T_1)}$$

$$but\ \alpha = \frac{\upsilon A}{4V}$$

$$\frac{log_e P_2 - log_e P_1}{(T_2 - T_1)} = \frac{\upsilon A}{4V}$$

$$A = \frac{4V(log_e P_2 - log_e P_1)}{\upsilon(T_2 - T_1)}$$

$$\alpha S = \frac{4V\left(log_e {}^{P_2}/_{P_1}\right)}{\upsilon(T_2 - T_1)}$$

$$\alpha = \frac{4V\left(log_e {}^{P_2}/_{P_1}\right)}{\upsilon S(T_2 - T_1)} - - - (2)$$

From equation (2), the absorption coefficient of the material fixed in the room can be calculated.

Factors affecting the Architectural acoustics and their remedies

Following factors affect the architectural acoustics.

1) Reverberation

- In a hall, when reverberation is large, there is overlapping of successive sounds which results in loss of clarity in hearing. On the other hand, if the reverberation is very small, the loudness is inadequate. Thus, the reverberation time for a hall should neither be too large nor too small.
- Experimentally it is observed that the time of reverberation depends upon the size of the hall, loudness of sound and on the kind of the music for which the hall is used.
- For a frequency of 512 Hz, the best time of reverberation lies between 1 and 1.5 sec for small halls and for large ones, it is up to 2-3 seconds.

Remedy: The reverberation can be controlled by the following factors.

i. By providing windows and ventilators which can be opened and closed to make the value of time of reverberation, optimum
ii. Decorating the walls by pictures and maps.
iii. Using heavy curtains with folds.
iv. By lining the walls with absorbent materials such as felt, fiber board etc.
v. Having full capacity of audience.

vi. By covering the floor with carpets.
vii. By providing acoustic tiles.

2) Loudness

With large absorption, the time of reverberation will be smaller and the intensity of sound may go below the level of hearing. Sufficient loudness at every point in the hall is an important factor for satisfactory hearing.

Remedy: The loudness may be increased by,

i. Using large sounding boards behind the speakers and facing the audience.
ii. Low ceilings are of great help to reflect the sound energy towards the audience.
iii. Providing additional sound energy with the help of equipments like loud speakers. For uniform distribution of intensity throughout the hall, the loudspeakers should be polished carefully.

3) Focusing

If there are focusing surfaces such as concave, spherical, cylindrical or parabolic ones on the walls or ceiling of the hall, they produce concentration of sound in particular regions, while in some other parts, no sound reaches at all. In this way, there will be regions of silence.

Remedy: For uniform distribution of sound energy in the hall,

i. There should be no curved surfaces. If such surfaces are present, they should be covered with absorbent material.
ii. Ceiling should be low.
iii. A paraboloidal reflected surface, with the speaker at the focus is also helpful in sending a uniform reflected beam of sound in the hall.

4) Echoes

An echo is heard when direct sound waves coming from the source, and it's reflected wave, reach the listener with a time interval of about 1/7 second. The reflected sound arriving earlier helps in raising the loudness while those arriving later produce echoes and confusion.

Remedy: Echoes may be avoided by covering the long distant walls and high ceiling with absorbent material.

5) Echelon effect

A musical note produced due to the combination of echoes, having regular phase difference is known as Echelon effect. The reflected sound waves from regularly spaced reflecting surfaces such as equally spaced stair cases or a set of railings produce musical note due to the regular succession of echoes of the original sound to the listener. This makes the original sound confused or unintelligible.

Remedy: Echelon effect can be avoided by forming the staircases with unusual spacing between them and covering them with sound absorbing materials like carpet.

6) Resonance

Sometimes, window panes loosely fitted wooden portions, wall separators and hollows, start vibrating by absorbing the sound produced in the hall. These may create sound. Certain tones of the original music and the created sound combine to produce interference such that the original sound gets disturbed.

Remedy: Resonance can be suppressed by hanging a large number of curtains in the hall.

7) Noise

Generally, there are three types of noise. They are (a) Air-borne noise (b) Structure borne noise (c) Inside noise.

(a) Air-borne noise: The noise that enters the hall from outside through open windows, doors and ventilators is known as air-borne noise.

Remedy:

i. By using heavy glass doors, windows or ventilators.
ii. By using double wall-doors and windows with insulating material in between them.
iii. Forming double wall construction.
iv. By fixing doors and windows at proper places.
v. Air conditioning the hall and sealing the openings perfectly.

(b) Structure-borne noise: The noise that reaches through the Structures of buildings is known as Structural noise. The activity around the building may cause a structural vibration of the building. Ex: footsteps, operating machinery, street traffic etc;

Remedy:

i. By using double walls with air space in between them.
ii. By using anti-vibration mounts.
iii. By properly insulating the equipments such as refrigerators, lifts, fans etc.,
iv. By using carpets on the floor.

(c) Inside noise: The noise produced inside big halls or offices due to equipment such as air conditioners, type writers and fans is called inside noise. This noise may be minimized as follows.

Remedy:

i. Placing the machinery on sound absorbent pads.

ii. Using noise-free air conditioners.

Covering the floor with carpets, walls, ceilings with sound absorbing materials

Principles to be observed in the acoustical design of an auditorium:

The basic requirements of an acoustically good hall are,

1) **The volume of the auditorium** is decided by the type of program to be conducted there and also the number of seats to be accommodated. A musical hall requires a large volume where as a lecture hall requires a smaller volume. In deciding the volume of the hall, its height plays an important role than its length and breadth. The ratio between the ceiling height and breadth should be 2:3. In deciding the volume of the hall, the following guidelines may be followed.
    i) 3.74-4.2 m^3 per seat in cinema theatres.
    ii) 2.8-3.7m^3 per seat in lecture halls.
    iii) 4.2-5.6m^3 per seat in musical halls.
2) **The shape of the wall and ceiling** should be so as to provide uniform distribution of sound throughout the hall. The design of a hall requires smooth decay and growth of sound. To ensure these factors, the hall should have scattering objects, walls should have irregular surface and walls must be fixed with absorptive materials.
3) **The reverberation of sound** in an auditorium is mainly due to multiple reflections at various surfaces inside the hall. The reverberation should be optimum i.e., neither too large nor too small. The reverberation time should be 1-2 seconds for music and 0.5-1 sec for speech. To control the reverberation, the sound absorbing materials are to be chosen carefully.
4) **The sound heard** must be sufficiently loud in every part of the hall and no echoes should be present.
5) **The total quality of the speech and music** must be unchanged i.e., the relative intensities of the several components of a complex sound must be maintained.
6) For the sake of **clarity,** the successive syllables spoken must be clear and distinct i.e., there must be no confusion due to overlapping of syllables.
7) There should be no **concentration of sound** in any part of the hall.
8) The boundaries should be sufficiently **sound proof** to avoid noise from outside.
9) There should be no **echelon effect**.
10) There should be no **resonance** within the building.
11) The hall must be full of **audience**.

Assignment –cum- tutorial questions

SECTION-A

Objective questions

1. What is nature of sound waves?
2. Define decibel.
3. What are the units of measurement of loudness and of intensity?
4. The preferred value of reverberation time is
   a) Zero b) optimum reverberation time c) infinity d) none of these
5. Intensity of a source of sound is increased by a factor of 100. Increase in the intensity level in decibels is
   a) 2 dB b) 10 dB c)20 dB d)40 dB
6. Ordered sound consists of
   a) Fundamental frequency b)harmonics and fundamental frequency c)only harmonics d)noise
7. Quality of sound is decided by
   a) loudness b)intensity c)number of overtones d)frequency
8. Intensity of sound depends
   a) directly on the distance from the source
   b) directly on the square of distance from the source
   c) inversely on the distance from the source
   b) inversely on the square of distance from the source
9. To avoid focussing of sound, one should not use
   a) concave walls b)convex walls c)plane walls d)straight walls
10. Total absorption is measured in units of
   a) $sabines^2$ b) 1/sabines c)sabines d) $1/sabines^2$

SECTION-B

DESCRIPTIVE QUESTIONS

Short Answer Questions:

1. Define reverberation.
2. Define reverberation time.
3. What is the intensity of sound? How is it measured?
4. Derive an expression for the growth of energy density of sound wave.
5. State the factors that affect acoustics of buildings and explain their effect in brief.
6. Suggest some remedies to improve the acoustics of buildings.
7. Suggest a method for measuring absorption coefficient.
8. What are the limitations of Sabine's formula?
9. What is echelon effect?

Long Answer Questions

1. Using Sabine's formula, explain how the sound absorption coefficient of a material is determined.
2. Deduce Sabine's formula for the reverberation time.
3. Explain various factors effecting architectural acoustics and their remedies.
4. Define absorption coefficient of a material and describe two methods to for its determination.

5. State the acoustic requirements of a good auditorium. Explain how these requirements can be achieved.

Questions testing the ability of students in applying the concepts

1. The volume of a room is 1500 m^3. The wall area of the room is 260 m^2, the floor area is 1400 m^2 and the ceiling area is 140 m^2. The average sound absorption coefficient for wall is 0.03, for the ceiling is 0.80 and the floor is 0.06. Calculate the average coefficient and the reverberation time.
2. A hall has a volume of 12500 m^3 and the reverberation time is 1.5 s. If 200 cushioned chairs are additionally placed in the hall, what will be the new reverberation time of the hall? The absorption coefficient of each chair is 1.0
3. A cinema hall has a volume of 7500 m^3. What should be the total absorption in the hall if the reverberation time of 1.5 S is to be maintained?
4. Reverberation time for a cubical chamber of 10m width is 2.68 s. Calculate its average absorption coefficient. If one of the walls is covered with acoustic tiles the reverberation time will decrease to 2 S. Calculate the sound absorption coefficient of acoustic tiles.
5. A hall has a volume of 1,20,000 m^3. It has a reverberation time of 1.5s. what is the average absorbing power of the surface if the total absorbing surface area is 25,00 m^3?

ENGINEERING PHYSICS
UNIT-III
CRYSTAL SYSTEMS

Objective:

1. Discuss the periodic arrangement of atoms in accordance with translation symmetry

Syllabus:

Lattice ,basis unit cell - Crystal systems - Important in Engineering Materials - Miller indices - Crystal planes – Von Lave formula for inter planar distance - Packing Fraction -X-Ray diffraction - Bragg's law

Outcomes:

At the end of the unit, the students are able to

1. define space lattice, basis and crystal parameters.
2. classify the crystal systems.
3. understand the Miller indices and plot various crystal planes.
4. apply the definitions to monoatomic SC, BCC and FCC structures.
5. derive Bragg’s Law of X-ray diffraction.

Introduction to Crystallography

Crystallography is the experimental science of determining the arrangement of atoms in crystalline solids**.**

Matter exists in three different states namely solids, liquids and gaseous. In liquid and gaseous states the atoms, molecules or ions move from one place to another and there is no fixed position of atoms in the substance. In solids the position of atom or molecules are fixed and they may or may not be present at regular intervals of distances in three dimensions.

If the atoms or molecules in a solid are periodically at regular intervals of distances in 3 dimensions then such solids are known as ***crystalline solids.***

If the atoms or molecules do not have such periodicity in the substances, then they are called ***Amorphous solid.***

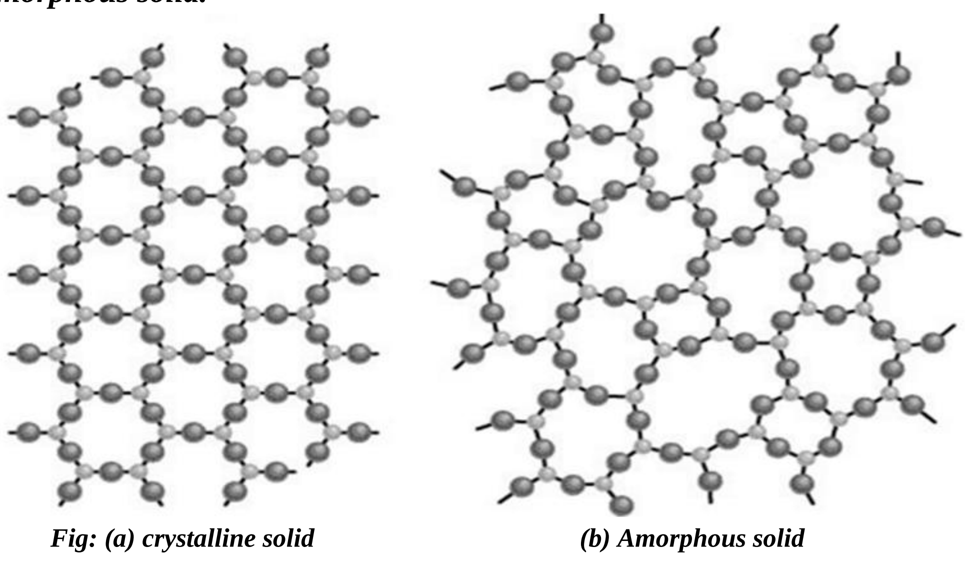


Fig: (a) crystalline solid ***(b) Amorphous solid***

If the periodicity of atoms or molecules, are extended throughout the solid such a solid known as ***single crystalline solid.***

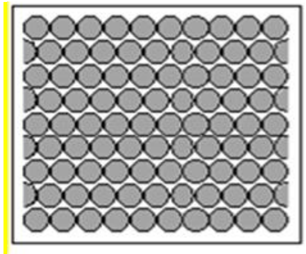


Fig: Single crystalline solid

The periodicity of atoms or molecules can be divided into small regions called grains. And these grains are large in different sizes in the solid. Such materials are known as ***polycrystalline solids.***

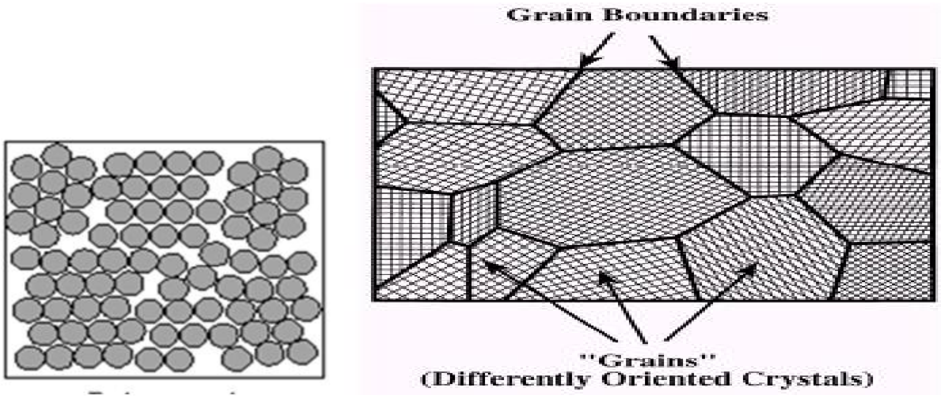


Fig: ***polycrystalline solid.***

Crystalline Solids	Amorphous solids
1. The crystalline solids posses' regular arrangement of ions or atoms or molecules.	1. These solids posses complete random arrangement of ions or atoms or molecules.
2. The solids have different physical properties (thermal, electrical conductivity, refractive index) in different directions.	2. These solids have some physical properties in all the directions.
3. These are anisotropic in nature.	3. These are isotropic in nature.
4. These solids posses elasticity.	4. These solids posses plasticity except rubber.
5. These posses more density. Eg: calcite, quartz, gold, silver, Al etc...	5. These posses less density. Eg: glass, plastic etc.

Space lattice (or) Crystal lattice

Imagine points in space about which atoms or molecules are located, such points in space are called lattice points and the totality of all such lattice points forms ***space lattice*** or ***crystal lattice.***

(or)

The geometric arrangement of lattice points, which describes the three dimensions arrangements of atoms or molecules or ions in a crystal, is called space lattice or *crystal lattice*.

If the points are arranged in a single or single column, then the lattice is solid to be ***linear lattice.*** If it is arranged in a two dimension then the lattice is called ***planar lattice.***

Basis

The crystal structure is formed by associating with every lattice point a set of assembly of atoms or molecules or ions identical in composition, arrangement, orientation is called as ***Basis.***

(Or)

A group of atoms or molecules identical in composition is called ***Basis.***

Lattice + basis → Crystal structures

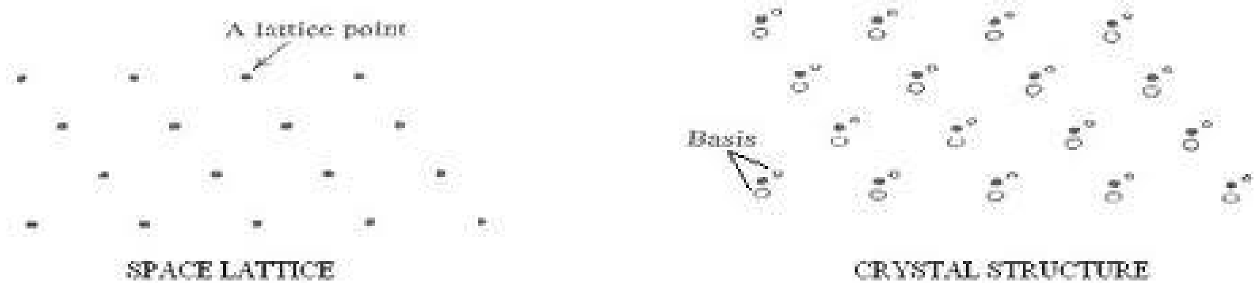


Fig: Space lattice + basis = crystal structure

In crystalline solids like aluminum, sodium, copper the basis is single atom. In case of NaCl, NaBr, KCl, KBr the basis is diatomic and in CaF_2 the basis is triatomic.

Unit Cell

The smallest portion of space which can generate the complete crystal structure by translation in three dimensions is called a "***Unit Cell"***.

(Or)

In every crystal some fundamental grouping of particles is repeated. Such fundamental grouping of particles is called a "***Unit Cell".***

Primitive Cell

It is the minimum volume cell which generates the crystal structure on translation in three dimensions. The number of lattice points per primitive cell is "one".

Multiple Unit Cell (or) Non-Primitive Cell

The unit cell which contains more than one lattice point per unit cell is known as ***Multiple Unit Cell (or) Non-Primitive Unit Cell.***

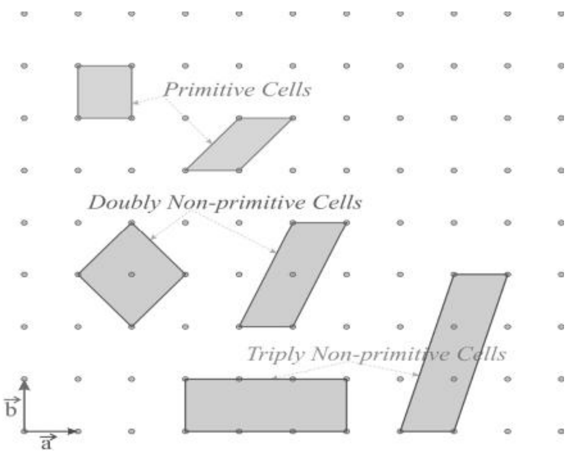


Fig: Unit cell in two dimensions

Lattice parameters of a Unit Cell

The basic lattice parameters of a unit cell are interfacial angles and primitives.

Primitives

The intercepts on x, y, z-axis in a unit cell, are called primitives. They are denoted by a, b, c. The primitives give the knowledge of the size of the unit cell. The interfacial angles give the shape of the unit cell.

α, β, γ ------> interfacial angles
a, b, c -------> primitives
X, Y, Z -------> crystallographic axis

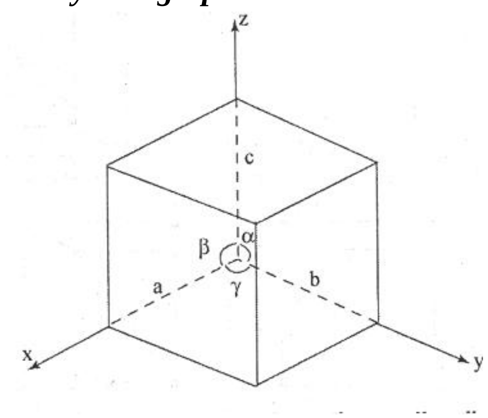


Fig: Unit cell and its lattice parameters

Crystal Systems

The crystal systems are divided into seven, based on the lattice parameters *i.e.*, primitives and interfacial angles,

The different crystal systems are

1. Cubic system
2. Tetragonal system
3. Orthorhombic system
4. Monoclinic system
5. Triclinic system
6. Rhombohedral system
7. Hexagonal system

1) **Cubic system**:- In this crystal system all the three edges of the unit cell are equal and right angles to each other

$a = b = c; \alpha = \beta = \gamma = 90^0$

Eg :- NaCl , CaF_2 etc.

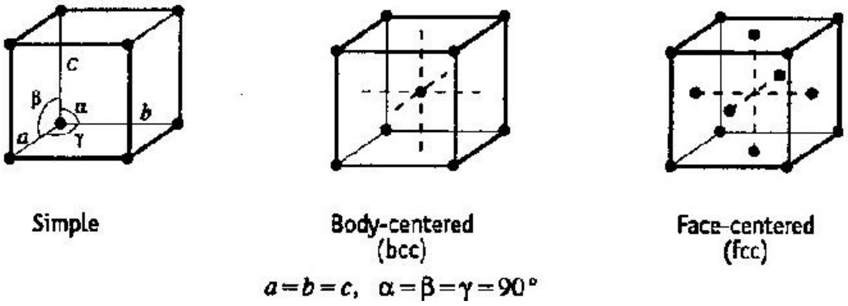
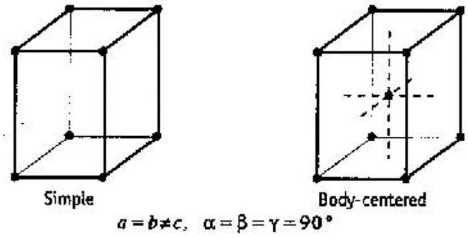


Fig: Cubic Crystal Systems

2) **Tetragonal system**: -- In this system all the two edges of the unit cell are equal while the third is different. The three axes are mutually perpendicular.

$a = b \neq c; \alpha = \beta = \gamma = 90^0$

Eg: - TiO2, NiSO4, Sno_4 etc.

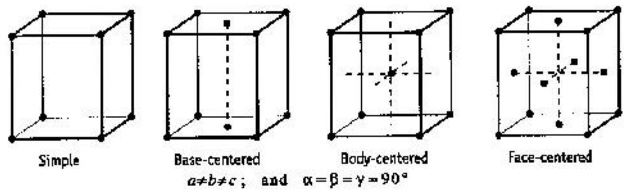


TETRAGONAL SYSTEM

3).**Orthorhombic system**:--In this crystal system the three edges are different but three axis are perpendicular to each other.

$a \neq b \neq c$; $\alpha = \beta = \gamma = 90^0$

Eg: $BaSO_4$, KNO_3

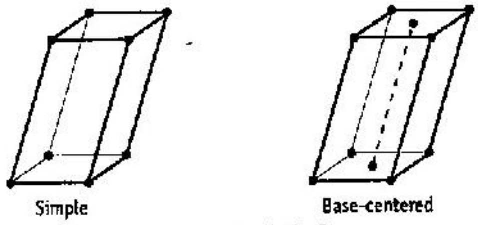


ORTHORHOMBIC SYSTEM

4).**Monoclinic system**: -- In this system the edges of the unit cell are different, two axis at right angles and third axis is obliquely inclined.

$a \neq b \neq c$; $\alpha = \beta = \gamma = 90^0 \neq \gamma$

Eg: $Na_2B_4O_710H_2O$ (Borax), $CaSO_42H_2O$ (Gypsum)



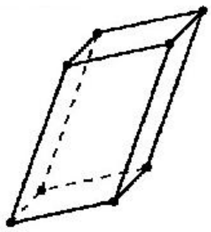
$a \neq b \neq c, \quad \alpha = \gamma = 90^\circ \neq \beta$

MONOCLINIC SYSTEM

5).**Triclinic system**: In this system the edges of unit cell are different and the 3 axis are obliquely inclined to each other

$a \neq b \neq c; \quad \alpha \neq \beta \neq \gamma \neq 90^0$

Eg : $K_2Cr_2O_7$,$CuSO_45H_2O$



$a \neq b \neq c$

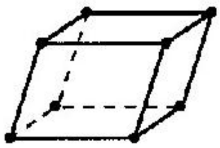
$\alpha \neq \beta \neq \gamma \neq 90^\circ$

TRICLINIC SYSTEM

6).**Rhombohedral system**: In this crystal system all the 3 edges of the unit cell are equal and are equally inclined to each other at an angle other than $90^{0.}$

$a = b = c; \quad \alpha = \beta = \gamma \neq 90^0$

Eg: Bismuth, calcite.



$a = b = c$

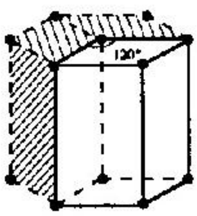
$\alpha = \beta = \gamma \neq 90^\circ$

IOMBOHEDRAL (TRIGONAL) SYSTEM

7). **Hexagonal system**: In this system the two axes of the unit cell are equal in length lie in one plane at 120^0 with each other and the third axis is perpendicular to this plane.

$a = b \neq c; \quad \alpha = \beta = 90^0; \gamma = 120^0$

Eg: SiO_2, Mg, Zn



$a = b \neq c$

$\alpha = \beta = 90^\circ, \quad \gamma = 120^\circ$

HEXAGONAL SYSTEM

Bravais Lattices

Bravais showed that there are 14 ways of arranging points in space lattice such that all the lattice points have exactly identical environment. These 14 different lattice types collectively called as Bravais lattices. The space lattices are classified based on the arrangement of atoms. They are

- Simple (or) Primitive (P)
- Body centered (I)
- Face centered (F)
- Base centered(C)

S.No	Crystal system	Bravais lattice	No. of lattice system	Axial length	Interfacial angles	examples
1	Cubic	P,I,F	3	a = b= c	$\alpha = \beta = \gamma = 90^0$	NaCl,
2	Tetragonal	P,I	2	$a = b \neq c$	$\alpha = \beta = \gamma = 90^0$	TiO_2
3	Orthorhombic	P,I,F,C	4	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^0$	KNo_3
4	Monoclinic	P,C	2	$a \neq b \neq c$	$\alpha = \gamma = 90^0 \neq \gamma$	$CaSo_42H_2o$
5	Triclinic	P	1	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^0$	$K_2Cr_2O_7$
6	Rhombohedral	P	1	a = b = c	$\alpha = \beta = \gamma \neq 90^0$	calcite
7	Hexagonal	P	1	$a = b \neq c$	$\alpha = \beta = 90^0\ \gamma =$	SiO_2

Simple Cubic Structure (SC)

The cubic cell that contains 8 atoms located at 8 corners is called simple cubic structure. In this structure all the atoms touch each other along their edges.

Effective number of atoms:

Effective number of atoms in unit cell is 1.
The eight corners have 8 atoms, each corner atom is shared by 8 unit cell.

Therefore $n = 8x\frac{1}{8}$

$n=1.$

Coordination Number:

Every atom is surrounded by six atoms at equal distances. For illustration, consider an atom at one of the corners, it is surrounded by 6 equidistant neighboring atoms. Hence the coordination number of simple cubic is 6.

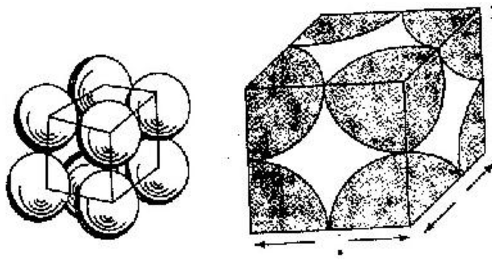


Figure: Simple Cubic Structure

Nearest neighbor distance

In this structure, atoms touch each other along the edge of the unit cell. Hence

a = 2r, where ‘a’ is lattice constant.

Packing fraction

It is defined as ***the ratio of volume of all atoms in the unit cell to the volume of unit cell.***

$$\text{Packing Fraction} = \frac{\text{Volume of all the atoms in the unit cell}}{\text{Volume of the Unit Cell}}$$

$$= n\,x\,\frac{\frac{4}{3}\pi r^3}{a^3}$$

$$= 1\,x\,\frac{\frac{4}{3}\pi r^3}{8r^3}$$

$$= \frac{\pi}{6}$$

$$= 0.52$$

$$= 52\%$$

Example: polonium.

Body Centered Cubic Structure (BCC)

In the body centered cubic structure, eight atoms located at the 8 corners and one atom is located at centre of body i.e. Cubic cell. In this structure, the corner atoms do not touch each other. But they are in contact with the body centre atom.

Effective number of atoms:

Effective number of atoms in BCC is 2. The cell has 8 atoms located at 8 corners; one atom located at centre of the body.

Therefore n $= (8x\frac{1}{8})+1$

$= 2.$

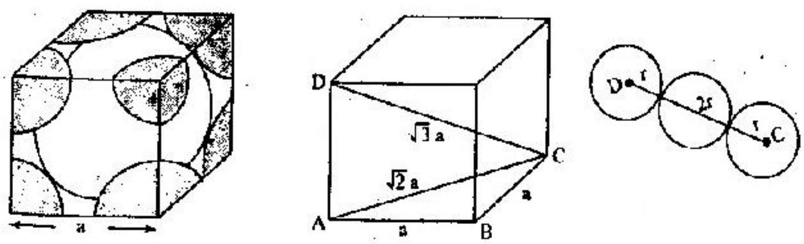


Figure: Body centered Cubic Cell

Coordination Number

For illustration, consider the atom at the centre, it is surrounded by 8 equidistant neighboring atoms. Hence the coordination number is 8.

Nearest neighbor Distance (a)

From the Δ ABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = a^2 + a^2$

$(AC)^2 = 2a^2$

From the Δ ACD,

$(AD)^2 = (AC)^2 + (CD)^2$

$(4r)^2 = 2a^2 + a^2$

$16r^2 = 3a^2$

$4r = \sqrt{3}\,a$

$r = \sqrt{3}\,a/4$

$a = 4r/\sqrt{3}.$

Packing Fraction

$$\text{Packing Fraction} = \frac{\text{Volume of all the atoms in the unit cell}}{\text{Volume of the Unit Cell}}$$

$$= n\,x\,\frac{\frac{4}{3}\pi r^3}{a^3}$$

$$= 2\,x\,\frac{\frac{4}{3}\pi r^3}{\frac{64r^3}{3\sqrt{3}}}$$

$$= \sqrt{\frac{3\,\pi}{8}} = 0.68 = 68\%.$$

Eg: Na, Fe etc.

Face Centered Cubic (FCC) structure

The Face centered cubic structure has eight atoms located at 8 corners and six atoms located at the centre of the faces. In this structure the corner atoms do not touch each other but they are in contact with the atoms located on the centre of its faces.

Effective number of atoms:

Effective Number in F.C.C is 4. i.e. 8 atoms are located at 8 corners, 6 atoms on sides shared by two sides. Hence,

$$n = (8\times\frac{1}{8}) + (6\times\frac{1}{2}) = 4.$$

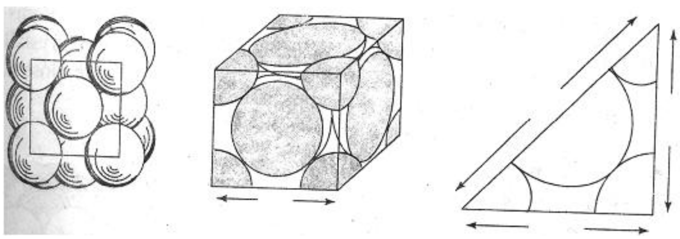


Figure: Face centered Cubic Cell

Coordination Number

Coordination number is 12. Consider an atom at face centre; it is surrounded by 12 atoms at the same distance. Therefore, Coordination Number in FCC is 12.

Nearest Neighbour Distance (a)

From the Δ ABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$16r^2 = a^2 + a^2$

$16r^2 = 2a^2$

$8r^2 = a^2$

$2\sqrt{2}r = a$

$$2r = \frac{a}{\sqrt{2}}$$

Packing Fraction

$$\text{Packing Fraction} = \frac{\text{Volume of all the atoms in the unit cell}}{\text{Volume of the Unit Cell}}$$

$$= n\,x\,\frac{\frac{4}{3}\pi r^3}{a^3}$$

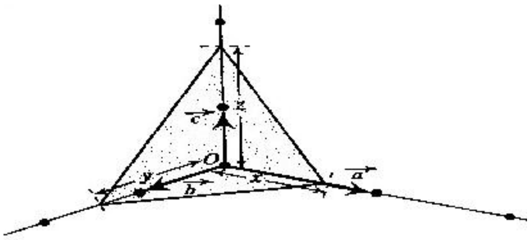
$$= 4\,x\,\frac{\frac{4}{3}\pi r^3}{16\sqrt{2}r^3} = \frac{\pi}{3\sqrt{2}} = 0.74 = 74\%.$$

Eg: Cu, Al, Ag etc.

Miller Indices

It is possible for defining a system of parallel and equidistant planes which can be imagined to pass through the atoms in a space lattice, such that they include all the atoms in the crystal. Such a system of planes is called crystal planes. Many different systems of planes could be identified for a given space lattice.

The position of a crystal plane can be specified in terms of three integers called Miller indices. If these are enclosed in "( )" as (h, k, l) then it represent a plane. If they are enclosed in "[ ]" as [h, k, l] then it represent the direction of crystal. Steps to determine Miller Indices for a given plane as shown in the figure. Take a lattice point as origin 'o' of crystallographic axis x, y, z in a space lattice. Let A, B, C be the crystal plane intercepts there axis of 4a, 4b, 3c.



PLANE ORIENTATION IN A LATTICE

Step (1): Determine the coordinate of intercepts made by the plan along the crystallographic axis

x	y	z
4a	4b	3c

In general

x	y	z
pa	qb	rc

(where p=4, q=4, r=3).

Step (2):-- Divide the intercepts with lattice point translational distances along the axis.

4a/a	4b/b	3c/c
4	4	3

In general

pa/a	qb/b	rc/c
p	q	r

Step (3): Determine the reciprocals of these numbers

i.e. 1/4 1/4 1/3

In general 1/p 1/q 1/r.

Step (4):-- Reduce the reciprocals to smallest integers and enclosed them in "( ) "[by multiplying with L.C.M]

1/4× 12   1/4× 12   1/3× 12

3   3   4

(3 3 4)

Miller Indices may be defined as the reciprocals of the intercepts may by the plane on crystallographic axis when reduced to smallest numbers

Important features of Miller indices,

1. Miller indices give the orientation of the crystal planes.
2. If a plane is parallel to any axis, the intercept of the plane on that axis is infinity. Hence the corresponding Miller index is zero.
3. A plane with negative intercept is represented with a bar on the corresponding Miller index.
4. Equally spaced parallel planes are represented with the same set of Miller indices. (h k l)

Distance of Separation between Successive planes

Let us consider a rectangular coordinate system with origin 'o' at one of the lattice points. Let (h k l) be the miller indices of the plane ABC which makes intercepts at OA, OB, OC along x, y, z axis respectively. Let ON be the normal from origin to the plane ABC, such that ON = d. The normal makes angles α, β, γ along x, y, z axis respectively.

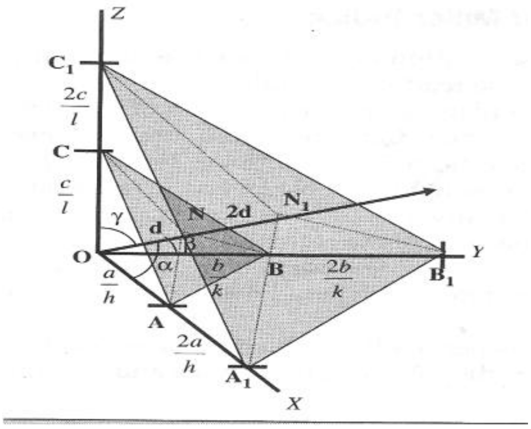


Fig: Inter planar Spacing

From figure, OA = a/h, OB = b/k, OC = c/l →(1)

From the definition of directional cosines,

$$\left.\begin{aligned}\cos\alpha &= \frac{ON}{OA} = \frac{d_1}{a/h}\\ \cos\beta &= \frac{ON}{OB} = \frac{d_1}{b/k}\\ \cos\gamma &= \frac{ON}{OC} = \frac{d_1}{c/l}\end{aligned}\right\} \quad \text{-----------} \ (2)$$

$\text{Cos}^2\alpha + \text{Cos}^2\beta + \text{Cos}^2\gamma = 1$ → (3)

Substitute eqn (2) in eqn (3),

$$d_1^2\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right)=1$$

$$d_1^2=\frac{1}{\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right)}$$

$$d_1=\frac{1}{\sqrt{\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}}} \quad \text{------------------- (4)}$$

Let the plane parallel to ABC be A' B' C'. The intercepts along x, y, z axis be OA' OB' OC' respectively. The normal meets the 2nd plane A'B'C' at N' such that ON' = d_2. The extension of d_1 to $d_2$the normal makes the same along α, β, γ with x, y, z axis respectively.

The intercepts of OA', OB', OC' are such that

$OA'=\frac{2a}{h}$ $OB'=\frac{2b}{k}$ and $OC'=\frac{2c}{l}$

$$\left.\begin{aligned}\cos\alpha &= \frac{ON'}{OA'}=\frac{d_2}{2a/h}\\ \cos\beta &= \frac{ON'}{OB'}=\frac{d_2}{2b/k}\\ \cos\gamma &= \frac{ON'}{OC'}=\frac{d_2}{2c/l}\end{aligned}\right\}$$

We know that,

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ → (5)

$$d_2^2\left(\frac{h^2}{4a^2}+\frac{k^2}{4b^2}+\frac{l^2}{4c^2}\right)=1$$

$$d_2=\frac{2}{\sqrt{\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}}}$$

Let the separation between the planes ABC & ABC be'd '.

$d = d_2 - d_1$

$= 2d_1 - d_1$

$= d_1$

$$d=\frac{1}{\sqrt{\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}}}$$

For cubic system a = b = c.

hence, $d=\frac{a}{\sqrt{h^2+k^2+l^2}}$

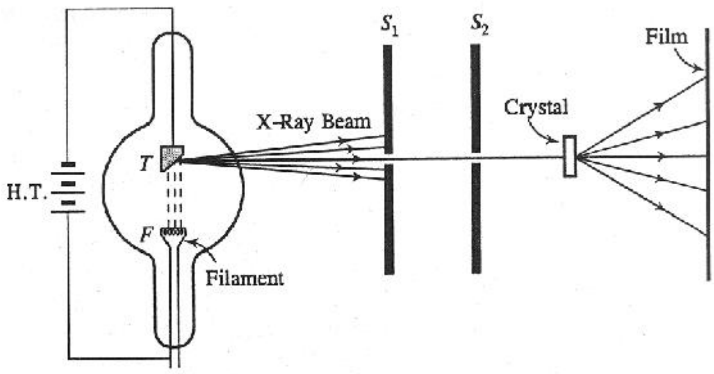
For Tetragonal system $a = b \neq c$.

$$d = \frac{1}{\sqrt{\frac{h^2+k^2}{a^2}+\frac{l^2}{c^2}}}$$

Laue method

S_1 & S2 are two lead screens in which two pin holes act as slits. X-ray beam from an X –ray tube is allowed to pass through these two slits S1 & S2 . The beam transmitted through S2 will be a narrow pencil of X – rays . The beam proceeds further to fall on a single crystal such that Zinc blended (ZnS) which is mounted suitably on a support . The single crystal acts as a 3 – dimensional diffraction grating to the incident beam. Thus, the beam undergoes diffraction in the crystal and then falls on the photographic film. The diffracted waves undergo constructive interference in certain directions, and fall on the photographic film with reinforced intensity. In all other directions, the interference will be destructive and the photographic film remains unaffected.

The resultant interference pattern due to diffraction through the crystal as a whole will be recorded on the photographic film (which requires many hours of exposure to the incident beam). When the film is developed, it reveals a pattern of fine spots, known as Laue spots.



X–RAY DIFFRACTION (LAUE's METHOD)

The distribution spots follow a particular way of arrangement that is the characteristic of the specimen used in the form of crystal to diffract the beam. The Laue spot photograph obtained by diffracting the beam at several orientations of the crystal to the incident beam are used for determining the symmetry and orientations of the internal arrangement of atoms, molecules in the crystal lattice . it is also used to study the imperfections in the crystal .

Powder Method (or) Debye Scherrer method:

X-ray powder method is usually carried out for polycrystalline materials. The given polycrystalline material is grounded to fine powder and this powder is taken in capillary tube. This tube is made up of non diffracting material, and fixed at the centre of cylindrical Debye Scherrer cylindrical camera as shown in figure.

The principle under this technique is that millions of tiny crystals in powder have completely random orientation all the possible diffraction phases are available for Bragg reflection to takes place. All the orientations are equally reflected ray will form a cone. whole axis likes along the direction of incident beam and whose semi vertical angle is twice the glancing angle for that particular planes.

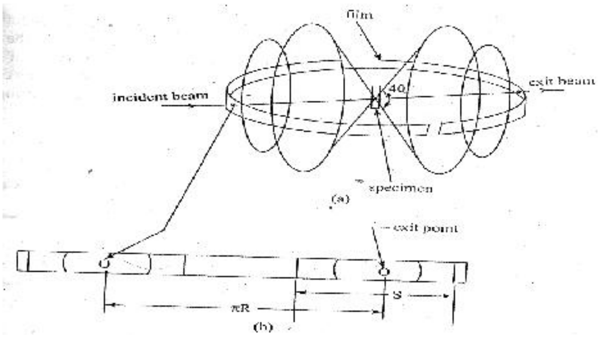


Fig: Powder method - apparatus

The different cones intercept the film in a series of concentric circular from the radial of these arcs. The angle can be calculated and hence the spacing between the atoms can be evaluated as shown in figure. The photographic film is in cylindrical shape, whose axis is perpendicular to the beam.

Let s be the distance of particular arc from the centre. Let R be the radius of camera. Then

$$4\theta = \frac{S}{R}$$

$$\theta = \frac{S}{4R}$$

If S_1, S_2, S_3, are the distances between symmetrical lines on the stretched film, then

$$\theta_1 = \frac{S1}{4R}$$

$$\theta_2 = \frac{S2}{4R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.$$

Using these values of θ_n in Bragg's equation

$$n\lambda = 2d \, \text{Sin}\theta_n.$$

Where,

n=1, 2, 3, …… is the Order of diffraction

d= Interplanar spacing

θ_n = Angle of diffraction for n^{th} order.

The inter-planar spacing d can be calculated.

Bragg's Law

Let λ be the wavelength of incident X-ray beam and angle of incidence be θ to Bragg's planes. Let inter planar spacing of crystal planes be'd'. The dots in the planes represent positions of atoms in the crystal. Every atom in the crystal is source of scattering for the X-ray incident on it.

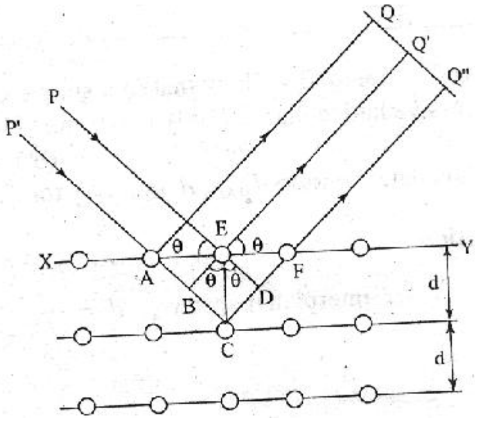


Fig: X-Ray Scattering by Crystal

A part of the incident x-ray beam PE, incident on atom at E in plane 1 is scattered along the direction EQ'. Similarly a part of incident x-rays P'C, fall on atom at C in plane 2 and is scattered in the direction CQ'' and it is parallel to EQ'. Let the beam PE and P'C make an angle θ with the Bragg's planes. This angle θ is called the angle of diffraction or glancing angle.

If the path difference between the two reflected rays is equal to integral multiple of incident wavelength λ, then constructive interference is observed.

$$Path\ Differenc\ ,(BC + CD) = n\lambda \ \rightarrow\ (1)$$

From the Δ^{le} BCE,

$$\mathbf{\sin\theta} = \frac{BC}{CE} = \frac{BC}{d}$$

$$BC = dsin\theta \ \rightarrow\ (2)$$

From the Δ^{le} EDC,

$$\mathbf{\sin\theta} = \frac{CD}{EC} = \frac{CD}{d}$$

$$CD = dsin\theta \ \rightarrow\ (3)$$

For constructive interference, from equations (1), (2) and (3)

$$CD + BC = n\lambda$$

$$dsin\theta + dsin\theta = n\lambda$$

$$2dsin\theta = n\lambda$$

The above equation is called Bragg's law of X-ray diffraction.

UNIT-III
Assignment –cum- tutorial questions
SECTION-A

1) X-ray are used for crystal diffraction studies because [ ]
   a) They have higher penetrating power
   b) Crystals are transparent to x-rays
   c) The inter atomic spacing is of the order of x-ray wavelength
   d) They have high resolving power
2) Inter atomic spacing is of the order of [ ]
   a) 2 to 3 A^0 b) 2 to 3μm c) 2 to 3nm d) 2 to 3mm
3) For simple cubic the ratio $\frac{1}{d_{100}}:\frac{1}{d_{110}}:\frac{1}{d_{111}}$ is given by [ ]
   a) $\mathbf{1}:\frac{1}{\sqrt{2}}:\sqrt{3}$
   b) $1:\sqrt{2}:\frac{\sqrt{3}}{2}$
   c) $1:\sqrt{2}:\frac{1}{\sqrt{3}}$
   d) $\sqrt{1}:\sqrt{2}:\sqrt{3}$
4) For body centered cubic the ratio $\frac{1}{d_{100}}:\frac{1}{d_{110}}:\frac{1}{d_{111}}$ is given by [ ]
   a) $1:\frac{1}{\sqrt{2}}:\sqrt{3}$
   b) $1:\sqrt{2}:\frac{\sqrt{3}}{2}$
   c) $1:\sqrt{2}:\frac{1}{\sqrt{3}}$
   d) $\sqrt{1}:\sqrt{2}:\sqrt{3}$
5) For face centered cubic ratio $\frac{1}{d_{100}}:\frac{1}{d_{110}}:\frac{1}{d_{111}}$ is given by [ ]
   1) $1:\frac{1}{\sqrt{2}}:\sqrt{3}$
   2) $1:\sqrt{2}:\frac{\sqrt{3}}{2}$
   3) $1:\sqrt{2}:\frac{1}{\sqrt{3}}$
6) A crystal is [ ]
   (a) A three dimensional representation of a solid
   (b) A three dimensional regular arrangement of U.C
   (c) A basis attached to the lattice point
   (d) All the above

7) The number of lattice points in a primitive cell is [ ]
   (a) 1 (b) ½ (c) 2 (d) 3/2

8).The atomic radius of BCC lattice is [ ]
   (a) $\frac{\sqrt{3}a}{4}$ (b) $\frac{\sqrt{3}a}{2}$
   (c) a/2 (d) $\frac{a}{2\sqrt{2}}$

9). Ratio of number of atoms per unit cell for SC, BCC, FCC crystal is [ ]

(a) 1:2:4 (b) 2:1:4

(c) 1:2:2 (d) 4:2:1

10). In a simple cubic lattice, $d_{100}:d_{110}:d_{111}$ is [ ]

(a) 6:3:2 (b) 2:3:6

(c) $\sqrt{2}:\sqrt{3}:\sqrt{6}$ (d) $\sqrt{6}:\sqrt{3}:\sqrt{2}$

SECTION-B

DESCRIPTIVE QUESTIONS

Short Answer Questions:

1. What are the differences between crystalline and amorphous materials?
2. Define basis, space lattice and unit cell
3. Is unit cell of SC lattice a primitive or not? Why? Apply the concept to body centered cell and face centered cell.
4. What are the differences between unit cell and primitive cell?
5. Define nearest neighour distance, atomic radius, coordination number and packing fraction.
6. What are miller indices?
7. State Bragg's law of X-ray diffraction.

Long Answer Questions:

1. Describe the seven crystal systems with diagrams.
2. Calculate the atomic radii for all the three types of cubic crystals.
3. Show that FCC is the most closely packed of all the three cubic structures by finding the packing factor.
4. Deduce the packing factors of SC, BCC and FCC structures.
5. What are Miller indices? How are they obtained?
6. Explain the significance of Miller indices.
7. Deduce the expression for the inter-planar distance in terms of Miller indices for cubic structure.
8. Derive Bragg's law of X-ray diffraction by crystals.
9. Discuss Laue method of diffraction.
10. Discuss powder method with the necessary diagrams.

Questions testing the ability of students in applying the concepts

1) Calculate the ratio $d_{100}:d_{110}:d_{111}$ for a single cubic structure.
2) The Bragg's angle in the 1^{st} order for [2, 2, 0] reflection from Ni (BCC) is 38.2^o. When x-rays of wavelength $\lambda=1.54$ A^o are employed in a diffraction experiment. Determine the lattice parameter of Ni.
3) Monochromatic x-rays of $\lambda=1.5$A^o is incident on a crystal phase having an inter planar spacing of 1.6A^o. Find the highest order for which Bragg's reflection maximum can be seen.
4) Calculate the glancing angle at (110) plane of a cubic crystal having axial length a=0.2nm.corresponding to the 2^{nd} order diffraction maximum for the x-rays of wavelength 0.065nm

5) The Bragg's angle for reflection from the (111) plane in a BCC crystal is 19.2^0. For an X-ray of wavelength 1.54 A^0. Compute the cube edge of the unit cell.
6) What is the angle at which the 3^{rd} order reflection of x-rays of 0.79A^0 wavelength can occur in a crystal of $3.04X10^{-8}$cm?
7) A beam of x-ray is incident on an ionic crystal with lattice spacing 0.313nm. Calculate the wavelength of x-rays, if the first order Bragg's reflection takes place at a glancing angle of $7^0 48'$.
8) Calculate the interplanar spacing for (3 2 1) plane in a simple cubic lattice whose lattice contact a = 4.2×10^{-10} m.
9) The atomic radius of copper is 1.278 A^0. It has atomic weight 63.54. Find the density of copper.

ENGINEERING PHYSICS

Unit IV:

Non-Destructive Testing using Ultrasonics

Objective:

- Assess the use of ultrasonics in NDT in evaluating the material characterization, flaw detection and data representation.

Syllabus:

Ultrasonic Testing-Basic principles-Transducer- Couplant and inspection standards- Inspection Methods-Pulse Echo Technique-Flaw detector- Different Types of scans- Applications

Outcomes:

At the end of the unit, the students are able to

1. interpret the basic principles of NDT using ultrsonics.
2. relate the knowledge of NDT to carry out inspection methods in ultrasonic testing.
3. analyse the different scans methods in data representation in the material inspection.

NON DESTRUCTIVE TESTING USING ULTRASONIC S

Ultrasonic Testing – Basic Principal – Transducer-Couplant and inspection standards –inspection methods –Pulse Echo Testing technique – Flaw Detector – Different Types of Scans – Applications

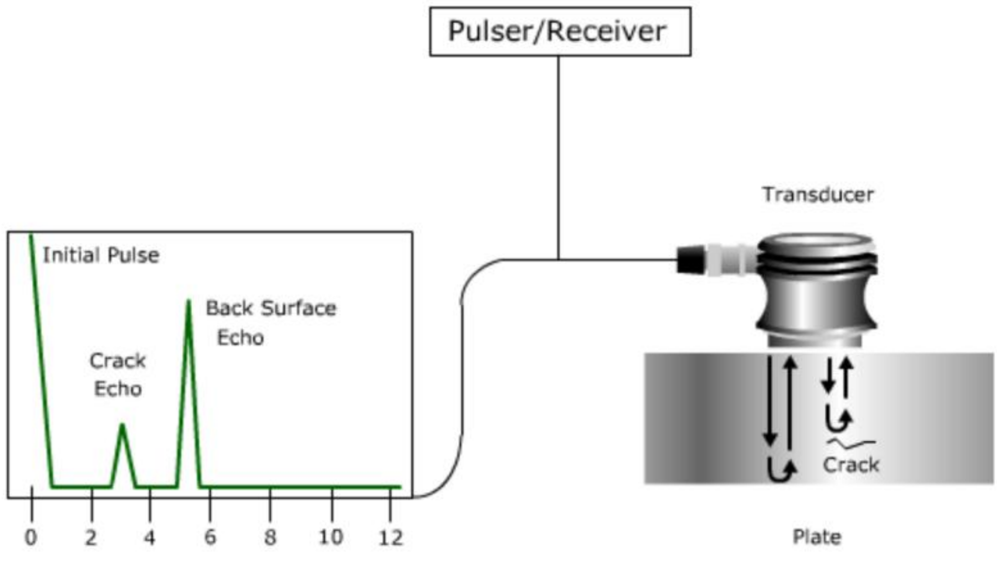
Introduction: Ultrasonic Testing

Ultrasonic Testing (UT) uses high frequency sound waves (*typically in the range between 0.5 and 15 MHz*) to conduct examinations and make measurements. Besides its wide use in engineering applications (*such as flaw detection/evaluation, dimensional measurements, material characterization, etc.*), ultrasonics are also used in the medical field (*such as sonography, therapeutic ultrasound, etc*.).

In general, ultrasonic testing is based on the capture and quantification of either the reflected waves (*pulse-echo*) or the transmitted waves (*through-transmission*). Each of the two types is used in certain applications, but generally, pulse echo systems are more useful since they require one-sided access to the object being inspected.

Basic Principles

A typical pulse-echo UT inspection system consists of several functional units, such as the pulser/receiver, transducer, and display devices. A pulser/receiver is an electronic device that can produce high voltage electrical pulses.



Driven by the pulser, the transducer generates high frequency ultrasonic energy. The sound energy is introduced and propagates through the materials in the form of waves. When there is a discontinuity (*such as a crack*) in the wave path, part of the energy will be reflected back from the flaw surface.

The reflected wave signal is transformed into an electrical signal by the transducer and is displayed on a screen. Knowing the velocity of the waves, travel time can be directly related to the distance that the signal traveled. From the signal, information about the reflector location, size, orientation and other features can sometimes be gained.

Advantages and Disadvantages

The primary advantages and disadvantages when compared to other NDT methods are:

Advantages

- It is sensitive to both surface and subsurface discontinuities.
- The depth of penetration for flaw detection or measurement is superior to other NDT methods.
- Only single-sided access is needed when the pulse-echo technique is used.
- It is highly accurate in determining reflector position and estimating size and shape.
- Minimal part preparation is required.
- It provides instantaneous results.
- Detailed images can be produced with automated systems.
- It is nonhazardous to operators or nearby personnel and does not affect the material being tested.
- It has other uses, such as thickness measurement, in addition to flaw detection.
- Its equipment can be highly portable or highly automated.

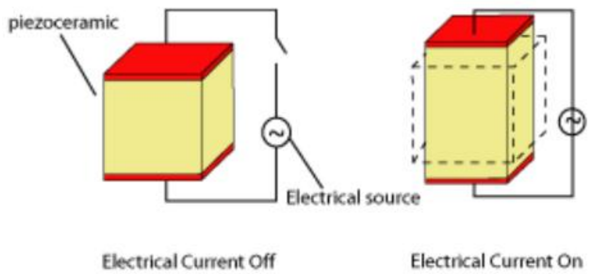
Disadvantages

- Surface must be accessible to transmit ultrasound.
- Skill and training is more extensive than with some other methods.
- It normally requires a coupling medium to promote the transfer of sound energy into the test specimen.
- Materials that are rough, irregular in shape, very small, exceptionally thin or not homogeneous are difficult to inspect.
- Cast iron and other coarse grained materials are difficult to inspect due to low sound transmission and high signal noise.
- Linear defects oriented parallel to the sound beam may go undetected.
- Reference standards are required for both equipment calibration and the characterization of flaws.

TRANSDUCERS

Piezoelectric Transducers

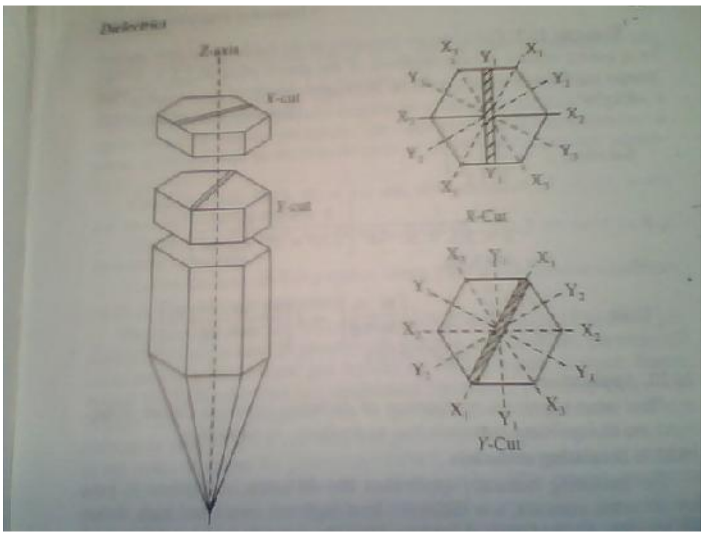
The conversion of electrical pulses to mechanical vibrations and the conversion of returned mechanical vibrations back into electrical energy is the basis for ultrasonic testing. This conversion is done by the transducer using a piece of piezoelectric material (*a polarized material having some parts of the molecule positively charged, while other parts of the molecule are negatively charged*) with electrodes attached to two of its opposite faces. When an electric field is applied across the material, the polarized molecules will align themselves with the electric field causing the material to change dimensions. In addition, a permanently-polarized material such as quartz (SiO_2) or barium titanate ($BaTiO_3$) will produce an electric field when the material changes dimensions as a result of an imposed mechanical force. This phenomenon is known as the piezoelectric effect.



The active element of most acoustic transducers used today is a piezoelectric ceramic, which can be cut in various ways to produce different wave modes. A large piezoelectric ceramic element can be seen in the image of a sectioned low frequency transducer. The most commonly employed ceramic for making transducers is lead zirconate titanate.

The thickness of the active element is determined by the desired frequency of the transducer. A thin wafer element vibrates with a wavelength that is twice its thickness. Therefore, piezoelectric crystals are cut to a thickness that is 1/2 the desired radiated wavelength. The higher the frequency of the transducer, the thinner the active element.

Quartz is Silicon dioxide (Sio_2] . It occurs naturally.The atoms are arranged as shown fig. They form a hexagon in the plane of the paper. The optical axis in quartz is z and is perpendicular to the xy plane. The line joining the opposite faces represents y axes. The line joining opposite corners are called x.A plate of quartz with its surface perpendicular to the X-axis is called on X-cut crystal a plate with its face perpendicular to the Y-axis is called Y-cut crystal. Piezoelectricity in quartz along the optic axis is absent. Piezoelectric effect is maximum along the X-axis minimum along Y-axis.

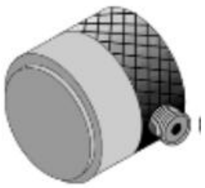


Transducer Types

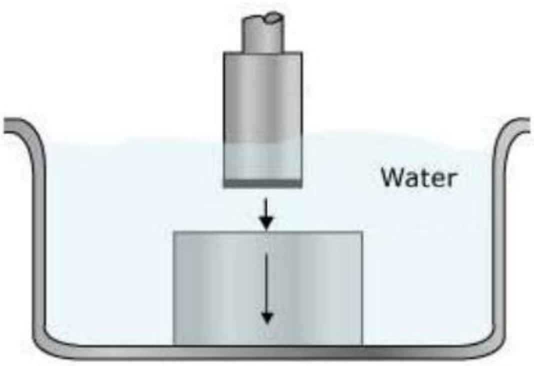
Ultrasonic transducers are manufactured for a variety of applications and can be custom fabricated when necessary. Careful attention must be paid to selecting the proper transducer for the application. It is important to choose transducers that have the desired frequency, bandwidth, and focusing to optimize inspection capability. Most often the transducer is chosen either to enhance the sensitivity or resolution of the system.

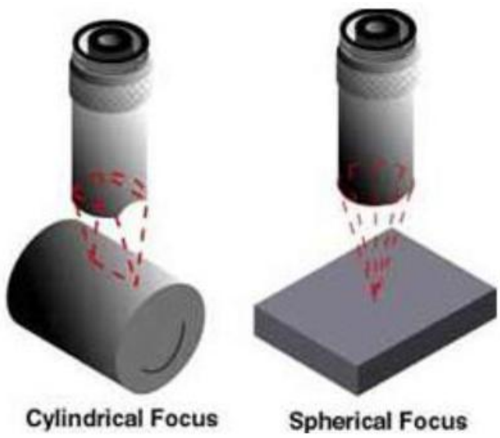
Transducers are classified into two major groups according to the application.

Contact transducers are used for direct contact inspections, and are generally hand manipulated. They have elements protected in a rugged casing to withstand sliding contact with a variety of materials. These transducers have an ergonomic design so that they are easy to grip and move along a surface. They often have replaceable wear plates to lengthen their useful life. Coupling materials of water, grease, oils, or commercial materials are used to remove the air gap between the transducer and the component being inspected.



Immersion transducers do not contact the component. These transducers are designed to operate in a liquid environment and all connections are watertight. Immersion transducers usually have an impedance matching layer that helps to get more sound energy into the water and, in turn, into the component being inspected. Immersion transducers can be purchased with a planer, cylindrically focused or spherically focused lens. A focused transducer can improve the sensitivity and axial resolution by concentrating the sound energy to a smaller area. Immersion transducers are typically used inside a water tank or as part of a squirter or bubbler system in scanning applications.

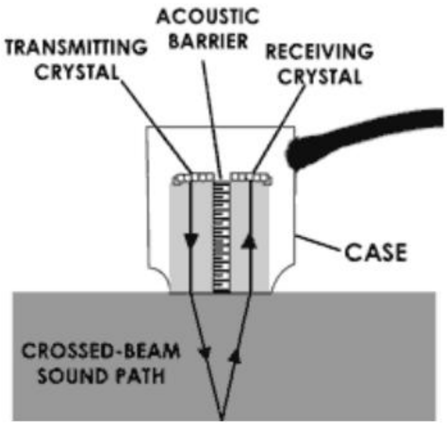




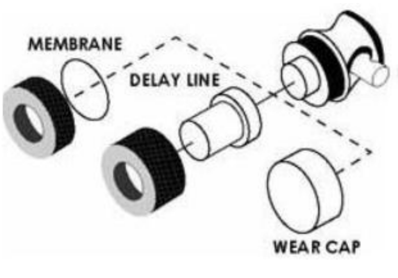
Other Types of Contact Transducers

Contact transducers are available in a variety of configurations to improve their usefulness for a variety of applications. The flat contact transducer shown above is used in normal beam inspections of relatively flat surfaces, and where near surface resolution is not critical. If the surface is curved, a shoe that matches the curvature of the part may need to be added to the face of the transducer. If near surface resolution is important or if an angle beam inspection is needed, one of the special contact transducers described below might be used.

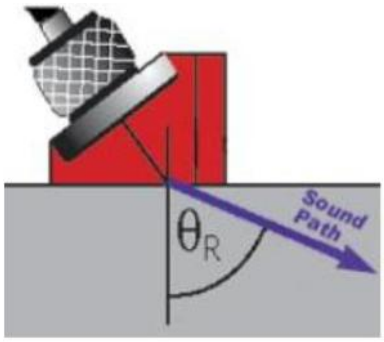
Dual element transducers contain two independently operated elements in a single housing. One of the elements transmits and the other receives the ultrasonic signal. Dual element transducers are especially well suited for making measurements in applications where reflectors are very near the transducer since this design eliminates the ring down effect that single-element transducers experience (*when single-element transducers are operating in pulse echo mode, the element cannot start receiving reflected signals until the element has stopped ringing from its transmit function*). Dual element transducers are very useful when making thickness measurements of thin materials and when inspecting for near surface defects. The two elements are angled towards each other to create a crossed-beam sound path in the test material.



Delay line transducers provide versatility with a variety of replaceable options. Removable delay line, surface conforming membrane, and protective wear cap options can make a single transducer effective for a wide range of applications. As the name implies, the primary function of a delay line transducer is to introduce a time delay between the generation of the sound wave and the arrival of any reflected waves. This allows the transducer to complete its "sending" function before it starts its "receiving" function so that near surface resolution is improved. They are designed for use in applications such as high precision thickness gauging of thin materials and delamination checks in composite materials. They are also useful in high-temperature measurement applications since the delay line provides some insulation to the piezoelectric element from the heat.

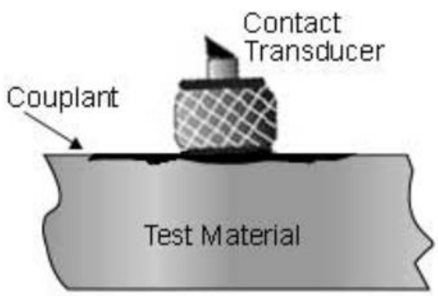


Angle beam transducers and wedges are typically used to introduce a refracted shear wave into the test material. Transducers can be purchased in a variety of fixed angles or in adjustable versions where the user determines the angles of incidence and refraction. In the fixed angle versions, the angle of refraction that is marked on the transducer is only accurate for a particular material, which is usually steel. The most commonly used refraction angles for fixed angle transducers are 45°, 60° and 70°. The angled sound path allows the sound beam to be reflected from the backwall to improve detectability of flaws in and around welded areas. They are also used to generate surface waves for use in detecting defects on the surface of a component.



Couplant

A couplant is a material (*usually liquid*) that facilitates the transmission of ultrasonic energy from the transducer into the test specimen. Couplant is generally necessary because the acoustic impedance mismatch between air and solids is large. Therefore, nearly all of the energy is reflected and very little is transmitted into the test material. The couplant displaces the air and makes it possible to get more sound energy into the test specimen so that a usable ultrasonic signal can be obtained. In contact ultrasonic testing a thin film of oil, glycerin or water is typically used between the transducer and the test surface. When shear waves are to be transmitted, the fluid is generally selected to have a significant viscosity.



When scanning over the part, an immersion technique is often used. In immersion ultrasonic testing both the transducer and the part are immersed in the couplant, which is typically water. This method of coupling makes it easier to maintain consistent coupling while moving and manipulating the transducer and/or the part.

Inspection standards

- Materials that are rough, irregular in shape, very small, exceptionally thin or not homogeneous are difficult to inspect., therefore they must be smooth,regular in shape, sufficient thick and homogeneous.
- Surface must be prepared by cleaning and removing loose scale, paint etc, although paint that is properly bounded to a surface usually need not be removed.
- Couplants are needed to provide effective transfer of ultrasonic wave energy between transducers and parts being inspected unless a non – contact technique is used.
- Inspected items must be water resistant, when using water based couplants that do not contain rust inhibitors.

Inspection methods

There are basically two methods to test an object using ultrasonic waves whether there are any defects are flaws. These two methods can be obtained by receiving the ultrasound waveform by reflection(pulse-echo) and attenuation(through – transmission).

In reflected mode, the transducer performs both the sending and receiving of the pulsed waves as the sound is reflected back to the device. Reflected ultrasound comes from an interface such as the back wall of the object or from an imperfection within the object. The diagnostic machine displays these results in the form a signal with amplitude representing the intensity of the reflection and the distance, representing the arrival time of the reflection.

In attenuation mode, a transmitter sends ultrasound through one surface and a separate receiver detects the amount that has reached it on another surface after travelling through the medium. Imperfections or other conditions in the space between the transmitter and receiver reduce the amount of sound transmitted, thus revealing their presence. Using the couplant increases the efficiency of the process by reducing the losses in the ultrasonic wave energy due to separation between the surfaces.

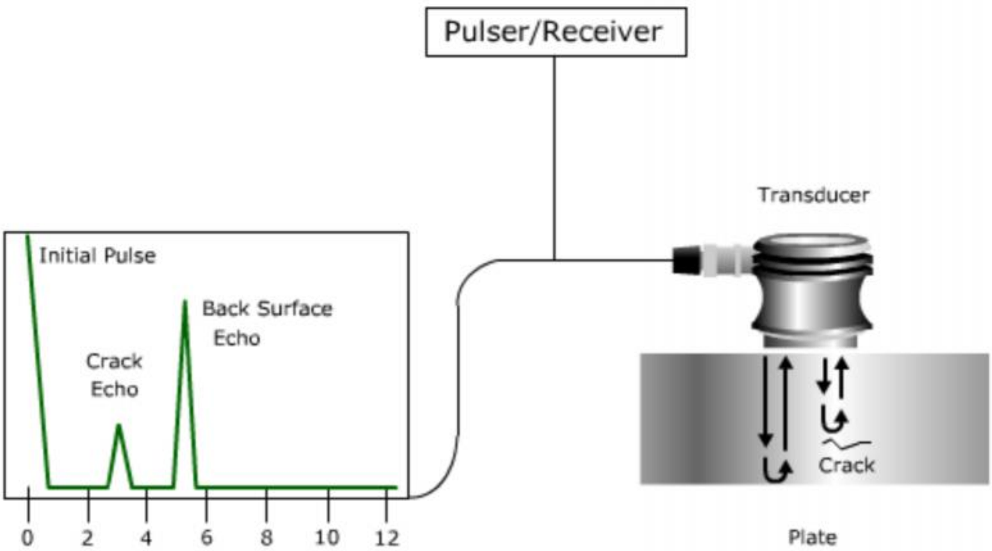
Pulse Echo testing technique

Pulse-echo ultrasonic measurements can determine the location of a discontinuity in a part or structure by accurately measuring the time required for a short ultrasonic pulse generated by a transducer to travel through a thickness of material, reflect from the back or the surface of a discontinuity, and be returned to the transducer. In most applications, this time interval is a few microseconds or less. The two-way transit time measured is divided by two to account for the down-and-back travel path andmultiplied by the velocity of sound in the test material. The result is expressed in the well-known relationship:

$$d=\frac{vt}{2}$$

Where is the distance from the surface to the discontinuity in the test piece, is the velocity of sound waves in the material, and is the measured round-trip transit time.

Precision ultrasonic thickness gages usually operate at frequencies between 500 kHz and 100 MHz, by means of piezoelectric transducers that generate bursts of sound waves when excited by electrical pulses. Typically, lower frequencies are used to optimize penetration when measuring thick, highly attenuating or highly scattering materials, while higher frequencies will be recommended to optimize resolution in thinner, non-attenuating, non-scattering materials. It is possible to measure most engineering materials ultrasonically, including metals, plastic, ceramics, composites, epoxies, and glass as well as liquid levels and the thickness of certain biological specimens. On-line or in-process measurement of extruded plastics or rolled metal often is possible, as is measurements of single layers or coatings in multilayer materials

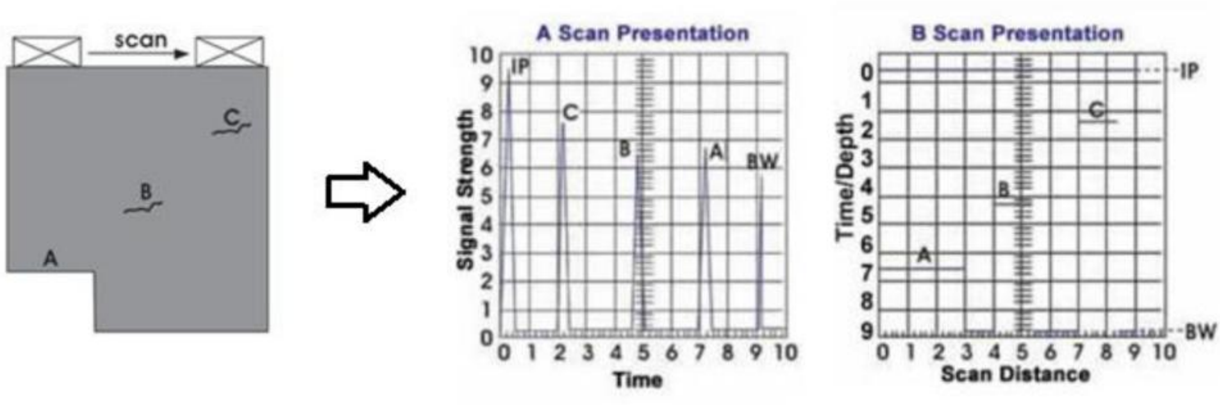


Data Presentation

Ultrasonic data can be collected and displayed in a number of different formats. The three most common formats are known in the NDT world as A-scan, B-scan and C-scan presentations. Each presentation mode provides a different way of looking at and evaluating the region of material being inspected. Modern computerized ultrasonic scanning systems can display data in all three presentation forms simultaneously.

A-Scan Presentation

The A-scan presentation displays the amount of received ultrasonic energy as a function of time. The relative amount of received energy is plotted along the vertical axis and the elapsed time (*which may be related to the traveled distance within the material*) is displayed along the horizontal axis. Most instruments with an A-scan display allow the signal to be displayed in its natural radio frequency form (RF), as a fully rectified RF signal, or as either the positive or negative half of the RF signal. In the A-scan presentation, relative discontinuity size can be estimated by comparing the signal amplitude obtained from an unknown reflector to that from a known reflector. Reflector depth can be determined by the position of the signal on the horizontal time axis.



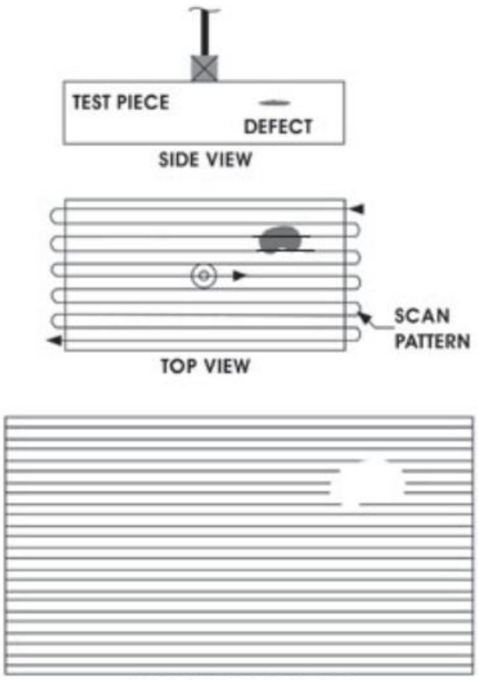
In the illustration of the A-scan presentation shown in the figure, the initial pulse generated by the transducer is represented by the signal IP, which is near time zero. As the transducer is scanned along the surface of the part, four other signals are likely to appear at different times on the screen. When the transducer is in its far left position, only the IP signal and signal A, the sound energy reflecting from surface A, will be seen on the trace. As the transducer is scanned to the right, a signal from the backwall BW will appear later in time, showing that the sound has traveled farther to reach this surface. When the transducer is over flaw B, signal B will appear at a point on the time scale that is approximately halfway between the IP signal and the BW signal. Since the IP signal corresponds to the front surface of the material, this indicates that flaw B is about halfway between the front and back surfaces of the sample. When the transducer is moved over flaw C, signal C will appear earlier in time since the sound travel path is shorter and signal B will disappear since sound will no longer be reflecting from it.

B-Scan Presentation

The B-scan presentation is a type of presentation that is possible for automated linear scanning systems where it shows a profile (cross-sectional) view of the test specimen. In the B-scan, the time-of-flight (travel time) of the sound waves is displayed along the vertical axis and the linear position of the transducer is displayed along the horizontal axis. From the B-scan, the depth of the reflector and its approximate linear dimensions in the scan direction can be determined. The B-scan is typically produced by establishing a trigger gate on the A-scan. Whenever the signal intensity is great enough to trigger the gate, a point is produced on the B-scan. The gate is triggered by the sound reflected from the backwall of the specimen and by smaller reflectors within the material. In the

B-scan image shown previously, line A is produced as the transducer is scanned over the reduced thickness portion of the specimen. When the transducer moves to the right of this section, the backwall line BW is produced. When the transducer is over flaws B and C, lines that are similar to the length of the flaws and at similar depths within the material are drawn on the B-scan. It should be noted that a limitation to this display technique is that reflectors may be masked by larger reflectors near the surface.

C-Scan Presentation



The C-scan presentation is a type of presentation that is possible for automated two-dimensional scanning systems that provides a plan-type view of the location and size of test specimen features. The plane of the image is parallel to the scan pattern of the transducer. C-scan presentations are typically produced with an automated data acquisition system, such as a computer controlled immersion scanning system.

Typically, a data collection gate is established on the A-scan and the amplitude or the time-of-flight of the signal is recorded at regular intervals as the transducer is scanned over the test piece. The relative signal amplitude or the time-of-flight is displayed as a shade of gray or a color for each of the positions where data was recorded. The C-scan presentation provides an image of the features that reflect and scatter the sound within and on the surfaces of the test piece.

High resolution scans can produce very detailed images. The figure shows two ultrasonic C-scan images of a US quarter. Both images were produced using a pulse-echo technique with the transducer scanned over the head side in an immersion scanning system. For the C-scan image on the top, the gate was setup to capture the amplitude of the sound reflecting from the front surface of the quarter. Light areas in the image indicate areas that reflected a greater amount of energy back to the transducer. In the C-scan image on the bottom, the gate was moved to record the intensity of the sound reflecting from the back surface of the coin. The details on the back surface are clearly visible but front surface features are also still visible since the sound energy is affected by these features as it travels through the front surface of the coin.

Assignment –cum- tutorial questions

I. Questions testing the remembering / understanding level of students

1. Give Examples of couplants____________________
2. The transducers of ultrasonic flaw detectors produce ultrasonic waves due to [ ]
   a) Magnetostricition effect b) Mechanical effect
   c) Piezoelectric effect d) none of the above
3. Couplant is used in ultrasonic flaw detectors to [ ]
   a) Increase the intensity of ultrasound
   b) Reduce the acoustic impedance mismatch between air and testpiece
   c) Increase the acoustic impedance mismatch between air and testpiece
   d) None of the above
4. The frequency of ultrasonic waves is [ ]
   a) below 20kHz b) above 20kHz c) above 20Hz d) none of the above
5. A constant testing of product without causing any damage is called [ ]
   a) Minute testing b) Destructive testing c) Non-destructive testing d) Random testing
6. The signal due to a reflected wave is called [ ]
   a) Transmitted wave b) Longitudinal wave c) Echo d) Peaco
7. The type of ultrasonic waves that exist in the solid are ______________________
8. Ultrasonic waves is not employed in [ ]
   a) Non-destructive testing b) Determination of elastic constant
   c) Determination of velocity of sound d) Determination of electrical conductivity
9. If the particles vibrate perpendicular to the propagation of the wave then it is called.[ ]
   a) Longitudinal waves b) Rayleigh waves c) Lamb waves d) Shear waves
10. In the production of ultrasonics ________________cut quartz crystals are used.

DESCRIPTIVE QUESTIONS

Short Answer Questions:

1. What are Piezoelectric transducers?
2. Write the properties, applications of ultasonics
3. What are ultrasonics? Mention the methods to produce ultrasonics?
4. Why couplants are used in ultrasonic testing ?
5. Give examples of Piezoelectric transducers?
6. How are ultrasonic waves classified?
7. Mention various methods of non destructive testing.

Long Answer Questions:

1. Explain the basic principles of ultrasonic testing.
2. What are Piezoelectric transducers? Explain Pulse-echo technique in non destructive ultrasonic testing.
3. What is non destructive testing ?Explain how flaw in solid can be detected by non destructive method using ultrasonics.
4. Explain the advantages and disadvantages of ultrasonic testing in NDT
5. Explain in detail A,B.C scans of data presentation in ultrasonic inspection.
6. What are couplants? Explain the requirement of a Couplant in ultrasonic testing.
7. Describe different types of probes in ultrasonic testing.
8. Analyze in detail the contact method, immersion methods of ultrasonic testing.

ENGINEERING PHYSICS

UNIT – V

Objective:

- Understand the properties of light

Syllabus:

Wave Optics

Interference: Introduction – Interference in thin films by reflection – Newton's rings.

Diffraction : Introduction – Fraunhofer diffraction - Fraunhofer diffraction at single slit– Diffraction grating – Resolving power of a grating

Polarization: Introduction – Types of Polarization – Double refraction – Quarter wave plate and Half Wave plate.

Learning Outcomes:

At the end of this chapter students are able to

- calculate path difference in thin films by reflection
- work out the radius of curvature of given plano convex lens
- derive expression for franhoffer single slit diffraction
- obtain expression for resolving power of grating.
- Sketch double refraction
- differentiate between half wave and quarter wave plate

ENGINEERING PHYSICS

Learning Material

Unit 5 PHYSICAL OPTICS

Interference

Principle of superposition: *When two or more waves overlap, the net disturbance at any point is the sum of the individual disturbances due to each wave.*

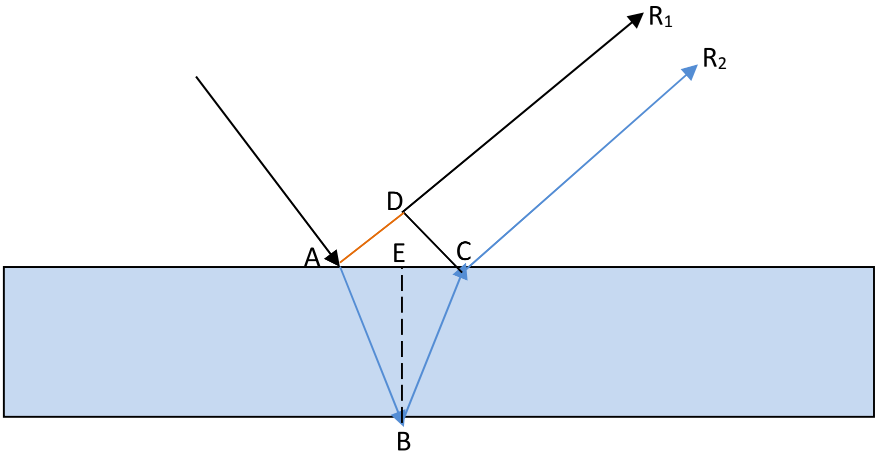
Interference in thin films

The colours of thin films, soap bubbles and oil slicks can be explained as due to the phenomenon of interference. In all these examples the formation of interference pattern is by the division of amplitude. For example. If a plane wave falls on a thin film then the wave reflected from the upper surface interferes with the wave reflected from the lower surface. Such studies have many practical applications such as production of non reflecting coatings.

Interference in plane parallel films due to reflected light

Let us consider a plane parallel film. Let light be incident at A part of the light is reflected towards R_1 and the other part is refracted into the film towards B. this second part is reflected at C and emerges as R_2 and is parallel to the first part.

At normal incidence, the path difference between ray 1 and ray 2 is twice the optical thickness of the film.



$\Delta = 2\mu t$

At oblique incidence the path difference is given by $\Delta = \mu(AB+BC) - AD$

$= \frac{2\mu t}{cosr} - AD$ $\quad [\cos r = \frac{BE}{AB} \quad AB = \frac{BE}{cosr} = \frac{t}{cosr} = BC]$

$= \frac{2\mu t}{cosr} - 2\mu t tanr sinr.$ $[\because AD = ACsini = 2AE.sini = 2t_{tanr.\mu sinr}]$

i.e., $\Delta = 2\mu t\{\frac{1}{cosr} - tanrsinr\}$ $= 2\mu t\{\frac{1-sin^2 r}{cosr}\}$ $= 2\mu t$.cosr which is known as cosine law.

where μ is the refractive index of the medium between the surfaces since for air μ=1,the path difference between ray1 and 2 is given by $\Delta = 2t$.cosr.

it should be remembered that a ray reflected at a surface backed by a denser medium suffers an abrupt phase change of π which is equivalent to a path difference $\lambda/2$

thus the effective path difference between the two reflected rays is($2\mu t$.cosr$\pm \lambda/2$)

we know that maxima occurs when effective path difference $\Delta = n\lambda$

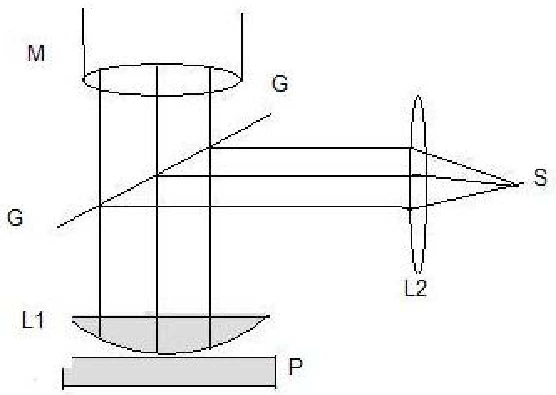
i.e., $2\mu t$.cosr$\pm \lambda/2 = n\lambda$

if this condition is fulfilled, the film will appear bright in the reflected light.

NEWTON'S RINGS

Newton's rings are formed due to the interference between the light waves reflected from the upper and lower surfaces of air film of variable thickness formed between the convex lens and the glass plate.

The experimental arrangement of Newton's rings is



Light from the source is rendered parallel by the convex lens L_2 placed between the source and the glass plate G.

The glass plate G which is inclined at an angle of 45^0 to the horizontal plane reflected the rays downward on to the lens L_1 the rays fall normally on the lens L_1

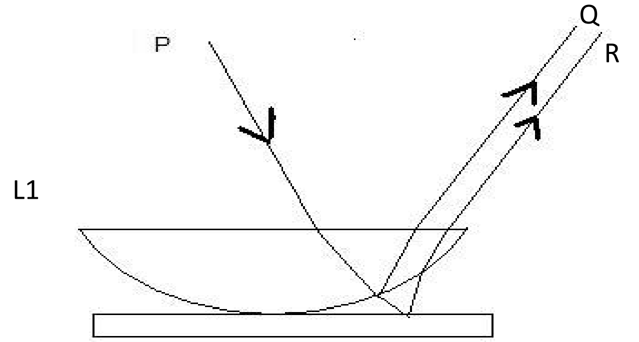
These rays are reflected back in the opposite direction by the two surfaces of the air film existing between the lens L_1 and glass plate P.

These reflected rays interfere. When the microscope M is focused on the air film interference fringes will be observed. Dark and bright circular fringes can be seen on the field of view of the microscope.

A black paper is usually placed underneath the plate P in order to absorb the light passing down through the glass plate thereby avoiding any reflection of light from the metallic bed of the instrument.

The formation of Newton's rings can be explained as

PQ is a monochromatic light ray which falls on the lens L_1. PQ undergoes reflection and refraction at R. The reflected ray RT undergoes no phase reversal. The refracted ray RS undergoes reflection at S and takes the path SU with the phase reversal of ‘P’. The rays RT and SU interfere since they are derived from the same ray PQ leading to Newton’s rings. Since the interference pattern is due to reflected light the path difference Δ is given by



$$\Delta = 2\mu t.\cos r + \lambda/2$$

For air film μ=1 and for normal incidence $r = 0^0$ therefore the path difference is given by

$$\Delta = 2t + \lambda/2$$

Case1: At the point of contact of the lens and plate t = 0

$\therefore \Delta = \lambda/2$ this is the condition for destructive interference i.e., minimum intensity hence the central spot of the ring is dark.

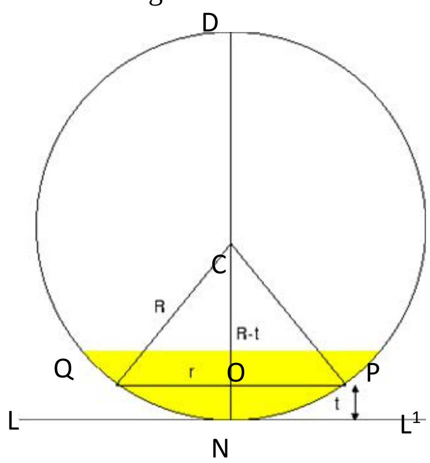
The condition for the formation of a bright ring is given by

$$2t + \frac{\lambda}{2} = n\lambda \text{ or } 2t = n\lambda - \lambda/2 = (2n-1)\,\lambda/2$$ where n =0,1,2,....

The condition of minimum intensity for a dark ring is

$$2t + \lambda/2 = (2n+1)\,\lambda/2 \text{ or } 2t = n\lambda$$ where n = 0,1,2,3,....

Now let us calculate the diameters of these fringes.



Let LoL^1 be the lens placed on the glass plate AB. The curved surface LoL^1 is part of the spherical surface with the centre at C. Let R be the radius of curvature and r be the radius of Newton’s ring corresponding to constant film thickness t.

From the property of the circle i.e., NP x NQ = NO x ND

i.e., r x r = t(2R-t) $=2Rt - t^2 = 2Rt$ i.e., $r^2 = 2Rt$ or $t = r^2/2R$

Thus for a bright fringe $\frac{2r^2}{2R} = \frac{(2n-1)\lambda}{2}$ or $r^2 = \frac{(2n-1)\lambda R}{2}$ replacing r by D/2 where D is the diameter we get $D_n = \sqrt{2\lambda R}\sqrt{2n-1}$ similarly for a dark fringe $\frac{2r^2}{2R} = n\lambda \; or \; r^2 = n\lambda R$

D_n^2 = 4nλR D_n=2$\sqrt{n\lambda R}$

Thus, the diameters of the rings are proportional to the square roots of the natural number. By measuring the diameter of the Newton's rings,it is possible to calculate the wavelength of light as follows. We have for the diameter of the nth dark fringe

D_n^2 = 4nλR

lly diameter for the (n+P)th dark fringe D_{n+p}^2 = 4(n+p)λR

$\therefore D_{n+p}^2 - D_n^2 = 4p\lambda R$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

λ can be calculated by using this formula

DIFFRACTION

When light falls on obstacles or small apertures whose size is comparable with the wavelength of light, there is a departure from straight lines propagation the light bends round the corners of the obstacles or apertures and enters in the geometrical shadow . This bending of light is called diffraction

Kinds of diffraction.

Diffraction phenomenon can be divided into following two general class

1 Fraunhofer's diffraction :- In this class of diffraction source and the screen or telescope (through which the image is viewed) are placed at infinity or effectively at infinity. In this case the wave front which is incident on the aperture or obstacle is plane .

II Fresnel's diffraction:- In this class of diffraction, source and screen are placed at finite distances from the aperture of obstacle having sharp edges. In this case no lenses are used for making the rays parallel or convergent. The incident wave fronts are either spherical or cylindrical.

FRAUNHOFER DIFFRACTION AT SINGLE SLIT

AB is a narrow slit of width e. Let a plane wave front WW^1 of monochromatic light of wave length λ propagating normally to the slit be incident on it. Let the diffracted light be focussed by means of a convex lens on a screen placed in the focal plane of the lens. According to Huygens-Fresnel, every point of the wave front in the plane of the slit is a source of secondary spherical wavelets, which spread out to the right in all directions. The secondary wavelets travelling normally to the slit, i.e., along the direction OPo, are brought to focus at P_o by the lens. Thus P_o is a bright central image. The secondary wavelets travelling at an angle θ with the normal are focussed at a point P_1 on the screen. The point P_1 is of the minimum intensity or maximum intensity depending upon the path difference between the secondary waves originating from the corresponding points of the wave front.

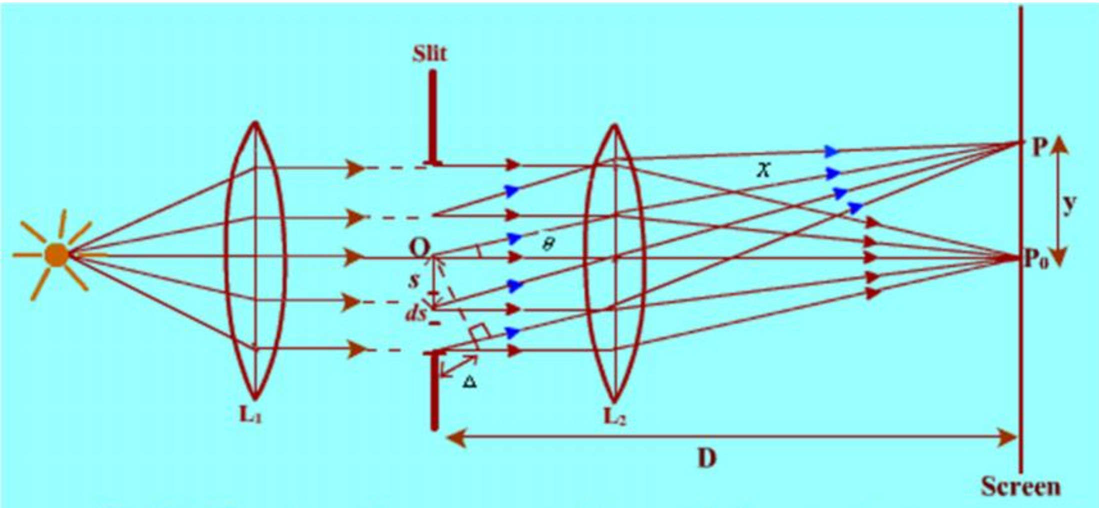


Fig: 2(b): Experimental arrangement and geometrical construction to understand the distribution of intensity on a screen in the single slit pattern

GENERAL MATHEMATICAL THEORY

In order to find out intensity at P_1, draw a perpendicular AC on BR. The path difference between secondary wavelets from A and B in direction θ

$$= BC = AB\sin\theta = e\sin\theta,$$

And corresponding phase difference $= \frac{2\pi}{\lambda}.e\ sin\theta$.

Let us consider that the width of the slit is divided into n equal parts and the amplitude of the wave from each part is a(because width of each part is same). The phase difference between any two consecutive waves from these parts would be

$\frac{1}{n}$(Total Phase) = $\frac{1}{n}(\frac{2\pi}{\lambda}.e\ sin\theta)$ = d (say)

Using the method of vector addition of amplitudes as discussed above, the resultant amplitude R is given by

$$R = a\frac{sin^{nd}/_2}{sin^{d}/_2} = a\frac{\sin(^{\Pi esin\theta}/_\lambda)}{sin(^{\Pi esin\theta}/_\lambda)} = a\frac{\sin\alpha}{sin^{\alpha}/_n} \text{ where } \alpha = {}^{\Pi esin\theta}/_\lambda$$

$$a\frac{\sin\alpha}{sin^{\alpha}/_n} = na\frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha}$$

(when n → ∞, a → 0 but product na =A remains finite)

Now the intensity is given by $I = R^2 = A^2(\frac{\sin\alpha}{\alpha})^2 = I_0(\frac{\sin\alpha}{\alpha})^2$

Intensity distribution

Principal maximum. The expression for resultant amplitude $R = A\frac{\sin\alpha}{\alpha}$

If α = 0 $\lim_{\alpha\to 0}\frac{sin\alpha\alpha}{\alpha} = 1$ (i.e., maximum value)

- ⇨ $\alpha = 0$ $\quad \Pi e sin\theta/\lambda = 0 \Rightarrow \theta = 0$
- ⇨ now maximum value of R *is* A and intensity is proportional to A^2. The condition θ =0 means that this maximum is formed by those secondary wavelets which travel normally to the slit. The maximum is known as as principle maximum.

Minimum intensity positions

The intensity will be minimum when sinα = 0. The values of α which satisfy this equation are

A =±π,±2π,±3π,±4π,..... etc =±mπ

Or $\Pi e sin\theta/\lambda$ =±mπ or esinθ =±mπ where m = 1,2,3,....etc

In this way we obtain the points of minimum intensity on either side of the principle maximum. The value of m=0 is not admissible, because for this value θ =0 and this corresponds to principle maximum.

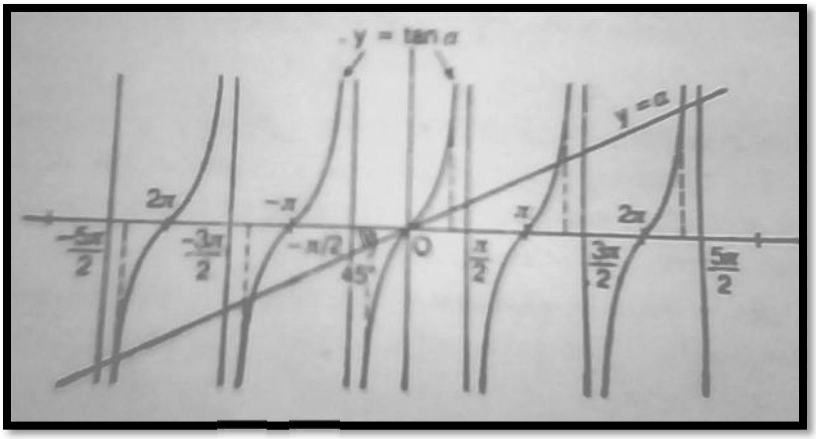
Secondary maxima. In addition to principal maximum at α = 0,there are weak secondary maxima between equally spaced minima. The positions can be obtained with the rule of finding maxima and minima of a given function in calculus. Differentiating the expression of I with respect to α and equating to zero we have

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{sin\alpha}{\alpha}\right)^2\right] = 0 \text{ or } A^2\frac{2sin\alpha}{\alpha}.\frac{(\alpha cos\alpha - sin\alpha)}{\alpha^2} = 0$$

Either sinα = 0 gives the values of α(except 0) for which the intensity is zero on the screen. Hence the position of maxima are given by the roots of the equation $\alpha cos\alpha - sin\alpha = 0$ or α = tanα

The values of α satisfying the above equation are obtained graphically by plotting the curves

Y = α and y = tanα are shown below



The point of intersections are α =0,±$\frac{3\pi}{2}$, ,±$\frac{5\pi}{2}$, etc.,

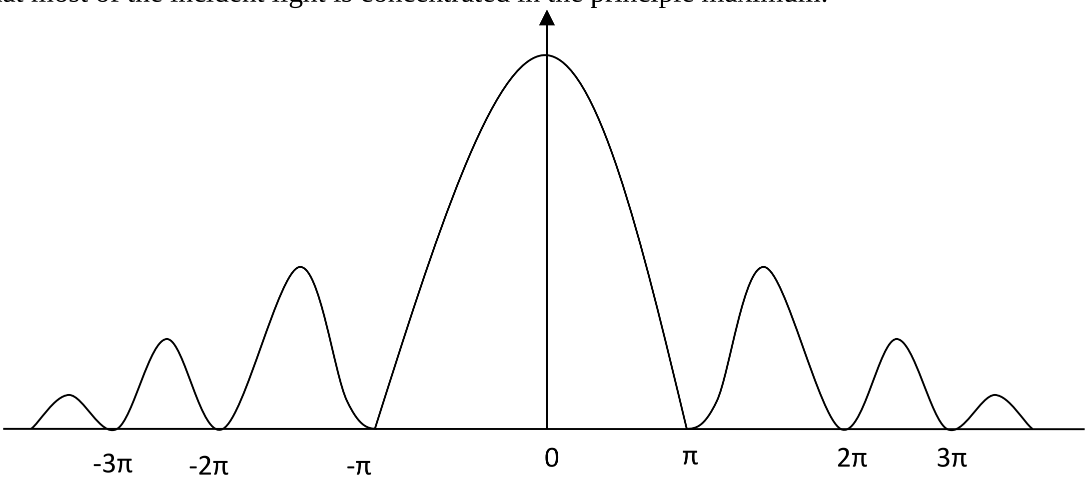
Or more exactly to α = 0, ±1.430π, ±2.462π, ±3.471π, etc

α = 0 gives principle maximum. Substituting approximate values in α in I =R^2 = $A^2(\frac{\sin\alpha}{\alpha})^2$

we get the intensities in various maxima $I_o=A^2$(Principle maximum)

$I_1 = A^2\left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{A^2}{22}$ (1st subsidiary maximum)

$I_2 = A^2\left[\frac{\sin(5\pi/2)}{(5\pi/2)}\right]^2 = \frac{A^2}{62}$ (2nd subsidiary maximum) and so on. From the expressions of I_0, I_1, I_2, it is evident that most of the incident light is concentrated in the principle maximum.



Intensity distribution graph

DIFFRACTION GRATING

Though David Rittenhouse invented diffraction grating in 1785, the important contributions came only later from Fraunhoffer. A diffraction grating is an optical device consists of a glass or polished metal surface over which thousands of fine, equidistant, closely spaced parallel lines are been ruled. Gratings are mainly of two types, the transmission grating and reflection grating. The ruling on a reflecting surface such as aluminium can make the reflection grating.

GRATING ELEMENT

Distance between two consecutive slits (lines) of a grating is called grating element. If 'a' is the width of transparent portion and 'b' is the width of opaque portion, then grating element'd' is given by; d = a + b if there are N number of lines per unit length, the grating element is given by d=1/N =1/(a+b) usually, the number N is 15000 lines per inch for a good quality grating.

Large scale productions of gratings are made by taking the cast of an actual grating on a transparent film like cellulose acetate. Cellulose acetate solution of proper strength is poured on the ruled surface and is allowed to dry. A strong thin film is formed which is detached from the parent grating and preserved by mounting it between two glass sheets.

PRINCIPLE

Its working principle is based on the phenomenon of diffraction. The space between lines act as slits and these slits diffract the light waves there by producing a large number of beams which interfere in such away to produce spectra.

The grating spectrum:-

The positions of the principle maxima are given by $d \sin\theta = m\lambda$; m= 0,1,2......(1)

This relation which is also called the grating equation can be used to study the dependence of the angle of diffraction θ on the wavelength λ . The zeroth order principle maximum occurs at $\theta = 0$ irrespective of the wavelength. Thus, if we are using a polychromatic source (e.g white light) then the central maximum will be of the same colour as the source itself. However, the m≠0, the angles of diffraction are different for different wave lengths and therefore various spectral components appear at different positions. Thus by measuring the angles of diffraction for various colours one can (knowing the value of m) determines the values of the wavelengths. It may be mentioned that the intensity is maximum for the zeroth order spectrum (where no dispersion occurs) and it falls off as the value of m increases.

If we differentiate equation (1) we would obtain $\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{dcos\theta}$ ----(2) from this result we can deduce the following conclusions

(1) Assuming θ to be very small (i.e., cos $\theta \approx 1$) we can see that the angle $\Delta\,\theta$ is directly proportional to the order of spectrum (m) for a given $(\Delta\lambda)$ so that for a given m, $\frac{\Delta\theta}{\Delta\lambda}$ is a constant. Such a spectrum is known as a normal spectrum and in this the difference in angle for two spectral lines is directly proportional to the difference in wavelengths. However for large θ, it can be easily shown that the dispersion is greater at the red end of the spectrum.
(2) Equation (2) tells us that $\Delta\,\theta$ is inversely proportional to d and therefore smaller the grating element the larger will be the angular dispersion.

Rayleigh's criterion for resolving power:

To express the resolving power of an optical instrument as a numerical value, lord Rayleigh proposed an arbitrary criterion. According to him, **two nearby images are said to be resolved if the position of the central maximum of one coincides with the first secondary minimum of the other and vice versa**.

Resolving power of grating

One of the important properties of a diffraction grating is its ability to separate spectral lines which have nearly the same wavelength. The resolving power of a diffraction grating is defined as the capacity to form separate diffraction maxima of two wavelengths which are very close to each other. This is measured by $\frac{\lambda}{\Delta\lambda}$ where $\Delta\lambda$ is the smallest difference in two wavelengths which are just resolvable by grating and λ is the wavelength of either of them or mean wavelength. the Rayleigh criterion can be used to define the limit of resolution. According to this criterion, if the principle maximum corresponding to the wavelength $\lambda + \Delta\lambda$ falls on the first minimum of the wavelength λ, then the two wavelengths λ and $\lambda + \Delta\lambda$ are said to be just resolved. If this common diffraction angle is represented by θ and if we are looking at the mth order spectrum, then the two wavelengths λ and $\lambda + \Delta\lambda$ will be just resolved if the following two equations are simultaneously satisfied.

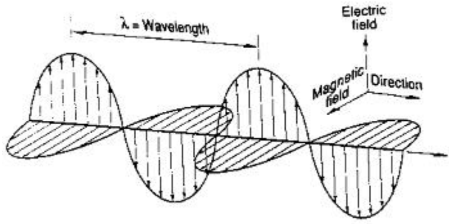
⇨ dsin θ = m$(\lambda + \Delta\lambda)$

And dsin $\theta = m\lambda + \frac{\lambda}{N}$ thus R = $\frac{\lambda}{\Delta\lambda}$ = mN. Which implies that the resolving power depends on the total number of lines in the grating-obviously on only those lines which are exposed to the incident beam. Further, the resolving power is proportional to the order of the spectrum.

Polarization

The phenomena of interference and diffraction which we discussed tell you that light is a form of wave motion.

To be precise it is a form of wave motion involving electric and magnetic fields. In general there are two forms of wave motion (a) longitudinal and (b) transverse. In the case of longitudinal waves the vibrations are always parallel to the direction of propagation. In the plane at right angles to the direction of propagation there is no motion.

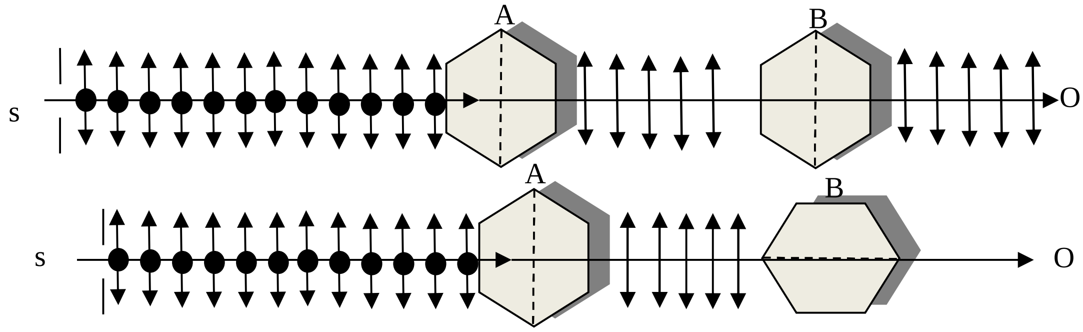


In the case of transverse wave motion the vibrations are always perpendicular to the direction of propagation thus in case of transverse wave motion two directions have to be specified. One the direction of propagation and the other that of direction of vibration. ***Maxwell's electromagnetic theory predicts the light waves to be transverse.***

NATURE OF LIGHT

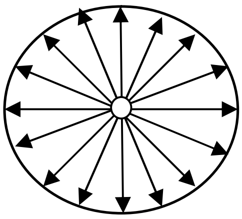
Consider a tourmaline crystal 'A' cut parallel to its crystallographic axis and held in the path of ordinary light such that light falls normally on it. If the crystal 'A' is rotated about SO, there is no change in intensity of the transmitted beam. Now, another similar crystal B is placed in the path such that the axis of B is parallel to the axis of A. Then all the light transmitted by A will be completely transmitted by B.

Now if crystal B is rotated about the beam axis, the intensity of light coming out of B decreases and becomes zero when the axes of A and B are perpendicular to each other. If b is rotated further, the intensity again increases and becomes maximum when both axes are parallel again. From the experiment, it is evident that light coming out of the crystal has acquired some property which prevents it being transmitted by the second crystal when its axis is perpendicular to the first one. That is, the light coming out of crystal has its oscillations confined to only one direction, the direction perpendicular to the direction of propagation of light. Hence, the light polarized by A will pass completely through B only if its axis is parallel to vibrations of light falling on B. It will be completely stopped if the axis of B is perpendicular to the vibrations of light falling on it. This proves that light waves are transverse waves.



POLARIZATION

Light is electromagnetic in nature. It consists of oscillating electric and magnetic fields. They are perpendicular to each other and also to the direction of propagation of the wave. In electromagnetic waves emitted by any common light source (electric bulb, sun etc) the electric vector at any given point is always perpendicular to the direction of propagation. But there are infinite number of directions which are perpendicular to the direction of propagation. So the electric vector changes its direction randomly which represents unpolarized or ordinary light.



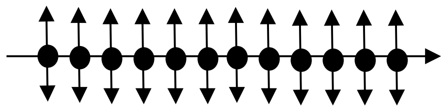


Figure 1: Unpolarized light

Types of Polarized light

Polarization is the process of restricting the vibrations of the electric field vector: if the electric field vector at a point always remains parallel to a fixed direction, that light is called linearly or plane polarized light. If the electric vector traces a circle, it is called circularly polarized light. In elliptically polarized light, the electric vector traces an ellipse.

Linearly polarized light

An electromagnetic wave (light wave) is said to be linearly polarized or Plane polarized if the plane containing the electric field vector is constant. That is the electric field vector at any point will be oscillating in a fixed direction perpendicular to the direction of propagation.

Fig: Linearly polarized light

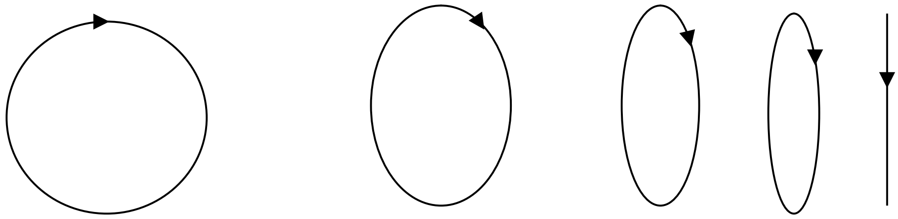
Circularly polarized light

In circular polarization, the electric field vector no longer oscillates in a plane as in linear polarization. However, the electric field vector is constant in magnitude and proceeds in the form of helix around the axis of propagation. Within one wavelength, the E vector completes one revolution. When looking towards the light source, if the tip of the vector rotates clockwise the light is said to be right circularly polarized and if it rotates anticlockwise it is left circularly polarized.

(a)Helical motion in the direction of propagation (b) Looking towards the light source

Elliptically Polarized light

Elliptical polarization is the most general type of polarization, which stands between linear and circular polarizations. Linear and circular are two extremes of elliptical polarization. In this polarization, the tip of the vector proceeds in the form of a flattened helix. The vector rotates and its magnitude changes



Circular , elliptical(flattened to different levels) and linear polarization

TYPES OF POLARIZED LIGHT – ANALYTICAL TREATMENT

The polarized light is, in general, elliptical and the linearly and circularly polarized ones are its special cases. For a wave travelling in the z-direction, the electric field components in the x and y-directions are: $E_x = E_1\sin\varpi t$ ----(1) $E_y = E_2\sin(\varpi t + c)$-----(2) where E_1 and E_2 are the amplitudes of E_x and E_y, respectively. The angle δ is the time phase angle by which E_y leads E_x. Expanding the sine term in equation (2), we get

$E_y = E_2(\sin\varpi t\cos\delta+\cos\varpi t\sin\delta)$------(3)

From equation (1), we have $\sin\varpi t = \frac{E_x}{E_1}$ and $\cos\varpi t = \sqrt{1-\left(\frac{E_x}{E_1}\right)^2}$ ---------(4)

Substituting the values of $\sin\varpi t$ and $\cos\varpi t$ in equation (3) we obtain

$\frac{E_y}{E_2} = \frac{E_x}{E_1}\cos\delta + \sqrt{1-\left(\frac{E_x}{E_1}\right)^2}\sin\delta$ ----(5) $\frac{E_y}{E_2} - \frac{E_x}{E_1}\cos\delta = \sqrt{1-\left(\frac{E_x}{E_1}\right)^2}\sin\delta$ -----(6)

Squaring and rearranging, we get

$$\frac{E_y^2}{E_2^2} - \frac{2E_xE_y}{E_1E_2}cos\delta + \frac{E_x^2}{E_1^2}cos^2\delta = sin^2\delta - \frac{E_x^2}{E_1^2}sin^2\delta$$

$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} - \frac{2E_xE_y}{E_1E_2}cos\delta = sin^2\delta$ ------(7)

This is the general equation of an ellipse. We shall now consider some special cases.

(i) When $\delta = 0$; $\sin\delta=0$, $\cos\delta=1$ from equation (7) we obtain

$$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} - \frac{2E_xE_y}{E_1E_2} = 0$$

$$\left(\frac{E_x}{E_1} - \frac{E_y}{E_2}\right)^2 = 0 \; or \; \left(\frac{E_x}{E_1} - \frac{E_y}{E_2}\right) = 0$$

$E_y=(\frac{E_2}{E_1})E_x$ ------(8)

This is the equation of a straight line and the wave is said to be linearly polarized.

(ii) When $\delta = \frac{\pi}{2}$ and $E_1 \neq E_2$; $\cos\delta=0$, $\sin\delta=1$. From equation (7), we obtain

$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} = 1$-----(9) which represents the equation of a symmetrical ellipse, and the wave is said to be elliptically polarized.

(iii) When $\delta = \frac{\pi}{2}$ and $E_1 = E_2 = E$. From equation(9), we have

$E_x^2 + E_y^2 = E^2$-------(10)

This is the equation of a circle. Here, the wave is said to be left circularly polarized. When $= \frac{-\pi}{2}$, the wave is right circularly polarized.

Methods of polarization

The methods by which polarized waves are produced are classified under the following headings

(i) Polarization by Reflection
(ii) Polarization by double refraction
(iii) Polarization by selective absorption and
(iv) Polarization by scattering

Double refraction

When a beam of light is passed through a transparent crystal like calcite($CaCO_3$) or quartz(SiO_2) it is split into two beams. Substances showing this property are called doubly refracting or birefringent. If experiments are carried out for various angles of incidence, one of the beam is found to obey the Snell's law and is called the O-ray (ordinary ray). The other beam does not obey Snell's law and is not even found to be in the plane of incidence. This is called the extraordinary ray or E-ray. It is found that

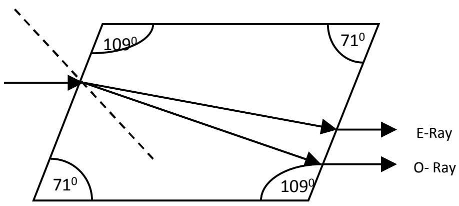


Figure: An illustration of double refraction along principle

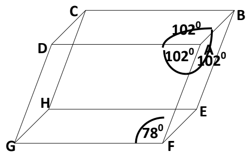
(i) O-ray travels in the crystal with the same speed in all directions. In other words the crystal is characterized by a single value of refractive index for the O-ray.
(ii) E-ray travels in the crystal with a speed that varies with direction. In other words refractive index for the E-ray varies with the direction and is in fact described by an ellipsoid.
(iii) Birefringence: The difference between the refractive index for O and E-rays is called birefringence.
(iv) In the case of calcite and quartz there is one direction in which there is no double refraction. This direction is called the optic axis. This direction is called the optic axis. In a class of crystals called biaxial crystals there are two directions in which there is no double refraction.
(v) O-ray and E-ray are polarized at right angles to each and the actual polarization depends on the direction of propagation. The vibration direction of O-ray is perpendicular to the plane defined by the direction of propagation and optic axis. The vibration direction of E-ray is in the plane defined by direction of propagation and the optic axis.
(vi) One important thing to note is that even though O-ray and E-ray are derived from the same beam, they will not exhibit interference fringes when they are made to interfere. This crucial experimental observations enabled Thomas Young to guess

That light waves were transverse in character.

Since O and E rays are polarized perpendicular to each other, if some means can be found to separate them, then this could be a way of producing polarized light from the unpolarized one. This is in fact achieved by using a Nicol prism.

Double Refraction in calcite crystal:

Calcite is colourless transparent crystal known as Iceland spar. It is hydrated calcium carbonate($CaCo_3$). It exists in several forms in nature and gives a rhombohedraon shape on cleavage or breakage. The six faces of rhombohedron are parallelogram having angles 78^0 and 102^0 respectively. At the two opposite corners, all the angles of faces meet at obtuse angle 102^0 known as blunt corners. At the remaining six corners, there are two acute angles(72^0) and one obtuse angle (102^0).



Optic axis: The direction along which the velocities of ordinary and extra ordinary rays are same is known as optic axis. The reference direction of the optic axis is given by the line passing through one of the blunt corners and making equal angle with all the faces meeting at that corner.

NICOL PRISM

The Nicol prism is an optical device made from a calcite crystal. This is used in many optical instruments for producing and analyzing polarized light. Nicol prism is made by cutting a calcite crystal along a diagonal and cementing it back together again with a special cement called Canada balsam. Canada balsam is a transparent substance. It is optically more dense than calcite for the E-ray and less dense for the O-ray(for sodiumlight n_0 =1.65836, $n_{canada\ balsam}$=1.55, n_e=1.48641). There exists therefore a critical angle of refraction for the O-ray but not for the E-ray. After both rays are refracted at the first crystal surface, the O-ray is totally reflected by the first Canada balsam surface. While the E-ray passes on through to emerge parallel to the incident light. Thus starting with ordinary unpolarized light, a Nicol prism transmits only the plane polarized light.

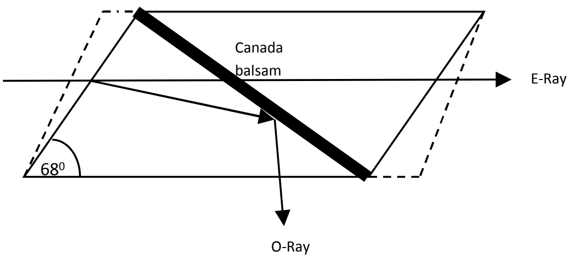


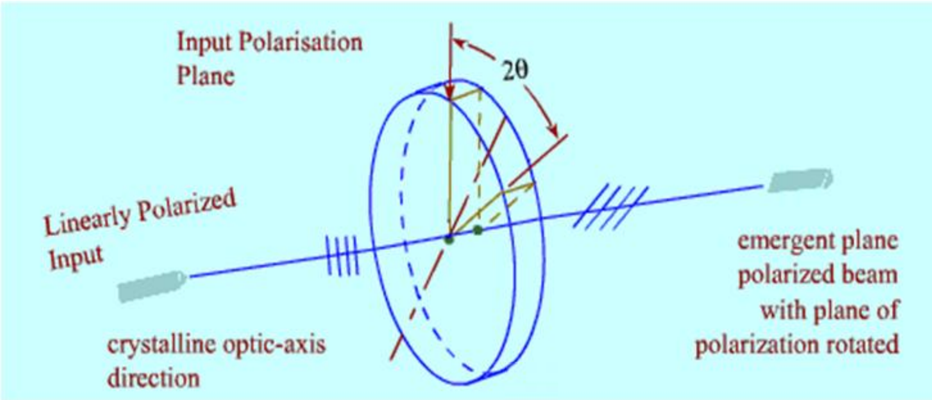
Figure : Nicol prism

Two Nicols lined up one behind the other is often used in optical microscopes for studying optical properties of crystals. The first Nicol which is used to produce the plane polarized light is called the polarizer and the second Nicol which is used to test the light is called the analyzer. In the parallel position light from the polarizer passes on through the analyzer. Upon rotating the analyzer through 90^0 no light is transmitted. In this case E-ray in the second Nicol is totally reflected. Thus when Nicols are crossed no light is transmitted.

HALF WAVE AND QUARTER WAVE PLATES

Quarter wave plate:

It is a plate of doubly refracting crystal of calcite or quartz. Its refracting faces are cut parallel to optic axis of the crystal to introduce a path difference of $\frac{\lambda}{4}$ or a phase difference of $\pi/2$ between extraordinary and ordinary components of the polarized light



Let us consider a doubly refracting crystal of thickness t, with refractive face cut parallel to the optic axis. If a plane polarized light of wavelength λ is incident normally on the plate, it breaks into ordinary and extraordinary components of the wave and travels in the same direction with different velocities. If the plate is of quartz a path difference of $(\mu_0 - \mu_e)t$ is introduced between ordinary and extraordinary component of light.

For quarter wave plate, the path difference = $\frac{\lambda}{4}$ therefore $(\mu_0 - \mu_e)t = \frac{\lambda}{4}$

$$t = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

Quarter wave retardation plates can be used to convert the linearly polarized input light into the circularly polarized or elliptically polarized light.

Half wave plate;

It is a doubly refracting crystal with face parallel to optic axis. It introduces a path difference of $\frac{\lambda}{2}$ or a phase difference of π between extraordinary and ordinary components of light. If t is the thickness of the plate and λ is the wave length of incident beam, then for a positive quartz crystal

$$t = \frac{\lambda}{2(\mu_0 - \mu_e)}$$

Half wave retardation plates are often used to rotate the plane of a linearly polarized input beam.

Assignment-Cum-Tutorial Questions

SECTION-A

I) Objective Questions

1) When a thin film of oil is illuminated with white light, multiple colours appear due to

A)Diffraction B) Polarization C) total internal reflection D)interference [ ]

2) In Newton's rings, the spacing between consecutive rings _____ with increase of order of rings

A)increases B) remain constant C) Decreases D)None [ ]

3)The capacity of an optical instrument to show separate images of very closely placed two objects is called

A)magnifying power B)Resolving power C) interference power D)diffracting power [ ]

4) In doubly refracting crystal, along optic axis

A) $\mu_0 > \mu_e$ B) $\mu_0 = \mu_e$ C) $\mu_0 < \mu_e$ D) $\mu_0 = \mu_e^2$ [ ]

5) The property that reveals the transverse nature of light is-------------.

6) Along optic axis of a crystal the ordinary and extraordinary ray velocities are----------------------.

7) In Fraunhoffer diffraction, the wave front of light used is ------------.

8) The fringe width in Newton's rings will remain constant

[True/False]

9) Polarization proves the transverse nature of light

[True/False]

10) In interference there is no loss of energy

[True/False]

11) Which of the following statement is true? [ ]

a) A thin film having zero thickness at one end and progressively increasing to a particular thickness at the other end is called a wedge shaped thin film.

b) A thin film having uniform thickness is called as a wedge shaped thin film

c) A thin film having zigzag thickness is called as a wedge shaped thin film

d) both b&c

12) Which of the following statement is true? [ ]

a) When an un-polarized light is propagating along the optic axis of an anisotropic crystal it does split into two polarized light i.e., double refraction
b) When an un-polarized light is propagating along the optic axis of an anisotropic crystal it does not split into two polarized light i.e., no double refraction.
c) When an un-polarized light is propagating along the optic axis of an isotropic crystal it does not split into two polarized light.
d) None of the above

13) The plane glass plate in Newton's rings arrangement is replaced by a plane mirror. Which one of the following statements is true? [ ]

(*a*) the radii of the fringes decrease

(*b*) the radii of the fringes increase

(*c*) the radii remain same

(*d*) no rings are observed.

14) Polaroid glass is used in sun glasses because [ ]

(*a*) it is cheaper

(*b*) it increases the light intensity to one and a half times on account of polarization

(*c*) it reduces the light intensity to half its value on account of polarization

(*d*) it produces irritation in the eye.

15) Match the following

a) Polychromatic source	[ ]	1) Sodium vapour lamp
b) Extended source	[ ]	2) Mercury vapour lamp
c) Monochromatic source	[ ]	3) Tube Light

16) Match the following

1) Uniaxial +ve crystal	[ ]	a) Calcite
2) Biaxial Crystal	[ ]	b) Quartz
3) Uniaxial –ve crystal	[ ]	c) Topaz

SECTION-B

II) Descriptive Questions

1) Explain how Newton's rings are formed in the reflected light. Derive an
expression for diameters of dark and bright rings.
2) Derive an expression for maxima and minima intensity due to interference
of reflected light from surface of a thin film.
3) Explain Fraunhofer Diffraction due to a single slit.
4) Explain Polarization by double refraction.
5) What is meant by resolving power of a grating? Derive an expression for it.
6) Explain the construction and working of a Quarter wave plate and Half wave plate.
7) Obtain an expression for the maximum order that can be obtained with a grating.

III) Problems:

1. Obtain the smallest angle of diffraction of the wavelength 7000^0 A in a grating having 15000 lines per inch.
2. A parallel beam of light of wavelength 6000 A^0 is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction into the plate is 50^0. Find the least thickness of the glass plate which will appear dark by reflection.
3. A plane transmission grating having 4250 lines per cm is illuminated with sodium light normally. In second order spectrum the spectral lines deviated by 30^0 are observed. Find the wavelength of the spectral line.
4. Plane polarized light passes through a quartz plate with its axis parallel to the face. Calculate the thickness of the plate so that the emergent light may be plane quartz «$_e = 1.553$, «$_0 = 1.542$; ª $= 5.5\times10^{-5} cm$
5. Find the highest order possible that can be seen with a grating having 15000 lines per inch . The wavelength of light used is 580 nm.
6. A diffraction grating has 1.26 x 10^4 rulings uniformly spaced over width w = 25.4 mm. It is illuminated at normal incidence by yellow light from a sodium vapour lamp with closely spaced wavelengths (589.00 and 589.59 nm)
7. Bright light of wavelength 585 nm is incident perpendicularly on a soap film (n=1.33) of thickness 1.21 μm, suspended in air. Is the light reflected by the two surfaces of the film closer to interfering fully destructively or fully constructively?

<u>SECTION-C</u>

C. Questions testing the analyzing / evaluating ability of students

1. Solar cells-devices that generate electricity when exposed to sunlight are often coated with a transparent, thin film of silicon monoxide (SiO, n = 1.45) to minimize reflective losses from the surface. Suppose that a silicon solar cell (n=3.5) is coated with a thin film of silicon monoxide for this purpose. Determine the minimum film thickness that produces the least reflection at a wavelength of 550nm, near the center of the visible spectrum.

ENGINEERING PHYSICS

UNIT-VI

LASERS

Objective:

- To comprehend principles of laser

Syllabus:

Lasers: Introduction – coherent sources – Characteristics of lasers – Spontaneous and Stimulated emission of radiation – Einstein"s coefficients – Population inversion – Ruby lasers - Helium Neon laser – Co_2 laser.

Learning Outcomes:

At the end of this chapter students are able to

- summarize the different characteristics of laser
- derive Einstein's coefficients
- analyze construction and working of Ruby, He- Ne, CO_2, with suitable diagrams
- list uses of lasers

BASIC CONCEPTS OF LASER

Absorption :

An atom/molecule residing in the lower energy state E_1 may absorb an incident photon and jump to the excited state E_2 as shown in Fig. 6.1. The transition is known as induced absorption or absorption.

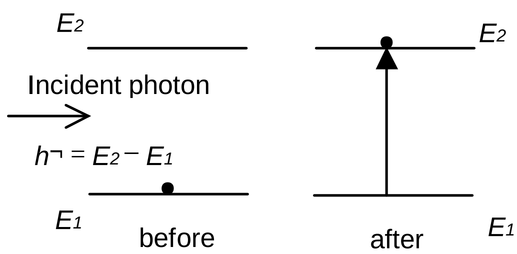


Fig. 6.1

The number of absorption transitions occurring in the material at any instant will be proportional to the number of atoms in the lower state E_1 and the number of photons in the incident beam. Normally, the number of atoms is greater in the lower energy state and the material absorbs incident energy. Therefore, the process of absorption leads to attenuation of radiation as light travels through the medium.

The number of atoms N_{ab} excited during the time Δt is given by, $N_{ab} = B_{12}\ N_1\ Q\ \Delta t$, where N_1 is the number of atoms in the state E_1, Q is the energy density of the incident beam, and B_{12} is the probability of an absorption transition.

Spontaneous Emission :

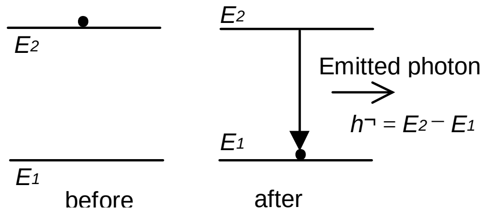


Fig. 6.2

The excited atom in the state E_2 may return to the lower state E_1 on its own out of its natural tendency to attain minimum potential energy condition. During the transition, the excess energy is released as a photon of energy h¬ = $E_2 - E_1$. This type of process in which photon emission occurs without any external influence is shown in Fig. 6.2. It is called spontaneous emission. Most of the common sources of light emit light through this process. The emission process is highly statistical and can not be controlled by external influences. The emitted photons travel in all directions with no consistent phase relationships. Hence the light is incoherent and also not monochromatic.

The number of spontaneous transitions taking place in the material during a given time interval depends only on the number of atoms lying in the excited state E_2. The number of spontaneous transitions N_{sp} taking place in the material during the time Δt is given by $N_{sp} = A_{21}\ N_2\ \Delta t$, where N_2 is the number of atoms lying in the excited state E_2 and A_{21} is the probability of a spontaneous transition from E_2 to E_1.

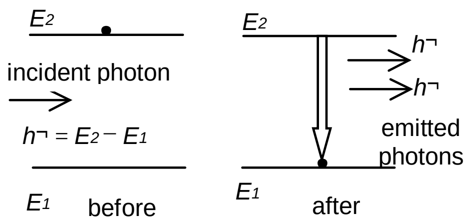


Fig. 6.3

Stimulated Emission:

An atom in the excited state need not wait for spontaneous emission to occur. There exists an alternative mechinism (Fig. 6.3) by which an excited atom can make a downward transition and emit light. The interaction of a photon of energy $h\neg = E_2 \; \mathrm{J} \; E_1$ with an excited atom forces the excited atom to drop to the lower energy state giving up a photon. This phenomenon of forced emission of photons is called stimulated emission.

The process of stimulated emission has remarkable and interesting features:

1. The emitted photon is identical to the incident photon in all respects. There exists correlation in frequency, in phase and in direction between the two photons.

2. The process is controllable from outside.

3. The most important feature is that multiplication of photons takes place in the process. One photon induces an atom to emit a second photon. These two photons travelling along the same direction de-excite two more atoms in their path producing a total of four photons. The number of photons thus builds up in an avalanche-like manner. The process of stimulated emission is the key to the operation of a laser.

The number of stimulated transitions N_{st} occuring in the material during the time Δt is given by

$N_{st} = B_{21} N_2 Q \Delta t$, where B_{21} represents the probability of a stimulated emission.

EINSTEIN COEFFICIENTS:

The probable rate of occurrence of the absorption transition from state 1 to state 2 depends on the properties of states 1 and 2 and is proportional to energy density u (ν) of the radiation of frequency ν incident on the same. Thus P_{12} α u (ν) or $P_{12} = B_{12} u(\nu)$ --(1)

The proportionality constant B_{12} is known a Einstein's coefficient of absorption of radiation.

The probability of spontaneous emission from state 2 to state 1 depends only on the properties of states 1 and 2. This is independent of energy density u(ν) of incident radiation. Einstein denoted the probability per unit time by $A_{21.}$ (P_{21}) spontaneous = $A_{21.}$ A_{21} is known as Einstein's coefficient of spontaneous emission of radiation. Here it should be noted that the probability of absorption transition depends upon energy density u(ν) of incident radiation. Whereas the probability of spontaneous emission is independent of it. Hence for equilibrium emission transition depending upon u(ν) must also exist. Actually these transitions are stimulated emission transitions.

The probability of stimulated emission transition from state 2 to state 1 is proportional to the energy density u(ν) of the stimulating radiation i.e. (P_{21}) stimulated = B_{21} u(ν) where B_{21} is Einstein's coefficient of stimulated emission of radiation. The total probability for an atom in state 2 to drop to the lower state 1 i.e. therefore $P_{21} = A_{21} + B_{21} u(\nu)$ ----2

RELATION BETWEEN EINSTEIN’S A AND B COEFFICIENTS:-

Consider an assembly of atoms in thermal equilibrium at temperature T with radiation of frequency ν and energy density u(ν). Let N_1 and N_2 be the number of atoms in energy states 1 and 2 respectively at any instant.

The number of atoms in state 1 that absorb a photon and rise to state 2 per unit time is given by $N_1P_{12} = N_1B_{12}\, u\, (\nu)$ ------- (3)

The number of photons in state 2 that can cause emission process (spontaneous + stimulated) per unit time is given by $N_2P_{21} = N_2\, [A_{21}+B_{21}u(\nu)]$ ------- (4)

For equilibrium, the absorption and emission must occur equally hence $N_1P_{12} = N_2P_{21}$

$N_1B_{12}\, u\, (\nu) = N_2\, [A_{21}+B_{21}u\, (\nu)]$ or $N_1B_{12}\, u\, (\nu) = N_2A_{21}+N_2B_{21}u\, (\nu)$ or $u\, (\nu)\, [N_1B_{12} – N_2B_{21}] = N_2A_{21}$

$$\therefore u(\nu) = \frac{N_2 A_{21}}{N_2 B_{21} - N_2 B_{21}} = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{N_1}{N_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1} \quad \text{------- (5)}$$

According to Boltzmann distribution law, the number of atoms N_1 and N_2 in energy states E_1 and E_2 in thermal equilibrium at temperature T is given by

$N_1 = N_o\, ^{e\text{-}E_1/kt}$ and

$N_2 = N_o\, ^{e\text{-}E_2/kt}$

Where N_0 = Total Number of atoms present and k = Boltzmann’s constant.

$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-(E_2-E_1)/kT} = e^{-h\nu/kT}$$

Or

$$\frac{N_1}{N_2} = e^{h\nu/kT} \qquad ---------------(6)$$

Substituting the value of N_1/N_2 from eq. (6) in eq. (5) we get

$$u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{\left[e^{h\nu/kT} \left(\frac{B_{12}}{B_{21}} \right) - 1 \right]} \qquad ----------------(7)$$

According to P lanck's radiation formula

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} . \frac{1}{\left[e^{h\nu/kT} \right]} \qquad -----------------(8)$$

Comparing equations (7) and (8) we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \, and \, \frac{B_{12}}{B_{21}} = 1 or B_{12} = B_{21}$$

Hence $B_{12} = B_{21}$, the probability of stimulated emission is same as induced absorption

A_{21}/B_{21} α ν^3 i.e the ratio of spontaneous emission and stimulated emission is proportional to ν^3. This shows that the probability of spontaneous emission increases rapidly with energy deference between two states.

Laser beam characteristics :

(I) Coherence :

A conventional light source produces incoherent light since it emits light waves of random wavelengths with no common phase relationships. On the other hand, the waves emitted by a laser source are in phase and are of the same frequency. Therefore, light generated by a laser is highly coherent. The coherence length in a laser beam is typically of the order of a few kilometers, whereas the coherence length of light radiated by conventional monochromatic source is of the order of a few millimeters or centimeters.

(2) Monochromaticity :

The light from normal monochromatic source spreads over a wavelength range of the order of $100A^0$ to 1000^0A. The laser light is highly monochromatic due to its temporal coherence property. The laser light spread expressed in terms of half width is of the order of a few angstroms ($<10A^0$) only.

(3) Divergence :

Light from conventional sources spreads out in the form of spherical wavefronts and hence it is highly divergent. The divergence of the laser beam is extremely small because of its high degree of spatial coherence.

(4) Directionality :

The conventional sources of light emit in all directions. Lasers emit light only in one direction as the photons travelling along the optical axis of the system are selected and augmented with the help of the optical resonator.

(5) Intensity :

The intensity of light from a conventional source decreases rapidly with distance as it spreads in the form of spherical waves. In contrast, a laser emits light in the form of a narrow beam and propagates in the form of plane waves. As the energy is concentrated in a very narrow region, its intensity would be tremendously high. The intensity of the laser beam stays nearly constant with distance as the laser light travels in the form of plane waves with very little divergence.

BASIC REQUIREMENTS OF LASER

1. Metastable state
2. Population inversion
3. Pumping
4. Optical resonator

(1) METASTABLE STATE:

The state in which possibility of finding electron more is called metastable state. The lifetime of metastable state is 10^{-3} sec.

(2) POPULATION INVERSION:

It is the process in which the population of a particular higher energy state is made more than that of a specified lower energy state. Normally the population of ground state is more than the excited state at equilibrium. Let us consider the number of atoms N per unit volume that exist in a given energy state E. this number called population N is given by Boltzmann's equation $N = N_o\ e^{-E/kBt}$. Here N_o is the population in the ground state (E=0_. K_b is the Boltzmann's constant and T is the absolute temperature.

It is clear from the above equation that population is maximum in the ground state and decreases exponentially as one goes to higher energy state. If N_1 and N_2 are the populations in two states, a lower state E_1 and a higher state E_2 we have $\frac{N_2}{N_1} = \frac{e^{\frac{-E_2}{kBT}}}{e^{\frac{-E_1}{kBT}}}$ from which it follows $N_2 = N_1 e^{-(E2-E1)/kBT}$ clearly $N_2 < N_1$ since $E_2 > E_1$. Since $N_1 > N_2$, whenever an electromagnetic wave is incident, there is net absorption of the radiation. For laser action to take place, it is absolutely necessary that stimulated emission predominate over spontaneous emission. This is, possible only if $N_2 > N_1$. This situation in which $N_2 > N_1$ is called population inversion.

3. PUMPING:

The act of exciting atoms from lower energy state to a higher energy state by supplying energy from an external source is called pumping. The following are the commonly used techniques.

(a) Optical pumping

(b) Electric discharges

(c) Atomic collisions

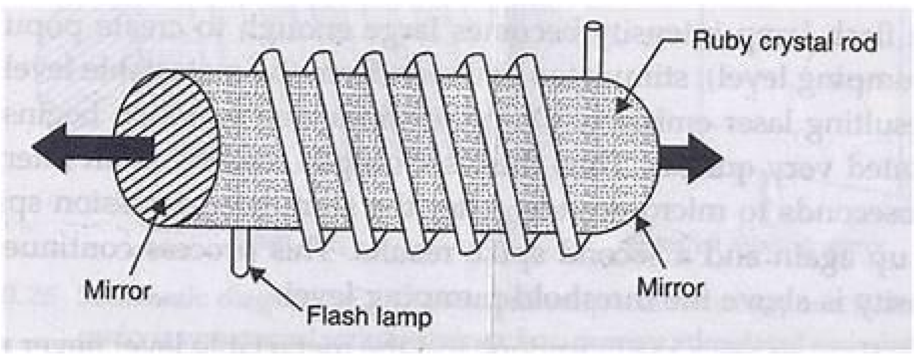
(d) Direct conversion and

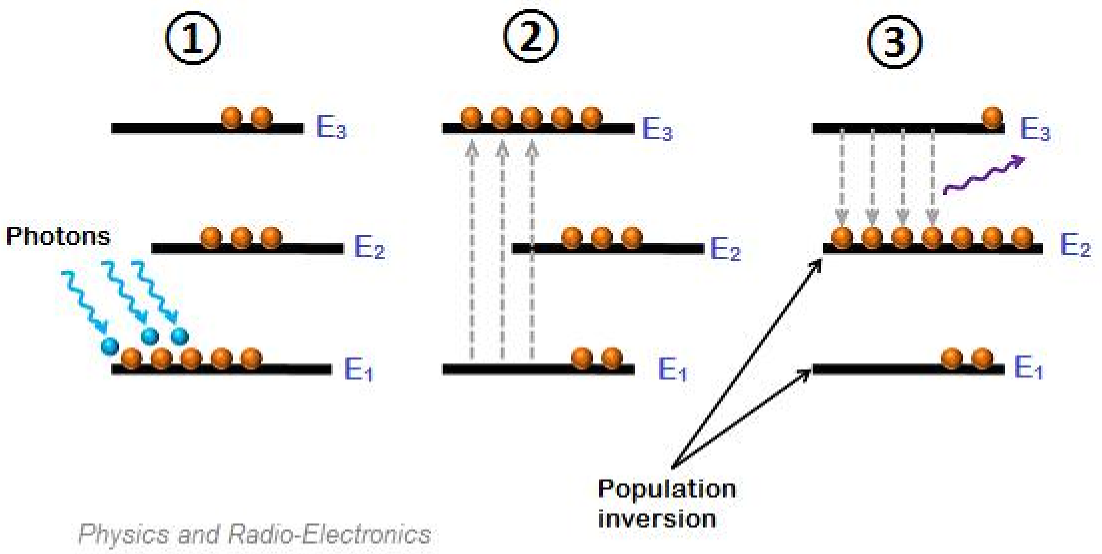
(e) Chemical reaction.

OPTICAL RESONATOR: A set of perfect reflector and partial reflector together are known as optical resonator.

Working of ruby laser

The ruby laser is a three level solid-state laser. In a ruby laser, optical pumping technique is used to supply energy to the laser medium. Optical pumping is a technique in which light is used as energy source to raise electrons from lower energy level to the higher energy level.
Consider a ruby laser medium consisting of three energy levels E1, E2, E3 with N number of electrons.
We assume that the energy levels will be E1 < E2 < E3. The energy level E1 is known as ground state or lower energy state, the energy level E2 is known as metastable state, and the energy level E3 is known as pump state.
Let us assume that initially most of the electrons are in the lower energy state (E1) and only a tiny number of electrons are in the excited states (E2 and E3)





Ruby laser is a three level solid state laser

When light energy is supplied to the laser medium (ruby), the electrons in the lower energy state or ground state (E1) gains enough energy and jumps into the pump state (E3).

The lifetime of pump state E_3 is very small (10^{-8} sec) so the electrons in the pump state do not stay for long period. After a short period, they fall into the metastable state E_2 by releasing radiationless energy. The lifetime of metastable state E_2 is 10^{-3} sec which is much greater than the lifetime of pump state E_3. Therefore, the electrons reach E_2 much faster than they leave E_2. This results in an increase in the number of electrons in the metastable state E_2 and hence population inversion is achieved.

After some period, the electrons in the metastable state E_2 falls into the lower energy state E_1 by releasing energy in the form of photons. This is called spontaneous emission of radiation.
When the emitted photon interacts with the electron in the metastable state, it forcefully makes that electron fall into the ground state E_1. As a result, two photons are emitted. This is called stimulated emission of radiation.
When these emitted photons again interacted with the metastable state electrons, then 4 photons are produced. Because of this continuous interaction with the electrons, millions of photons are produced.
In an active medium (ruby), a process called spontaneous emission produces light. The light produced within the laser medium will bounce back and forth between the two mirrors. This stimulates other electrons to fall into the ground state by releasing light energy. This is called stimulated emission. Likewise, millions of electrons are stimulated to emit light. Thus, the light gain is achieved.

The amplified light escapes through the partially reflecting mirror to produce laser light.

Construction and Working of Helium - Neon Laser

Gas lasers usually employ a mixture of two gases, say A and B. Atoms of A are initially excited and they in turn transfer their energy to atoms of B which are the actual active centres.

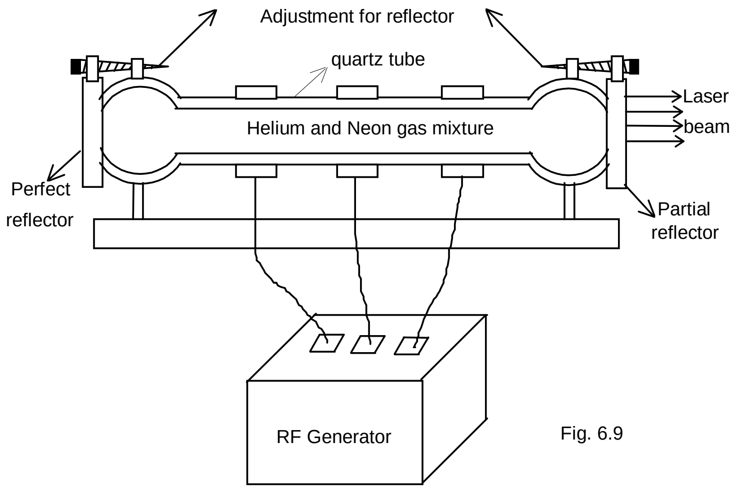


Fig. 6.9

CONSTRUCTION :

The schematic of a typical He-Ne laser is shown in Fig. 6.9. It consists of a fused quartz tube with a diameter of about 1.5cm and length of about *80cm*. This tube is filled with a mixture of helium and neon gases in the ratio of *10:1*. There is a majority of helium atoms and a minority of neon atoms. At one end of the tube, there is a perfect reflector while at the other end is a partial reflector. The active material is excited by means of a high frequency generator.

WORKING :

He-Ne laser employs a four-level pumping scheme. The energy level diagram is shown in Fig 6.10. When the power is switched on, the high frequency electric field ionizes some of the atoms in the mixture of helium and neon gases. Since the electrons have a smaller mass, they acquire a higher velocity due to the electric field. The helium atoms are more readily excitable than neon atoms because they are lighter. The energetic electrons excite helium atoms through collisions to the excited states F_2 and F_3. These two states are metastable states. The excited helium atoms return to the ground state by transfering their energy to neon atoms through collisions. This is the main pumping mechanism in He-Ne system. Neon atoms are the active centres and the role of helium is to excite neon atoms and cause population inversion.

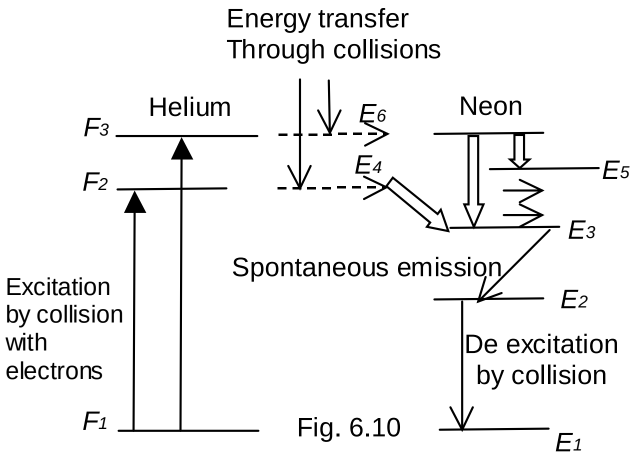


Fig. 6.10

The E_6 and E_4 levels of neon atoms are also metastable states. Therefore, as the collisions go on, neon atoms accumulate in these two excited states. At ordinary temperatures, E_5 and E_3 levels of neon are less populated and a state of population inversion is achieved between E_6 and E_5, E_3 levels and between E_4 and E_3 levels. Consequently, three laser transitions can occur.

$E_6 \rightarrow E_3$ transition gives a laser beam of red colour of wavelength 6328 A^0.

$E_4 \rightarrow E_3$ transition produces infrared laser beam at *1.15«m* wavelength.

$E_6 \rightarrow E_5$ transition produces infrared laser beam at *3.39«m* wavelength.

The He-Ne laser operates in CW mode as the neon atoms are excited to upper levels continuously through collisions. In the He-Ne laser, the terminal levels of lasing transitions are sparsely populated. As such, the transition of neon atoms that must be excited to the upper levels can be much less. Hence the power required for pumping is low and efficiency is high.

CO_2 LASER:

The carbon dioxide laser invented in the year 1963 by kumar patel. Carbon dioxide lasers are the highest power continuous wave lasers that are currently available.

The CO_2 laser produces a beam of infrared light with the principal wavelength bands centering around 9.4 and 10.6 micrometer.

Any laser is made up of three main parts

Active Medium

Pumping

Optical Cavity

Active Medium:

The active medium is CO_2 gas. As, He is used for excitation of Ne atoms in the He-Ne laser, in CO_2 laser for efficient excitation of CO_2 molecules, N_2 molecules are used. Addition of He to the gas mixture enhances the efficiency. The ratio of pressures of CO_2:Ne:He is 1:4:5. The tube is about 2.5cm in dia & 5m in length.

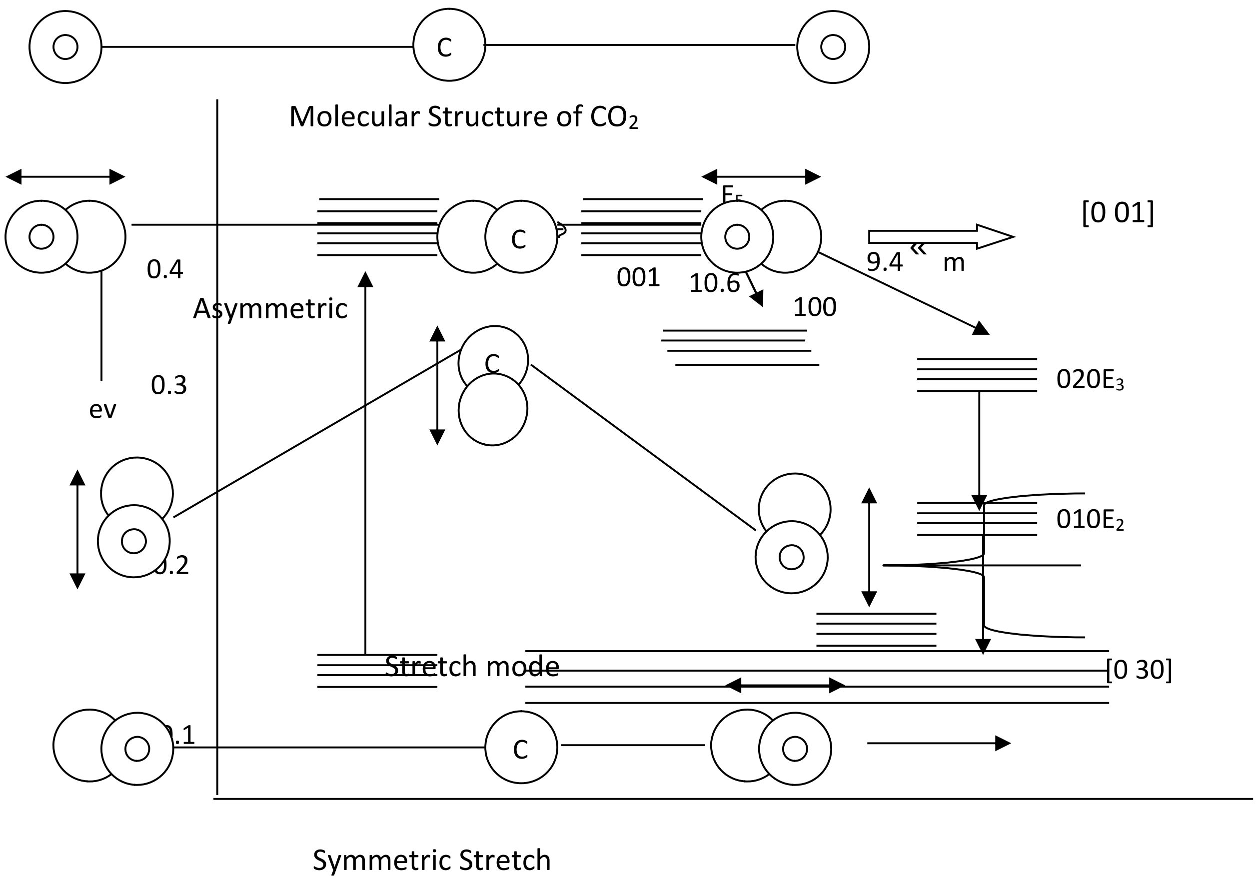
Pumping:

In CO_2 laser the excitation is produced by electric discharge excited N_2 molecules transfer energy to the CO_2 molecules in resonant collisions.

Optical Cavity:

Because CO_2 lasers operate in the infrared region, special materials are necessary for their construction. The mirrors are made of coated silicon, molybdenum or gold while windows and lenses are made of either germanium (or) Zine Selenide.

Vibration energy levels in co_2 Molecule.



Laser Action in CO_2 Laser:

1. When a discharge is passed through tube the nitrogen molecules are excited and are raised to higher excited state.
2. The excited energy of nitrogen molecules is transferred to carbon dioxide molecules through collisions and carbon-dioxide molecules are raised to their excited vibrations energy level E_5 [0 01] from their ground state.
3. The energy level E_5 is a meta stable state energy level. Hence there is a population inversion.
4. Stimulating photons of wavelength 10.6 « m and 9.6 « m induce CO_2 molecules to undergo stimulated emission by laser transitions E_5 to E_4 giving laser of 10.6 « m & from E_5 to E_3 9.6 « m.
5. CO_2 molecules from E_4 and E_3 are returned to their ground through fast decay and diffusion.

Applications Of Lasers:

(1) Laser output is a beam of highly intense parallel rays, which permits the light to traverse large distances with very little divergence. Using external optics, this beam can be focussed to very smalll dimensions so that it can be used for industrial applications like cutting, drilling, welding, machining etc.

(2) Because of the extremely high temperatures obtainable at the focus of a laser beam, laser is an excellent tool for triggering chemical and photochemical reactions.

(3) Laser output is a highly coherent beam which finds important application in holography. Holography is a process for producing three-dimensional images. Conventional photography records the two-dimensional image of a three-dimensional scene. It contains information about the intensity of the light wave that

produced the image. Holography records the three-dimensional image of an object. The image contains information of both phase and intensity of the light wave. A hologram is made by using reference beams and interference beams.

Holography has several applications which are becoming increasingly wide spread. Credit cards, for example, some times use holograms for identification purposes. Other applications of holography include head-up displays for the instrument panels in high-preformance fighter planes, laser scanners at supermarket checkout counters, computerized data storage and retrieval systems, methods for high-precision biomedical measurements and others.

(4) Another important application of laser is in the field of communications. Since optical frequencies are extremely large as compared to radio waves and microwaves, a light beam acting as a carrier wave is capable of carrying far more information over long distances in comparison to radio waves and mocrowaves. Laser light offers unique advantages over conventional incoherent light. The advantages are:

(a) The spectral width of a laser is much smaller and hence information carrying capacity is much larger.

(b) The turbulent nature of the atmosphere produces no effect on laser light because of its very little divergence and high directionality.

(c) Another important advantage is of its immunity from jamming and interception.

(5) Semiconductor diode lasers are used to reproduce music in CD players, laser printers, laser copiers, optical floppy discs, optical memory cards etc. They are also used in measuring instruments like range finders, strain gauges etc.

(6) Laser light reflection is widely used in applications ranging from measuring the speed of automobiles to reading price information from bar codes at the supermarket.

(7) Medical applications of lasers include delicate eye surgery, removal of kidney stones, removal of tooth and gum decay and in the treatment of cancer.

(8) The large coherence length and high output intensity coupled with a low divergence enables the laser to find applications in precision length measurements using interferometric techniques.

(9) Laser tracking systems are used to determine the trajectory of a moving object like a rocket, to determine the daily positions of a heavenly object or an artificial satellite. One of the main advantages of a laser tracking system over a microwave radar system is that a laser tracking system has a smaller size and its cost is also much less. In a microwave radar system, one has to incorporate corrections because of the presence of ionosphere and also because of the presence of water vapour in the troposphere. These corrections are much easier to incorporate in the case of an optical beam.

(10) Lasers are also used in isotope seperation. Laser isotope seperation (LIS) process makes it possible to enrich uranium on a larger scale besides producing isotopes which are used in medicine, science and technology.

Assignment-Cum-Tutorial Questions

SECTION-A

I) Objective Questions

1. MASER Means________________
2. The number of atoms in tine lower energy state is _____that of the higher energy state. [ ]
   a) morethan b) less than c) equal to d) none
3. The life time of an atom in the excited state is [ ]
   a) 10^{-3} sec b) 10^{-8} sec c) 10 sec d) none
4. The expression for number of atoms in any energy state at temperature T is called [ ]
   a) $N = N_0 e^{E/k_B T}$ b) $N_0 = N e^{-E/k_B T}$ c) $N = N_0 e^{-E/k_B T}$ d) None
5. The Einstein coefficient for absorption is equal to the [ ]
   a) Einstein coefficient for spontaneous emission
   b) Einstein coefficient for stimulated emission
   c) Both a and b d) none

Descriptive Questions

1. What is meant by a laser? Explain the characteristics of lasers.
2. With the help of suitable diagrams explain the principle, construction and working of He-Ne gas laser.
3. Derive the relation between the probabilities of spontaneous emission and stimulated emission in terms of Einstein Coefficients.
4. What are the various applications of lasers?
5. Describe the construction and working of a ruby laser.
6. What is CO_2 laser explain it?

Multiple Choice Questions:

1. Define the terms (a) Metastable state (b) Population Inversion
2. Match the following : ( )

List I	List II
A) Optical fibre	1) Vibrational modes
B) Lasers	2) Dielectric wave guide
C) CO_2 laser	3) $N_2 > N_1$
D) Population Inversion	4) Blood less surgery

	A	B	C	D
a)	1	2	3	4
b)	2	4	1	3
c)	4	1	2	3
d)	1	2	4	3

3. Match the following

1) Graded Index Fiber	[ ]	A) dielectric wave guides
2) Optical Fiber	[ ]	B) pulsed mode
3) Ruby lasers	[ ]	C) supports multimode only

4. The ratio of Einstein's co-efficients $\frac{A_{21}}{B_{21}} =$ ( )

A) $\frac{8®hv^3}{C^3}$ **B)** $\frac{8®hv^3}{C^2}$ **C)** $\frac{8®hv^3}{C^2}$ **D)** $\frac{2®hv^3}{C^3}$

5. Find out the correct equation of population

A) $\frac{N}{N_0} = \exp\left[\frac{hv}{K_B T}\right]$ **B)** $\frac{N_0}{N} = \exp\left[\frac{hv}{K_B T}\right]$ **C)** $\frac{N}{N_0} = \exp\left[\frac{-hv}{K_B T}\right]$ **D)** $\frac{N_0}{N} = \exp\left[\frac{-hv}{K_B T}\right]$

6. Example for indirect band gap semiconductor is ( )
A) Arsenic B) Phosporous C) Silicon D) a and b

7. Calculate the Wavelength of radiation emitted by an LED made up of a semiconducting material with band gap energy 2.8eV. ( )
A) 44 m B) 4430 x 10^{-8} cm C) 4830 x 10^{-8} cm D) 499 x 10^{-8} cm

Problems:

1. Calculate the wavelength of emission from GaAs material whose energy band gap E_g = 1.44 eV. (Planck's constant = 6.625 x 10^{-34} J^{-s}).
2. Energy gap of a semiconductor 3 eV. Calculate wave length of emitted photons

***